Specialist Mathematics

Written examinations 1 and 2 – October/November

Introduction

Specialist Mathematics examination 1 is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology.

Specialist Mathematics examination 2 is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students are required to respond to multiple-choice questions in Section 1 of the paper and to extended answer questions, involving multi-stage solutions of increasing complexity, in Section 2 of the paper.

Structure and format

Examination 1

The examination will consist of short answer questions which are to be answered without the use of technology.

The examination will be out of a total of 40 marks.

A formula sheet will be provided with the examination. Details of the formulas to be provided are published with the examination. The formula sheet will be the same for examinations 1 and 2.

Examination 2

The examination will consist of two sections. Section 1 will consist of 22 multiple-choice questions worth 1 mark each and Section 2 will consist of extended answer questions, involving multi-stage solutions of increasing complexity worth 58 marks, a total of 80 marks.

A formula sheet will be provided with the examination. Details of the formulas to be provided are published with the examination. The formula sheet will be the same for examinations 1 and 2.

Version 3 – June 2013

Approved materials

Examination 1

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- A calculator is not allowed in this examination.
- Notes are not permitted in this examination.

Note: protractors, set squares, aids for curve sketching are no longer required for this examination and have been **removed** from the list of approved materials.

Examination 2

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- One bound reference that may be annotated. The reference may be a textbook.
- Protractors, set squares, aids for curve sketching.
- A CAS calculator **or** CAS software, and, if desired, one scientific calculator. The memories of calculators need not be cleared for this examination.

The VCAA publishes details of approved technology for use in mathematics examinations annually. Details of approved calculators are published in the October *VCAA Bulletin*. The current list may be found at the VCE Specialist Mathematics Study page on the VCAA website. Details concerning VCAA approved reference are published in the *VCE and VCAL Administrative Handbook*.

Other resources

Teachers should refer to the Examinations section of the *VCE and VCAL Administrative Handbook*, *VCE Mathematics Assessment Handbook*, the VCE Specialist Mathematics Study page on the VCAA website and to the *VCAA Bulletin VCE, VCAL and VET* for further advice during the year.

Sample examinations

The sample examination papers for Specialist Mathematics examinations 1 and 2 address content that remains unchanged and new content areas. Detailed information regarding the changes can be found in Supplement 2 of the March 2005 *VCAA Bulletin*, No. 23.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2006

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Written examination 1

Day Date 2006

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour)**

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 7 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 3 – June 2013

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

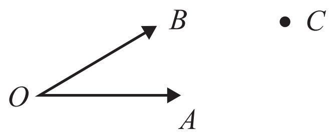
Question 1 (4 marks)

A crate of mass 100 kg rests on a rough horizontal factory floor. The coefficient of friction between the floor and the crate is $\frac{1}{7}$. A storeman applies a horizontal force of F newtons to the crate in order to move it to a different location.

a. Show that if $F = 120$, the crate will not move. 2 marks

b. Find the acceleration of the crate if $F = 190$. 2 marks

Question 2 (4 marks)



$OACB$ is a parallelogram with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. Given that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{OC}$, prove that $OACB$ is a rhombus.

Question 3 (5 marks)

Consider the function with rule $f(x) = \arctan(x) + x\arctan(x)$.

a. Show that $f'(x) = \arctan(x) + \dfrac{x+1}{x^2+1}$. 2 marks

b. Given that the graph of f has a single point of inflection, find the exact coordinates of this point of inflection. 3 marks

Question 4 (5 marks)

Consider the relation $2y - xy^2 = 8$.

a. Find an expression for $\frac{dy}{dx}$ in terms of x and y. 3 marks

b. **Hence** find the exact value of $\frac{dy}{dx}$ when $y = 2$. 2 marks

Question 5 (4 marks)

a. Verify that $z = 3i$ is a solution of the equation $z^3 - 2z^2 + 9z - 18 = 0$. 1 mark

b. Find all solutions over C of the equation $z^3 - 2z^2 + 9z - 18 = 0$. 3 marks

Question 6 (4 marks)

a. Show that $\frac{d}{dx}\left(\cos^{-1}\left(\sqrt{3x}\right)\right) = \frac{-\sqrt{3}}{2\sqrt{x-3x^2}}$, where $0 < x < \frac{1}{3}$. 2 marks

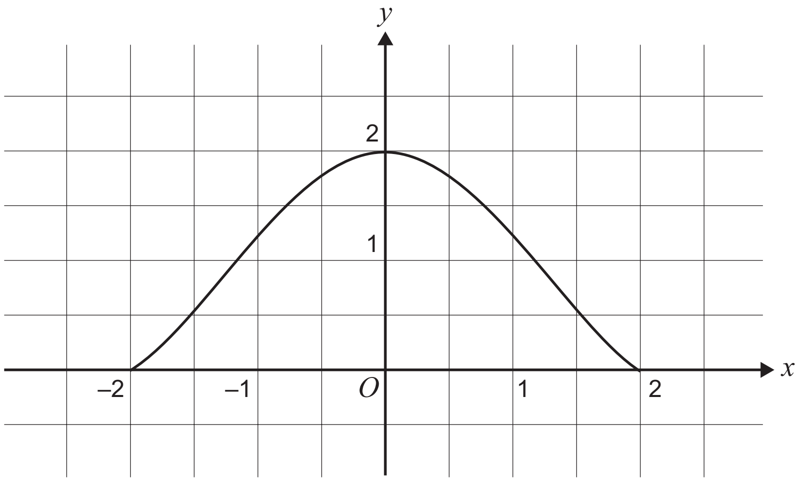
b. Hence find the exact value of $\int_{\frac{1}{6}}^{\frac{1}{4}} \frac{1}{\sqrt{x-3x^2}}\,dx$. 2 marks

Question 7 (4 marks)

Solve the differential equation $\frac{dy}{dx} = \sin^3(x)\cos^2(x)$ given that $y(0) = 0$.

Question 8 (3 marks)

The graph of $y = \frac{16}{x^2 + 4} - 2$ is shown for $-2 \le x \le 2$.



The region enclosed by the curve and the coordinate axes in the first quadrant is rotated about the y axis to form a solid of revolution. Express the volume of this solid as a definite integral and **hence** find the exact volume of the solid.

Question 9 – continued

Question 9 (7 marks)

a. Show that the curve with equation $y = \dfrac{3}{x^2 + 4x + 5}$ has no vertical asymptotes. 1 mark

b. Given that $\int \dfrac{3}{x^2 + 4x + 5}\,dx = A\tan^{-1}(x + B) + c$, where A, B and c are real constants, find A and B. 2 marks

c. **Hence** find the area enclosed by the curve with equation $y = \dfrac{3}{x^2 + 4x + 5}$, the x-axis and the lines $x = \sqrt{3} - 2$ and $x = a$, where a is the x-coordinate of the turning point of the curve. 2 marks

d. Find the values of c for which $5\tan^{-1}(x + 7) + c > 0$ for all x. 2 marks

END OF QUESTION AND ANSWER BOOK

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Victorian Certificate of Education
2006

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STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Written examination 2

Day Date 2006

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (2 hours)**

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 26 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graph of $y = \dfrac{-x^2+1}{2x}$ has

A. no straight line asymptotes.

B. $y = 2x$ as its only straight line asymptote.

C. $x = 0$ as its only straight line asymptote.

D. $y = 0$ and $y = -\frac{1}{2}x$ as its only straight line asymptotes.

E. $x = 0$ and $y = -\frac{1}{2}x$ as its only straight line asymptotes.

Question 2

The x-axis is tangent to an ellipse at the point (1, 0) and the y-axis is tangent to the same ellipse at the point (0, –2).

Which one of the following could be the equation of this ellipse?

A. $\dfrac{(x-1)^2}{4} + (y+2)^2 = 1$

B. $\dfrac{(x+1)^2}{4} + (y-2)^2 = 1$

C. $(x-1)^2 + \dfrac{(y+2)^2}{4} = 1$

D. $(x+1)^2 + \dfrac{(y-2)^2}{4} = 1$

E. $(x-2)^2 + \dfrac{(y+1)^2}{4} = 1$

Question 3

Which one of the following is **not** equal to $\tan\left(\frac{\pi}{5}\right)$?

A. $\dfrac{\sin\left(\frac{\pi}{5}\right)}{\cos\left(\frac{\pi}{5}\right)}$

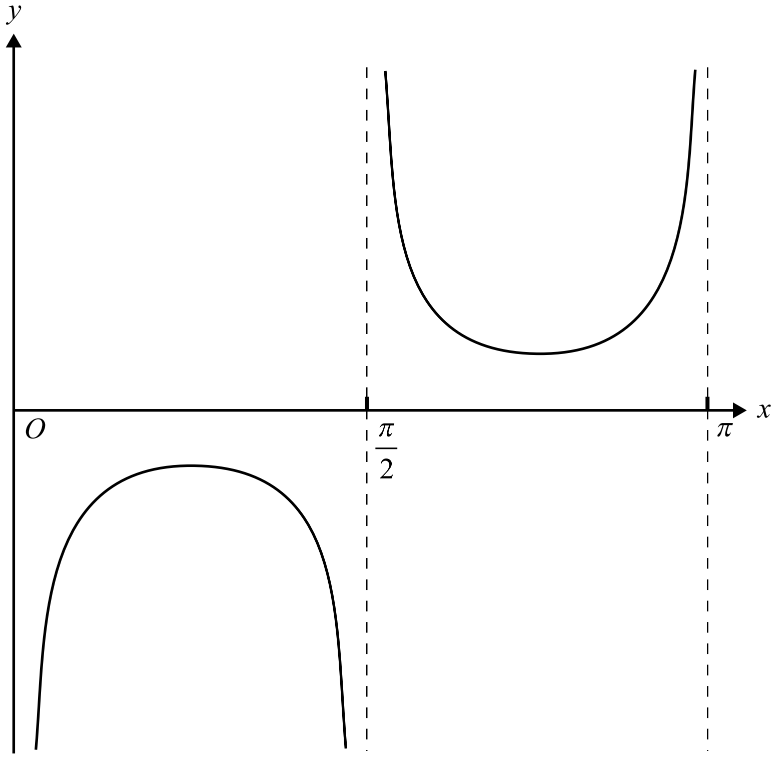
B. $\dfrac{1}{\cot\left(\frac{\pi}{5}\right)}$

C. $\cot\left(\frac{3\pi}{10}\right)$

D. $\dfrac{2\tan\left(\frac{\pi}{10}\right)}{1-\tan^2\left(\frac{\pi}{10}\right)}$

E. $\dfrac{2\tan\left(\frac{2\pi}{5}\right)}{1-\tan^2\left(\frac{2\pi}{5}\right)}$

Question 4



The graph of $y = -\sec(a(x - b))$ is shown above for $0 \le x \le \pi$.
The values of a and b could be

A. $a = 1,\ b = \frac{\pi}{2}$

B. $a = 1,\ b = \frac{\pi}{4}$

C. $a = 2,\ b = \frac{\pi}{2}$

D. $a = 2,\ b = \frac{\pi}{4}$

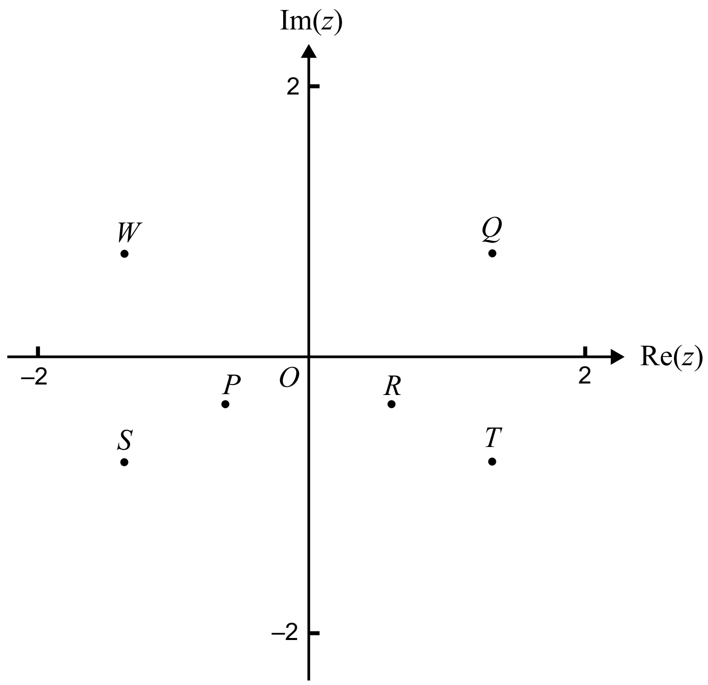
E. $a = 2,\ b = -\frac{\pi}{4}$

Question 5

$P(z)$ is a polynomial in z of degree 4 with real coefficients.
Which one of the following statements **must** be **false**?

A. $P(z) = 0$ has no real roots.
B. $P(z) = 0$ has one real root and three non-real roots.
C. $P(z) = 0$ has one (repeated) real root and two non-real roots.
D. $P(z) = 0$ has two real roots and two non-real roots.
E. $P(z) = 0$ has four real roots.

Question 6



The point W on the Argand diagram above represents a complex number w where $|w| = 1.5$.

The complex number w^{-1} is best represented by the point

A. P

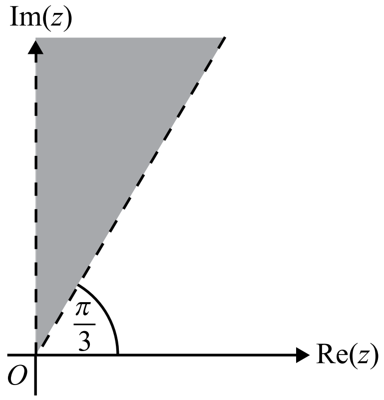
B. Q

C. R

D. S

E. T

Question 7



The shaded region (with boundaries excluded) of the complex plane shown above is best described by

A. $\left\{z\text{: Arg}(z) > \dfrac{\pi}{3}\right\}$

B. $\left\{z\text{: Arg}(z) > \dfrac{\pi}{3}\right\} \cup \left\{z\text{: Arg}(z) < \dfrac{\pi}{2}\right\}$

C. $\left\{z\text{: Arg}(z) > \dfrac{\pi}{3}\right\} \cap \left\{z\text{: Arg}(z) < \dfrac{\pi}{2}\right\}$

D. $\left\{z\text{: Arg}(z) > \dfrac{\pi}{2}\right\} \cup \left\{z\text{: Arg}(z) < \dfrac{\pi}{3}\right\}$

E. $\left\{z\text{: Arg}(z) > \dfrac{\pi}{2}\right\} \cap \left\{z\text{: Arg}(z) < \dfrac{\pi}{3}\right\}$

Question 8

With a suitable substitution, $\int_{0}^{\frac{\pi}{3}} \cos^2(x)\sin^3(x)\,dx$ can be expressed as

A. $\int_{\frac{1}{2}}^{1} u^2\left(1-u^2\right)du$

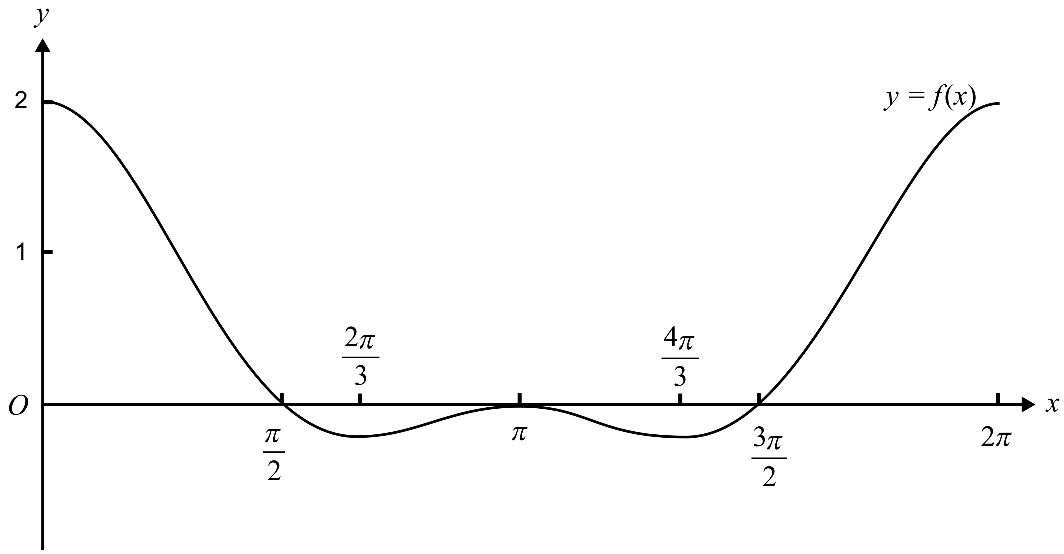
B. $\int_{1}^{\frac{1}{2}} u^2\left(1-u^2\right)du$

C. $\int_{0}^{\frac{\pi}{3}} u^2\left(1-u^2\right)du$

D. $-\int_{0}^{\frac{\pi}{3}} u^2\left(1-u^2\right)du$

E. $-\int_{0}^{\frac{\sqrt{3}}{2}} u^2\left(1-u^2\right)du$

Question 9



The graph of $y = f(x)$ is shown above.

Let $F(x)$ be an antiderivative of $f(x)$.

The stationary points of the graph of $y = F(x)$ could be

A. local maximums at $x = 0$, π and 2π, and local minimums at $x = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$

B. stationary points of inflexion at $x = 0$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$ and 2π, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$

C. stationary points of inflexion at $x = 0$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$ and 2π, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

D. a stationary point of inflexion at $x = \pi$, a local maximum at $x = \frac{\pi}{2}$, and a local minimum at $x = \frac{3\pi}{2}$

E. a stationary point of inflexion at $x = \pi$, a local minimum at $x = \frac{\pi}{2}$, and a local maximum at $x = \frac{3\pi}{2}$

Question 10

The gradient of the curve $x^2 + y^2 = 9$ at the point in the third quadrant where $x = -1$ is

A. $-2\sqrt{2}$

B. $-\frac{1}{2\sqrt{2}}$

C. 1

D. $\frac{1}{2\sqrt{2}}$

E. $2\sqrt{2}$

Question 11

If $a < b < 0$, then $\int_a^b \frac{1}{x}\,dx$ is equal to

A. $\log_e(b) - \log_e(a)$

B. $|\log_e(b) - \log_e(a)|$

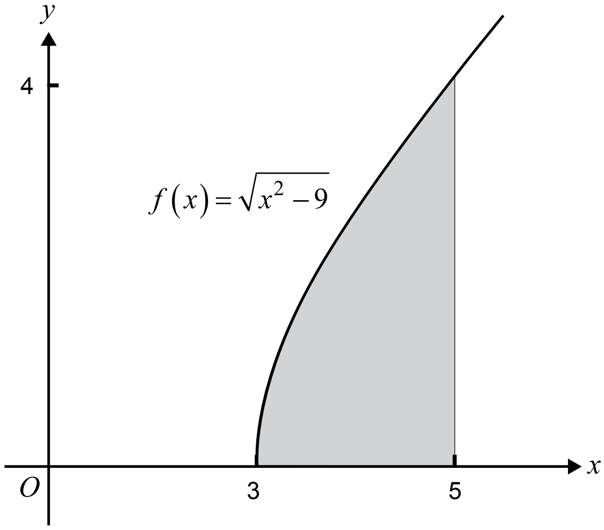
C. $\log_e|b| - \log_e(a)$

D. $\log_e(b) - \log_e|a|$

E. $\log_e|b| - \log_e|a|$

Question 12

The graph of $f:[3,\infty)\to R$, where $f(x)=\sqrt{x^2-9}$, is shown below.
The shaded region is bounded by this graph, the x-axis, and the line with equation $x = 5$.



The shaded region is rotated about the y-axis to form a solid of revolution.
The volume of this solid, in cubic units, is given by

A. $\pi\int_0^4 \left(y^2-9\right)dy$

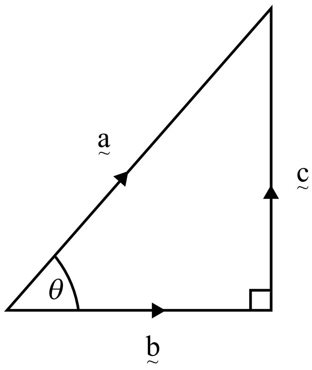
B. $\pi\int_0^4 \left(34-y^2\right)dy$

C. $\pi\int_0^4 \left(y^2+9\right)dy$

D. $\pi\int_0^4 \left(16-y^2\right)dy$

E. $\pi\int_0^4 \left(5-\sqrt{y^2+9}\right)^2 dy$

Question 13



The right-angled triangle shown above has sides represented by the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.
Which one of the following statements is **false**?

A. $|\underset{\sim}{b}|^2 + |\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2$

B. $\underset{\sim}{b} \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = |\underset{\sim}{b}|^2$

C. $\underset{\sim}{b} \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = |\underset{\sim}{b}||\underset{\sim}{c}|$

D. $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos(\theta)$

E. $\underset{\sim}{a} \cdot \underset{\sim}{c} = |\underset{\sim}{a}||\underset{\sim}{c}|\sin(\theta)$

Question 14

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = 2\sin(t)\underset{\sim}{i} + \cos(t)\underset{\sim}{j}$, $0 \le t \le \pi$.
The Cartesian equation of the path of the particle is

A. $y = \cos\left(\sin^{-1}\left(\frac{x}{2}\right)\right), \quad 0 \le x \le 2$

B. $y = \sqrt{1 - \frac{x^2}{4}}, \quad 0 \le x \le 2$

C. $\frac{x^2}{2} + y^2 = 1, \quad -2 \le x \le 2$

D. $\frac{x^2}{4} + y^2 = 1, \quad -2 \le x \le 2$

E. $\frac{x^2}{4} + y^2 = 1, \quad 0 \le x \le 2$

Question 15

A body of mass 5 kg slides from rest down a smooth plane inclined at an angle of 30° to the horizontal.

The acceleration, in m/s^2, of the body down the plane has magnitude

A. $\dfrac{\sqrt{3}g}{2}$

B. $\dfrac{g}{2}$

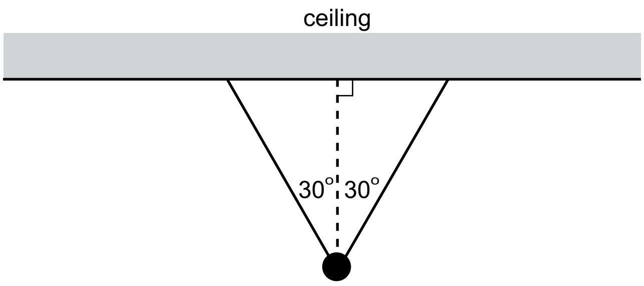
C. 0

D. $\dfrac{5\sqrt{3}g}{2}$

E. $\dfrac{5g}{2}$

Question 16

A 10 kg mass is suspended in equilibrium from a horizontal ceiling by two identical light strings. Each string makes an angle of 30° with the vertical as shown below.



The magnitude, in newtons, of the tension in each string is equal to

A. $5g$

B. $10g$

C. $20g$

D. $\dfrac{10g\sqrt{3}}{3}$

E. $\dfrac{20g\sqrt{3}}{3}$

Question 17

A body of mass 5 kg is acted upon by three concurrent coplanar forces $\underset{\sim}{R}$, $\underset{\sim}{S}$ and $\underset{\sim}{T}$, where $\underset{\sim}{R} = 2\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{S} = \underset{\sim}{i} + 10\underset{\sim}{j}$ and $\underset{\sim}{T} = 3\underset{\sim}{i} + 3\underset{\sim}{j}$. The forces are measured in newtons.

The magnitude of the acceleration of the body, in m/s^2, is

A. 2

B. 4

C. 6

D. 8

E. 10

Question 18

A balloon is rising vertically at a constant speed of 21 metres per second. A stone is dropped from the balloon when it is *h* metres above the ground. The stone strikes the ground 10 seconds later.

Assuming that air resistance is negligible, the value of *h* is

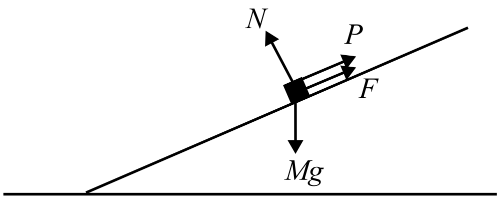
A. 210
B. 280
C. 490
D. 700
E. 770

Question 19

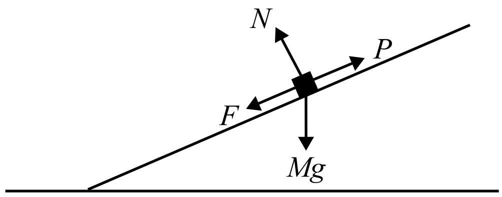
A body of mass *M* kg is on a rough plane inclined at an angle to the horizontal. The body, which is on the point of sliding down the plane, is held in equilibrium by a force of magnitude *P* applied parallel to the plane. There is a normal reaction of magnitude *N* and a frictional force of magnitude *F*. All forces are measured in newtons.

Which one of the following diagrams shows the forces acting on the body?

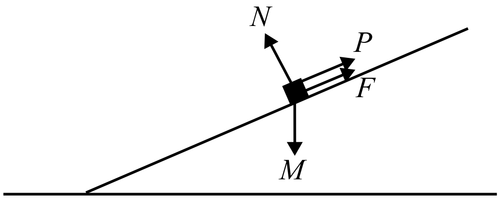
A.



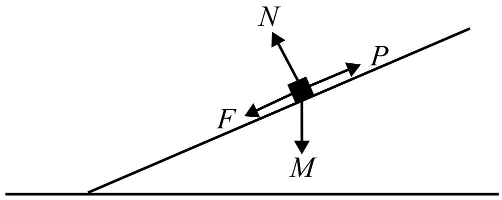
B.



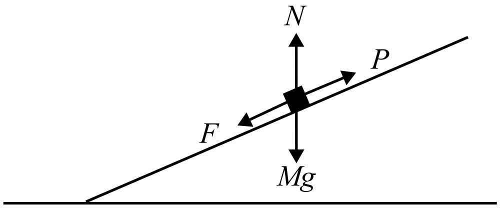
C.



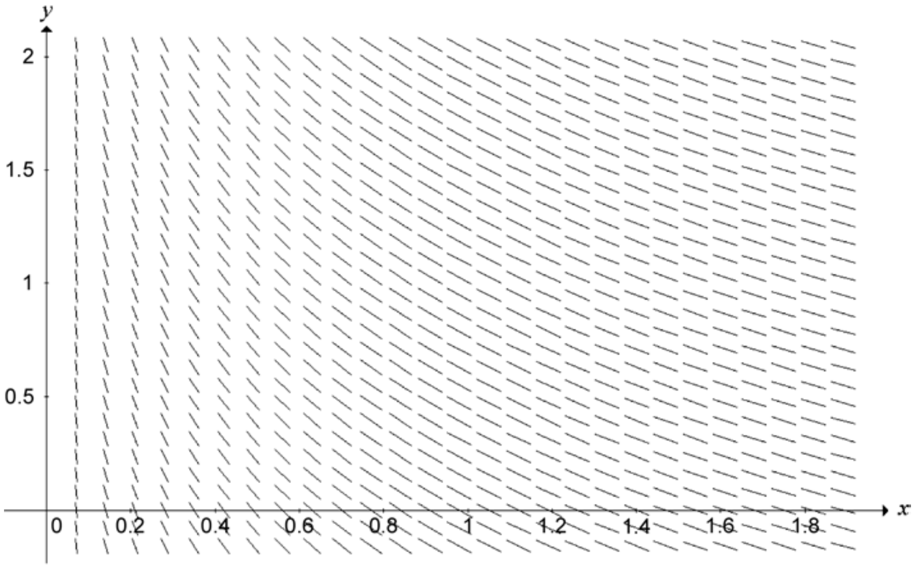
D.



E.



Question 20



The direction (slope) field for a certain first order differential equation is shown above.
A solution to this differential equation could be

A. $y = \frac{1}{x}$

B. $y = -\log_e(x)$

C. $y = e^{-x}$

D. $y = \frac{1}{x^2}$

E. $y = \log_e(-x)$

Question 21

A jug of water at a temperature of 20°C is placed in a refrigerator. The temperature inside the refrigerator is maintained at 4°C.

When the jug has been in the refrigerator for t minutes, the temperature of the water in the jug is y°C. The rate at which the water's temperature decreases is proportional to the excess of its temperature over the temperature inside the refrigerator.

If k is a positive constant, a differential equation involving y and t is

A. $\frac{dy}{dt} = -k(y - 20); \quad t = 0, \ y = 4$

B. $\frac{dy}{dt} = -k(y + 4); \quad t = 0, \ y = 20$

C. $\frac{dy}{dt} = -k(y - 4); \quad t = 0, \ y = 16$

D. $\frac{dy}{dt} = -k(y + 4); \quad t = 0, \ y = 24$

E. $\frac{dy}{dt} = -k(y - 4); \quad t = 0, \ y = 20$

Question 22

Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = \cos\left(\frac{x}{2}\right)$, with initial condition $y = 2$ when $x = 0$.

The approximation obtained for y when $x = 0.4$ is given by

A. $2 + 0.4\cos(0.1)$

B. $2 + 0.4\cos(0.2)$

C. $2.2 + 0.2\cos(0.1)$

D. $2.2 + 0.2\cos(0.2)$

E. $2 + 0.2\cos(0.1) + 0.2\cos(0.2)$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Two particles R and S move so that their position vectors, $\underset{\sim}{r}$ and $\underset{\sim}{s}$ respectively, are given by

$$\underset{\sim}{r} = \left(2 - 2\cos(t)\right)\underset{\sim}{i} + \left(1 + \sin(t)\right)\underset{\sim}{j} \text{ and } \underset{\sim}{s} = \sin(t)\,\underset{\sim}{i} + 2\cos(t)\,\underset{\sim}{j}$$

where t seconds, $t > 0$, is the time elapsed since the start of their motions.

a. Evaluate $\underset{\sim}{r} \cdot \underset{\sim}{s}$ and **hence** deduce the exact time when the position vectors of the two particles are first at right angles.

4 marks

b. Find the cartesian equation, in terms of x and y, of the path of the particle R.

3 marks

c. Show that the particles R and S move at the same speed at any given time, but never have the same velocity.

6 marks

Total 13 marks

Question 2

A dragster racing car accelerates uniformly over a straight line course and completes a 'standing' (that is, starting from rest) 400 metres in eight seconds.

a. **i.** Find the acceleration (in m/s^2) of the dragster over the 400 metres.

1 mark

ii. Show that the dragster reaches a speed of 100 m/s at the end of the 400 metre course.

1 mark

At the 400 metre mark, the dragster stops accelerating. At this instant, the dragster's brakes are applied and, in addition, a small parachute opens at the rear to slow the car down.

The retarding force applied by the brakes (including friction) is 5000 N. The retarding force due to the parachute is $0.5v^2$ N where v m/s is the velocity of the car x metres beyond the 400 metre mark. The mass of the dragster (car and driver) is 400 kg.

b. **i.** If a m/s^2 is the acceleration of the dragster during the retardation stage, write down the equation of motion for the dragster during this stage.

1 mark

ii. By choosing an appropriate derivative form for acceleration, show that a differential equation relating v to x is

$$\frac{dv}{dx} = -\frac{\left(10^4 + v^2\right)}{800\,v}$$

2 marks

iii. Hence find, to the nearest metre, the distance the dragster takes to stop from the instant the brakes are applied.

3 marks

c. Write a definite integral representing the time taken, T seconds, to bring the dragster to rest from the 400 metre mark and **hence** find the value of T correct to two decimal places.

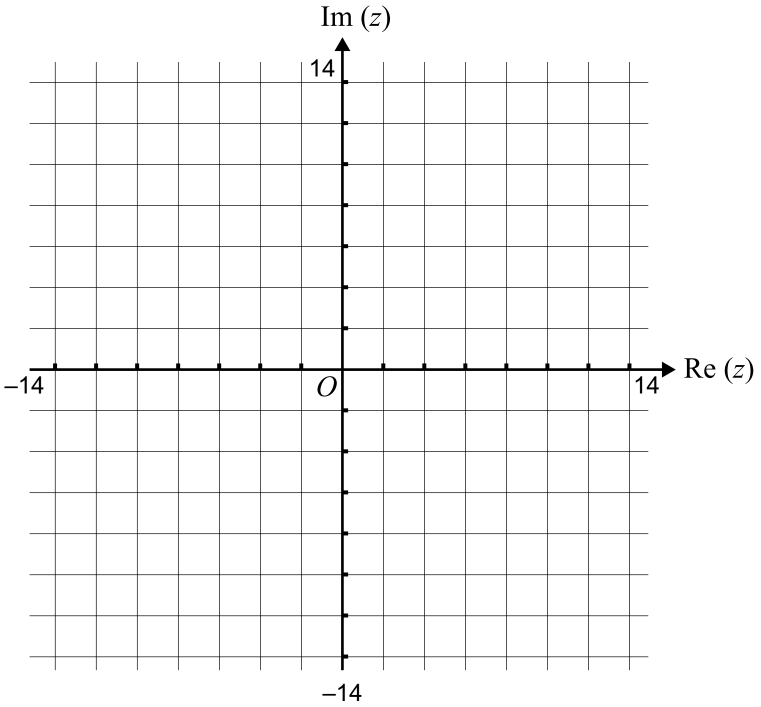
3 marks

Total 11 marks

Question 3

Let $v = 6 + 8i$ and $w = 7 + i$.

a. Plot the points corresponding to v and w on the diagram below, labelling them as V and W respectively.



1 mark

b. Let S be defined by $S = \{z : |z| = 10, z \in C\}$.

i. Verify that $v \in S$.

1 mark

ii. Sketch S on the Argand diagram in part **a**.

1 mark

c. Let u be such that $u + i\bar{w} = \bar{w}$. Find u in cartesian form.

2 marks

d. Sketch, on the Argand diagram in **a.**, $T = \{z : |z| \le 10\} \cap \{z : |z - u| = |z - v|\}$

2 marks

e. Use a vector method to prove that $\angle OWV$ is a right angle.

3 marks

Total 10 marks

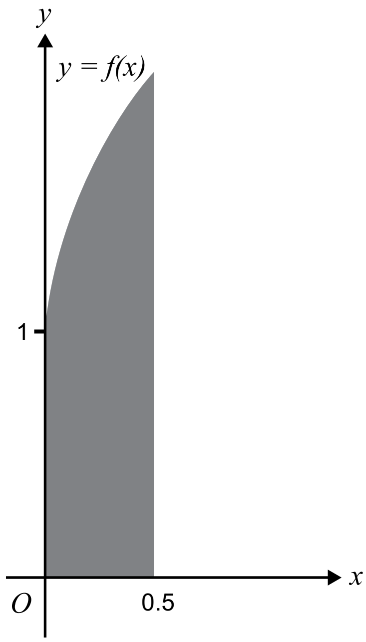
Question 4

Consider the function f with rule $f(x) = \dfrac{2x^{\frac{1}{2}}\left(1-x^2\right)^{\frac{1}{4}}}{} + \dfrac{1}{\left(1-x^2\right)^{\frac{1}{4}}}$.

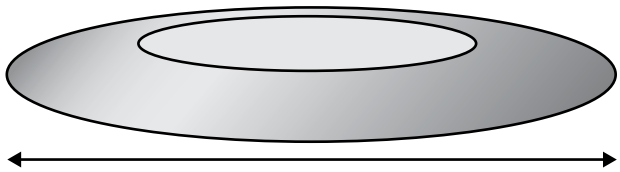
a. State the largest domain for which f is defined.

1 mark

A solid platform for a statue is constructed by rotating, about the x-axis, the region enclosed by the curve $y = f(x)$, the line $x = 0.5$, and the coordinate axes. Lengths are measured in metres.



The platform is laid flat on a horizontal surface as shown below (diagram not to scale).



b. Let D metres be the diameter of the base of the platform. Find D correct to three significant figures.

1 mark

c. Find, correct to the nearest degree, the angle of slope of the platform at any point where it meets the horizontal surface.

3 marks

d. **i.** Two of the three terms in the expansion of $(f(x))^2$ are $4x\sqrt{1-x^2}$ and $\dfrac{1}{\sqrt{1-x^2}}$.
Find the third term.

1 mark

ii. Write a definite integral which gives the volume of the platform in cubic metres.

2 marks

The material used to make the base is sold in packages containing one cubic metre.

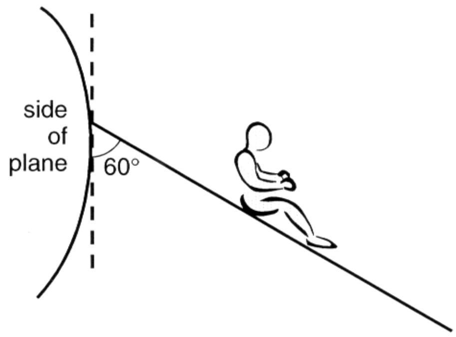
iii. Find the volume of the platform in cubic metres correct to two significant figures and **hence** write down the number of packages needed.

2 marks

Total 10 marks

Question 5

A plane is forced to make an emergency landing. After landing, the passengers are instructed to exit using an emergency slide, of length 6 metres, which is inclined at the angle of 60° to the vertical.



Passenger Jay has mass 75 kilograms. The coefficient between Jay and the slide is $\frac{1}{5}$.

a. Clearly mark and label on the diagram the forces acting on Jay as he slides down.

1 mark

b. Show that the acceleration a m/s^2 of Jay down the slide is given by $a = \frac{g}{2}\left(1 - \frac{\sqrt{3}}{5}\right)$.

3 marks

c. **i.** Jay starts from rest at the top of the slide. Find the time, in seconds, that he takes to reach the end of the slide. Give your answer correct to two decimal places.

2 marks

ii. Show that Jay reaches the end of the slide with a speed of 6.2 m/s.

1 mark

The plane came to rest so that the end of the slide is 2 metres vertically above level ground.

d. How far horizontally from the end of the slide does Jay land? Give your answer to the nearest tenth of a metre.

4 marks

e. Find the magnitude of Jay's momentum, in kg m/s, when he hits the ground. Give your answer correct to three significant figures.

3 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

March 2012

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum:	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion:	$\underset{\sim}{R} = m\underset{\sim}{a}$
friction:	$F \leq \mu N$

END OF FORMULA SHEET