2005 Specialist Maths: GA 3: Written examination

GENERAL COMMENTS

The number of students who sat for the 2005 examination was 5625, compared to 6150 in 2004. As in 2004, students had to answer five questions worth a total of 60 marks. Each question was worth between nine and 14 marks.

The data suggests that, on average, students' performances were better than that of students in 2004. The mean and median scores, out of a possible 60, were both 30.0 and 30, compared with 26.2 and 27.2 in 2004. Also, 4.37% of students scored 90% or more of the marks, compared with only 0.81% in 2004. Thirteen students scored full marks, compared with three in 2004. Hence, it would appear that there were sufficient marks 'available' for students to show what they could do, but that it was challenging to get full marks. At the lower end of the scale, about 1.1% of students scored four or less marks compared with 4.7% in 2004.

The average scores for the five questions, expressed as percentages of the total marks available, were about 49%, 52%, 47%, 43% and 53% respectively. The smaller percentages were not unexpected for Questions 3 and 4, as 3d. and 4c. were quite challenging; many students had difficulty coping with 3d. and 4c. as they were questions which required significant insight. However, the averages for several question parts (Questions 1a., 1cii., 2cii., 2ciii., 3c., 4d. and 5c.), some of which were quite straightforward, were lower than expected, which was disappointing and unexpected. Clearly, the latter parts of some questions were challenging for many students.

There were five 'show that…' questions and four 'hence'-type questions on the 2005 paper. Teachers should remind students that the former question type is intended to help keep them on track and enable access to subsequent parts of the same question; while in the latter question type, previously derived results must be used. For 'show that…' questions it is essential for students to show all steps that led to the given answer—the assessor needs to be convinced that the student's working would lead them to the given answer. As the answer is given, the working is the important part of the solution, because virtually everyone ends up with the given answer. In this type of question, the given answer often appeared without substantial working preceding it. Students need to pay attention to methods of setting out this sort of solution. This also applies to questions that require a verification to be performed.

There was scope for effective use of graphics calculators on the 2005 examination paper, and many students were able to use the technology quite successfully. The most obvious examples of this were in Questions 1d.–1eii., where the graph could not be easily sketched without a calculator and the points of intersection with $x = 15$ could not be readily solved analytically (and there was only one mark for each of the latter parts). Other places where the technology could be used to the student's advantage were Questions 3c. (finding the area after using calculus), 3d. (finding an approximate value for x having found h), and the latter part of 4d. Students could have used one of the 'solve' features of a graphics calculator to find some of the above the solutions.

It is necessary to remind students again that appropriate working must be shown in questions worth more than one mark, and that assessors must be able to see the steps that students used to reach the solution. Marks are allocated for methods and procedures, not just for numerical or algebraic answers.

The phrase 'use calculus to…' was employed on the 2005 paper in Question 3c., the intention being that students would show the appropriate antiderivatives in the course of their working. A large proportion of students gave the correct answer (2.18) without showing the appropriate antiderivatives in the steps towards a solution. These students received one mark if they had correctly set up the definite integral; if they had not done this, they scored no marks. In questions where a numerical approximation is sufficient and an analytical solution is not required, the phrase 'use calculus to…' would not be employed and less marks would be allocated.

When instructions such as 'find the exact…' are given, the question must be worked analytically rather than numerically. It needs to be emphasised that if an exact answer is required, full marks cannot be obtained if only a decimal approximation is given.

As was the case in some past examinations, students often struggled with questions that involved the more advanced aspects of complex numbers. In this examination, many students had difficulty with parts of Question 2. Perhaps more intensive revision work on complex number analysis questions would be beneficial leading up to the examination period, especially as this material is often covered early in the course. Care also needs to be taken with some calculator work. Basic errors were often seen in Question 1d. and Question 4d.

SPECIFIC INFORMATION

Question 1a.

Marks	**0**	**1**	**Average**
%	80	20	**0.2**

$$\frac{x}{10+10t}$$

This question was surprisingly poorly answered. Most students did not appear to know what was meant by 'concentration'.

Question 1b.

Marks	**0**	**1**	**2**	**3**	**Average**
%	47	11	5	37	**1.4**

as given

This question wasn't answered as well as might have been expected. Many students had difficulty starting this question, but those who did start were often successful. Many students did not fully show how the terms in the differential equation came about. Some of those who could not answer 1a. were able to complete this question.

Question 1ci.

Marks	**0**	**1**	**2**	**3**	**4**	**Average**
%	27	12	14	14	33	**2.2**

as given

This question was reasonably well attempted. Most students applied the product rule successfully; however, some used the quotient rule and most of those got into difficulty. Many did not use the chain rule on the log term. Only a minority managed to combine the terms that did not involve log and Tan^{-1} to obtain the required result. Many students got lost in the required algebra, and a small number used an antiderivative of some terms when applying the product rule. Some students verified the derivative on the assumption that x was a solution to the differential equation, even though this property of x was not mentioned until the next part of the question.

Question 1cii.

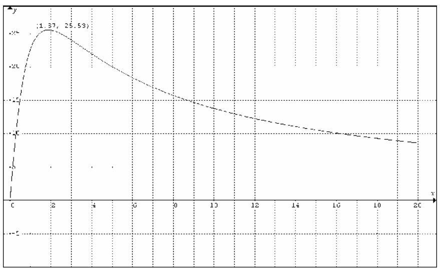
Marks	**0**	**1**	**Average**
%	72	28	**0.3**

as given

This was surprisingly poorly answered, with many students unable to substitute the information given into the differential equation and set out a reasonably convincing verification. Many made this question much harder than it was.

Question 1d.

Marks	**0**	**1**	**2**	**3**	**Average**
%	18	17	21	45	**2.0**



This question was generally well done although many students did not pay attention to detail. A vertical scale, labelling and positioning of the turning point and consideration of the domain were often ignored or handled poorly. A number of students had their calculator in degree mode when tackling this technology specific question.

Question 1ei.

Marks	0	1	Average
%	46	54	0.6

0.485

This question was generally well done by those who had had their calculators in radian mode. Some rounded to two decimal places instead of three.

Question 1eii.

Marks	0	1	Average
%	49	51	0.5

8.17

This question was generally well done by those who had their calculators in radian mode.

Question 2ai.

Marks	0	1	2	Average
%	6	14	80	1.8

$\text{cis}\left(\frac{\pi}{3}\right)$

Generally well done but, as this was a two-mark question, students were expected to show the working that led to their correct result.

Question 2aii.

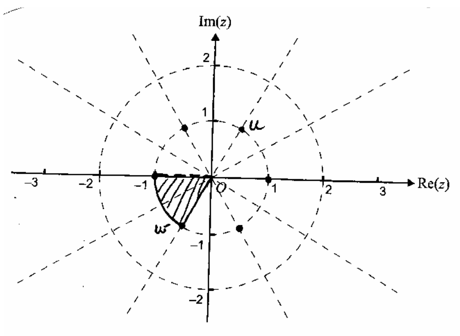
Marks	0	1	Average
%	18	82	0.9

as given

This question was well done by those who followed the 'hence' instruction and worked with polar form to apply De Moivre's theorem.

Question 2aiii.

Marks	0	1	2	3	Average
%	29	16	18	36	1.7



This question was quite well done, with most students plotting six complex roots. Some did not label correctly and several confused $-u$ with $\overline{u}$. Others put u on the real axis.

Question 2b.

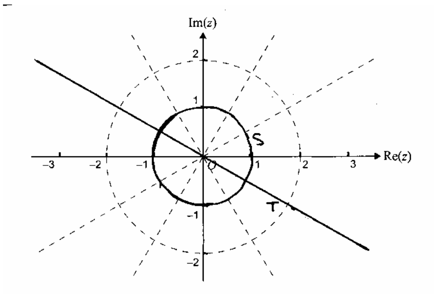
Marks	0	1	2	3	Average
%	26	46	15	13	**1.2**

Answer on graph for 2aiii.

This question proved to be quite difficult for most students. Many knew that the region was somewhere inside the circle $|z| = 1$, but most had difficulty refining the region any further. Most did not have the correct sector and very few had the correct boundary.

Question 2ci.

Marks	0	1	Average
%	37	63	**0.7**



This question was reasonably well answered. Many drew the circle $|z| = 1$ but did not label it as required.

Question 2cii.

Marks	0	1	2	Average
%	47	24	29	**0.9**

Answer on graph for **2ci**.

This question was not particularly well done. Many students realised that they had to draw a straight line, but the correct orientation and labelling of the line eluded them.

Question 2ciii.

Marks	0	1	2	Average
%	65	12	23	**0.6**

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

This question was not particularly well done, with many students showing a lot of complicated working instead of simply reading the answers off the graph.

Question 3a.

Marks	0	1	2	Average
%	22	19	59	**1.4**

1.9875

This question was quite well done. Some students used a rectangle rule approach or triangles instead. Students were expected to show that they had used the prescribed method.

Question 3b.

Marks	0	1	2	3	Average
%	14	16	14	56	**2.2**

$A = 3,\ B = -3$

This question was generally well done. However, some students insisted on using an incorrect partial fractions format of their own, instead of the one provided. Some did not use brackets, which led to errors.

Question 3c.

Marks	0	1	2	3	4	5	Average
%	24	42	3	5	5	22	**1.9**

2.18

This question was done poorly by the majority of students. The most common errors were notation errors (for example, the lack of *dx*). Large numbers of students did not realise that they had to split up $\frac{x+3}{x^2+1}$ to continue. Students who managed to find the correct answer without using calculus (that is, no antiderivative was shown) were eligible for two marks (for setting out the integral and for splitting up the fractions).

Question 3d.

Marks	0	1	2	3	Average
%	68	5	11	16	**0.8**

52

This question was not well done. Many students took the simplistic approach and gave an answer of 50. Many of those who tried to allow for the ‘overlap’ used 0.55 as the value of h at $x = 2$ rather than $\frac{4}{7}$.

Question 4a.

Marks	0	1	Average
%	31	69	**0.7**

as given

This question was well done. A minority erroneously simply verified that the speed was 12.

Question 4b.

Marks	0	1	2	3	Average
%	14	14	20	52	**2.2**

as given

This question was reasonably well done with most students allowing for the ‘constant vectors’ of integration. There was the usual sprinkling of sign errors when the first constant of integration was incorporated. A minority tackled the problem by differentiation of the position vector, and very few of these were awarded full marks.

Question 4c.

Marks	0	1	2	3	Average
%	76	7	2	14	**0.5**

as given

Most students had difficulty starting this question, with very few realising that they needed to find the time at which the position of the skier intersected with $y = -x$. Some used $y = x$ and a minority attempted to solve the problem using constant acceleration formulae.

Question 4d.

Marks	0	1	2	3	Average
%	55	15	10	21	**1.0**

22.5

This question was not done particularly well considering that the value of T was given and students merely had to substitute this value into the velocity vector and then find the magnitude of that vector. Many were unable to substitute accurately (brackets were often a problem), and quite a few students substituted into the displacement vector. The correct numerical evaluation of $\left|\dot{\underset{\sim}{r}}(T)\right|$ proved to be challenging for many. Constant acceleration formulae approaches also occasionally appeared.

Question 5a.

Marks	0	1	2	Average
%	20	12	68	**1.5**

24.5

This question was well done. The major error appearing here was friction acting down the plane. Some students misinterpreted the problem and set about finding μ , treating the situation as one of limiting equilibrium.

Question 5b.

Marks	0	1	2	3	Average
%	25	40	6	29	**1.4**

0.67

This question was not answered as well as was expected. Several sign errors were seen in the equations of motion. Again, having friction acting down the plane was the major problem. Incorrect values, even negative values, for μ were often seen.

Question 5c.

Marks	0	1	2	3	4	Average
%	32	12	12	25	19	**1.9**

14.4 (kg)

Those who got started generally made a good attempt at this question. A reasonable number of students set up the equations of motion correctly. However, some used a mass of 5kg in some of the terms instead of m, some left out g, and the omission of the friction term was a frequent occurrence.