2009 Mathematical Methods GA 3: Examination 2

GENERAL COMMENTS

There were 8517 students who sat the 2009 Mathematical Methods examination 2. Students' scores extended over the entire range of possible scores. Student responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate their knowledge.

The mean for 2009 was 40. The median score for the paper was 39 marks. Of the whole cohort, 9% of students scored 86% or more of the available marks, and 20% scored 76% or more of the available marks. The mean score for the multiple-choice section was 13 out of 22.

There was some evidence that a significant number of students did not allocate time efficiently across the two sections of the paper. Students should carefully plan their approach to the whole paper and avoid spending too much time on Section 1.

As stated in the instructions on the examination paper, students must show appropriate working for questions worth more than one mark. This was done better than in previous years but a number of students still gave answers only or did not give sufficient working for questions such as Questions 3a. and 3b. Students should attempt to write out an expression or equation, even if they think their answer to a previous question is incorrect, because marks can be awarded, for example, in Questions 1dii., 2d. and 4eii. Students must ensure that they provide sufficient working in 'show that' questions such as Question 2aii.

Students must ensure they read questions carefully so that they give answers over the required interval. Students must ensure they use the units given in the question, especially in questions such as Question 2bii. and Question 4. They do not need to convert units unless the question asks for the answer to be in a specific unit. It was pleasing to see that many students used the correct variables this year.

It was still apparent that some students were not clear on the meaning of the word 'exact' when it appeared in a question. Many students gave the exact answer and then wrote down an approximate answer as their final response. This occurred in Questions 1c., 1di., 1dii. and 4cii.

Students should retain sufficient decimal place accuracy in computation to ensure that they can provide numerical answers to a specified accuracy; for example, in Question 3ci., the conditional probability uses results of previous calculations.

Students should take care when drawing graphs. If they are required to sketch over part of a graph they need to ensure it is done clearly, as in Question 1b. The use of a distinguishing colour here is recommended. Care needs to be taken with closed and open circles for end points.

It was pleasing to see better use of notation and better decimal place usage than in previous years.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	9	3	19	67	2	0	
2	1	3	92	3	1	0	This was the best answered question on the paper.
3	8	84	3	5	1	0	
4	2	27	2	59	10	0	
5	2	16	23	57	1	0	A graph of the function using technology would have given students the correct answer quickly.
6	6	4	25	59	7	0	
7	12	71	6	3	8	0	
8	5	12	72	6	5	0	
9	12	8	9	25	46	1	The curve of $y = f(x)$ has been translated 2 units to the right and 3 units up. The image of the point (1, 5) is (3, 8). Hence the gradient of $y = f(x-2)+3$ at the point (3, 8) will be 2. The equation of the tangent at (3, 8) is $y = 2x+2$.
10	6	63	19	8	5	0	
11	12	8	15	54	10	1	
12	3	4	21	65	7	0	
13	5	18	65	6	5	1	
14	8	22	11	44	15	0	
15	7	12	8	3	69	0	
16	3	7	73	7	10	0	
17	27	47	9	9	6	1	For independent events $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ Let $A = \{1, 3, 5, 7, 9, 11\}$ and $B = \{1, 4, 7, 10\}$ $\Pr(A) = \frac{1}{2}$ and $\Pr(B) = \frac{1}{3}$ $\Pr(A \cap B) = \frac{1}{6}$ $\Pr(A) \times \Pr(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ Hence $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$. Many students chose option B, which listed mutually exclusive events, for which $\Pr(A \cap B) = 0$.
18	16	56	8	14	5	1	
19	15	5	16	4	60	0	
20	10	28	17	12	33	0	The number of solutions to $\lvert a\cos(x)\rvert = \lvert a\rvert$ where $x \in [-2\pi, 2\pi]$ is 9. The horizontal line $y = \lvert a\rvert$ touches the graph of $y = \lvert a\cos(x)\rvert$ at the two end points and the seven turning points.
21	9	41	24	12	14	1	$f'(x) > 0$ for $x \in (-3, 2)$, $f'(x) = 0$ at $x = 2$ and $f'(x) < 0$ for $x > 2$. Hence the graph of *f* has a local maximum at $x = 2$.

22	13	13	25	39	9	1	The inverse must be considered. $\int_0^3 (e^x + 1)dx = e^3 + 2$

Section 2

Question 1

1a.

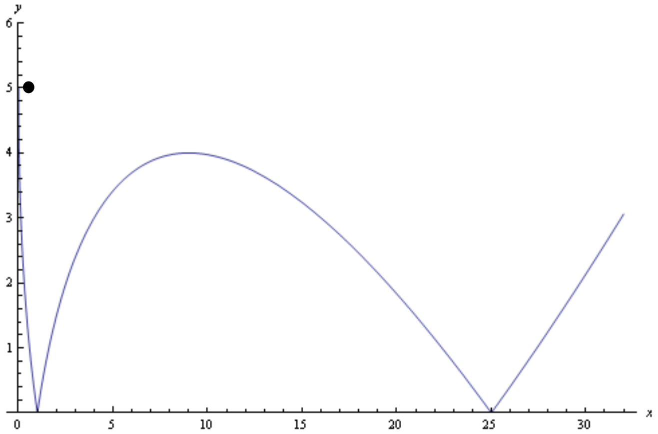
Marks	0	1	2	3	Average
%	22	12	9	58	2.1

$$f'(x) = 0, \Rightarrow = \frac{6}{2\sqrt{x}} - 1 = 0$$
$$\Rightarrow x = 9$$
$$\Rightarrow y = 4$$

Most students missed out on marks because they failed to find the value of y or because of difficulty with algebra.

1b.

Marks	0	1	2	Average
%	13	37	50	1.4



This question was generally answered well. Students were required to draw over the middle section of the original graph and have the closed circle at the end point (0, 5). A number of students did not draw a smooth curve. Some had the end point (0, 5) below the turning point (9, 4). The cusps were drawn well. Students were not expected to label the graph nor the stationary point.

1ci.

Marks	0	1	2	3	Average
%	17	15	10	58	2.1

$$\int_1^{25} (6\sqrt{x} - x - 5)dx = \left[4x^{\frac{3}{2}} - \frac{x^2}{2} - 5x\right]_1^{25} = 64$$

This question was quite well done, with most students heeding the instruction to use calculus.

1cii.

Marks	0	1	Average
%	51	49	0.5

$$AD = \frac{64}{24} = \frac{8}{3}$$

Many students gave $\frac{64}{25}$ as the answer. An exact answer was required, not 2.67.

1di.

Marks	0	1	Average
%	21	79	0.8

$m = \frac{3-0}{16-25} = -\frac{1}{3}$

Some students gave two answers to this question. Others gave -3 as the answer, using $m = \frac{x_2 - x_1}{y_2 - y_1}$ or simplified $-\frac{3}{9}$ to -3. An exact answer was required, not -0.3.

1dii.

Marks	0	1	2	Average
%	42	17	42	1

$\frac{6}{2\sqrt{a}} - 1 = -\frac{1}{3} \Rightarrow a = \frac{81}{4} = 20.25$

Many students were able to equate $f'(a)$ to their answer to Question 1di. but some failed to see the connection. Some students gave lengthy solutions which often led to incorrect answers, for example, $\sqrt{a} = \frac{9}{2}$ therefore $a = \sqrt{\frac{9}{2}}$. Others integrated or attempted to find the equation of the line passing through the points *P* and *B*.

1ei.

Marks	0	1	2	Average
%	18	64	19	1

$f(g(x)) = 6\sqrt{(x^2)} - x^2 - 5$ or $f(g(x)) = \begin{cases} 6x - x^2 - 5, \text{ for } x \geq 0 \\ -6x - x^2 - 5, \text{ for } x < 0 \end{cases}$

Many students showed a poor understanding of the absolute value function and incorrectly simplified $\sqrt{(x^2)}$ to x.

1eii.

Marks	0	1	2	Average
%	92	3	4	0.1

$h'(x) = \frac{df(g(x))}{dx} = \frac{6|x|}{x} - 2x$ or $h'(x) = \begin{cases} 6 - 2x, \text{ for } x > 0 \\ -6 - 2x, \text{ for } x < 0 \end{cases}$

Other equivalent forms of the answer were accepted. Most students were able to get $-2x$. Students who incorrectly simplified $\sqrt{(x^2)}$ to x and differentiated this were not awarded any marks. A common incorrect answer was $h'(x) = 6 - 2x$. A few students correctly used the chain rule to differentiate $\sqrt{x^2}$.

Question 2

2ai.

Marks	0	1	2	3	Average
%	26	24	4	46	1.7

(2, 0) lies on the curve $\Rightarrow 0 = \frac{1}{200}(8 + 4b + c)$

$\frac{dy}{dx} = \frac{1}{200}(3x^2 + 2bx)$

Stationary point at $x = 4 \Rightarrow 0 = \frac{1}{200}(48 + 8b)$

Equivalent forms of the equations were acceptable. Many students were able to get the first equation but some incorrectly substituted (4, 0). For a standard type of question, this was poorly done.

2aii.

Marks	0	1	2	Average
%	51	4	45	1

$0 = \frac{1}{200}(48 + 8b) \Rightarrow b = -6$

$0 = \frac{1}{200}(8 + 4b + c)$, $b = -6 \Rightarrow 0 = \frac{1}{200}(8 - 24 + c) \Rightarrow c = 16$

For 'show that' questions students must make sure they show sufficient working. Again, this was poorly done.

2bi.

Marks	0	1	2	3	Average
%	37	24	8	31	1.4

$P\ (2 - 2\sqrt{3}, 0)$, $M\ (2 + 2\sqrt{3}, 0)$

A few students attempted to 'solve' a cubic polynomial equation using the quadratic formula. Some did not put their answers in coordinate form or label the coordinates correctly. Exact answers were required but not always given.

2bii.

Marks	0	1	Average
%	60	40	0.4

$\frac{2}{25} = 0.08$ km = 80 m

If the depth was given as 80 m, the unit was required. The positive value was required. Common incorrect answers were 0.8 km or 800 m and 16 km. It is important to relate the answer from the algebra to the original question.

2biii.

Marks	0	1	Average
%	43	57	0.6

$\frac{2}{25} = 0.08$ km = 80 m

This question was fairly well done, with similar problems relating to units as in Question 2bii.

2biv.

Marks	0	1	Average
%	79	21	0.2

$2\sqrt{3}$ or $2000\sqrt{3}$ m

Students were often confused about adding or subtracting the x coordinates of P and N. An exact answer was required.

2c.

Marks	0	1	Average
%	70	30	0.3

$k = \left(= -\dfrac{120}{\log_e 7}\right)$

A common error was to use $d = 6.2$ instead of 0.

2d.

Marks	0	1	2	Average
%	52	30	18	0.7

$$v = 0 \Rightarrow 0 = \frac{120}{\log_e\left(\frac{1}{7}\right)} \log_e\left(\frac{d+1}{7}\right) \Rightarrow \frac{d+1}{7} = 1 \Rightarrow d = 6 \text{ so distance} = 0.2$$

$$v = 0 \Rightarrow 0 = \frac{120}{\log_e\left(\frac{1}{7}\right)}, \log_e\left(\frac{d+1}{7}\right) \Rightarrow d = 6 \text{ km}$$

Distance from $Q = 6.2 - 6 = 0.2$ km.

Of those who attempted the question, many students were able to set up the equation with the value of k. Some students continued to solve the equation by hand but struggled with the algebra and the logarithms. Students often gave the final answer as 6 km, the distance from P. Students should reread questions before moving on to the next question.

Question 3

Questions 3a. and 3b. were done quite well. Some working had to be shown to get full marks. A labelled diagram with the correct area shaded was sufficient working.

3a.

Marks	0	1	2	Average
%	22	16	62	1.4

$X \sim N(67, 1)$, $\Pr(X < 68.5) = 0.9332$, correct to four decimal places

3b.

Marks	0	1	2	Average
%	25	15	59	1.4

$X \sim N(67, 1)$, $\Pr(65.6 < X < 68.4) = 0.8385$, correct to four decimal places

3ci.

Marks	0	1	Average
%	62	38	0.4

$\dfrac{0.8385...}{0.9332...} = 0.8985$, correct to four decimal places

This question was not answered well. Many students did not recognise that the question involved conditional probability. Some students gave 0.8385 as the answer (the same as for Question 3b.). Probabilities greater than 1 were common.

3cii.

Marks	0	1	2	Average
%	82	10	8	0.3

$Y \sim \text{Bi}(4, 0.101486)$, $\Pr(Y \geq 1) = 0.3482$, correct to four decimal places

Many students recognised that the binomial distribution was required; however, they used the wrong probability. Common incorrect answers were 0.5057 and 0.9993. Students must make sure they work to more decimal places than what is required in the answer.

3d.

Marks	0	1	2	3	Average
%	61	15	3	20	0.8

$\Pr(X < 68.4) = 0.995$, when $x = 68.4$, $z = 2.5758$

So $2.5758 = \dfrac{68.4 - 67}{\sigma}$, $\sigma = 0.54$ mm, correct to two decimal places

Other methods were acceptable – some students solved an equation involving σ, while others used trial and error. Some clever use of the calculator was evident. Many students used 0.99 as the required probability instead of 0.995, obtaining an answer of 0.60. Some used 0.99 as their z value. Other students used 0.05 instead of 0.005, obtaining an answer of 0.85 and some used $\mu - 3\sigma \le x \le \mu + 3\sigma$, which does not lead to a correct answer.

3e.

Marks	0	1	2	Average
%	24	2	74	1.5

$0.8 \times 0.8 \times 0.8 = 0.512 = \dfrac{64}{125}$

This question was done well. Some students tried to use a transition matrix.

3f.

Marks	0	1	2	3	Average
%	28	10	6	56	1.9

$0.8 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.15 + 0.2 \times 0.15 \times 0.8 = 0.176 = \dfrac{22}{125}$

This question was quite well answered. Tree diagrams were popular. Some students tried to use the binomial distribution. Others had all the working correct but did not evaluate the expression correctly.

Question 4

4ai.

Marks	0	1	Average
%	72	28	0.3

Triangles similar, $\dfrac{h}{r} = \dfrac{8}{4} \Rightarrow h = 2r$

This question was not answered well. For 'show that' questions sufficient working must be shown. Many students assumed $h = 2r$ and substituted the values into the equation, writing $h = 8$, $r = 4$, $8 = 2 \times 4$; this is not correct reasoning.

4aii.

Marks	0	1	Average
%	64	36	0.4

$V = \dfrac{1}{3}\pi r^2 h,\ r = \dfrac{h}{2} \Rightarrow V = \dfrac{\pi h^3}{12}$

The wording in this question, 'at time t', confused many students and a common incorrect answer was $V = \dfrac{\pi h^3}{12} t$. Some students used the volume of a cylinder instead of a cone.

4b.

Marks	0	1	2	Average
%	52	15	32	0.8

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{9\pi}{4} \times \frac{4}{\pi h^2} = \frac{9}{h^2}$$

Many students recognised that this question was a related rates of change question. Other solutions were acceptable but quite time consuming.

4ci.

Marks	0	1	Average
%	47	53	0.6

When $h = 2, \frac{dh}{dt} = \frac{9}{4}$ m/h

This question was reasonably well done. Some students had the units as m/s or m but were not penalised. Others simplified $\frac{9}{4}$ to $\frac{3}{2}$.

4cii.

Marks	0	1	2	Average
%	58	4	38	0.8

When $\frac{dh}{dt} = \frac{9}{8}, \frac{9}{h^2} = \frac{9}{8} \Rightarrow h = 2\sqrt{2}$ m

Some students wrote that $\frac{1}{2}$ of $\frac{9}{4}$ was $\frac{9}{2}$ and some left their answer as $h = \pm 2\sqrt{2}$. An exact answer was required.

4di.

Marks	0	1	Average
%	56	44	0.5

$$\frac{dt}{dh} = \frac{h^2}{9}$$

This question was reasonably well done. Some students did not relate the question to Question 4b.

4dii.

Marks	0	1	2	Average
%	72	12	16	0.5

$$t = \int \frac{h^2}{9} dh = \frac{h^3}{27} + c.$$

When $h = 0$, $t = 0$, so $t = \frac{h^3}{27}$, $h = 3t^{\frac{1}{3}}$.

This question was not answered well. Many students failed to consider a constant of integration and others left their answer as $t = \frac{h^3}{27}$.

4ei.

Marks	0	1	Average
%	70	30	0.3

Height of statue above ground level = $14 - t$

Some students were able to answer this question even though they did not attempt other parts of the question. Common incorrect answers were $6-t$ and $8-t$.

4eii.

Marks	0	1	2	Average
%	78	12	10	0.3

$14-t=3t^{\frac{1}{3}}$, $\Rightarrow t=8$ so the statue first touches the acid at 5.00 pm

Some students were able to equate their answers to Question 4dii. to 4eii. Of those who did, some left the answer as $t=8$, and missed out on a mark. It is important that students relate answers back to the question.