Further Mathematics

Written Examination 2 – October/November

Introduction

Further Mathematics Examination 2 is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students are required to respond to extended questions, involving multi-stage solutions of increasing complexity, covering the Data analysis core area of study and three selected modules from the Applications area of study in relation to Outcomes 1 to 3.

Structure and format

The examination will consist of a set of extended questions for the core and a set of extended questions for each of the six application modules. Students will be required to answer all questions on the core and three selected application modules. Each set of questions will be worth 15 marks and the examination will be out of a total of 60 marks.

A formula sheet will be provided with the examination.

Approved materials

The following materials are permitted in this examination:

- Normal stationery; this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- One approved graphics calculator or CAS and, if desired, one scientific calculator.
- One bound reference that may be annotated. The reference may be typed or handwritten. The reference may be a textbook.

The VCAA publishes details of approved technology for use in mathematics examinations annually in the October *VCAA Bulletin*. Details concerning VCAA approved reference material for use in Further Mathematics examinations are published in the *VCE and VCAL Administrative Handbook*.

Note: protractors, set squares, aids for curve sketching are not required for this examination and have been **removed** from the list of approved materials.

Other resources

Teachers should refer to the Examination section of the *VCE and VCAL Administrative Handbook, VCE Mathematics Assessment Handbook*, the VCE Further Mathematics Study page on the VCAA website and to the *VCAA Bulletin VCE, VCAL and VET* for further advice during the year.

Sample questions

The following sample examination questions provide a full set of extended questions for the new Application Module 6: Matrices.

Questions from previous Further Mathematics examinations (2000–2005) for the Data analysis core and application modules 1–5 continue to be relevant.

Sample questions

Module 6: Matrices

Question 1 (4 marks)

The system of equations

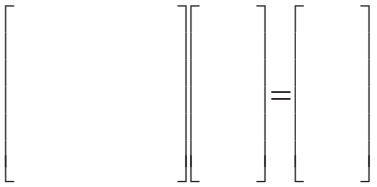
$2x + y + 7z = 9556$

$3x + y + 4z = 5899$

$5x + 2y + z = 3155$

can be used to estimate the number of cats, x, rats, y, and lizards, z, on an island used as a nature reserve.

a. Write this system of simultaneous linear equations in matrix form. 1 mark



b. Write down the inverse matrix that can be used to solve this system of simultaneous linear equations. 1 mark

c. Solve the system of simultaneous linear equations and hence estimate the number of cats, rats and lizards on the island. 2 marks

Question 2 (8 marks)

Each year migratory birds nest at one of three sites, *A*, *B* or *C*, on the island. While the birds generally attempt to nest at the same site each year, this does not always happen.

It is known that, in general, from one year to the next year

- 85% of birds that nested at Site *A* this year return to Site *A* next year
- 5% of birds that nested at Site *A* this year move to Site *B* next year
- 10% of birds that nested at Site *A* this year move to Site *C* next year
- 80% of birds that nested at Site *B* this year return to Site *B* next year
- 15% of birds that nested at Site *B* this year move to Site *A* next year
- 5% of birds that nested at Site *B* this year move to Site *C* next year
- 90% of birds that nested at Site *C* this year return to Site *C* next year
- 5% of birds that nested at Site *C* this year move to Site *A* next year
- 5% of birds that nested at Site *C* this year move to Site *B* next year.

a. Enter this information (converting percentages to decimals) into the transition matrix *T* as indicated below. 2 marks

$$T = \begin{array}{c} \text{This week} \\ \begin{array}{ccc} S & E & N \end{array} \\ \left[\begin{array}{ccc} & & \\ & & \\ & & \end{array}\right] \begin{array}{l} S \\ E \\ N \end{array} \end{array} \quad \text{Next week}$$

In 2006, 4000 birds nested at Site *A*, 2500 nested at Site *B* and 3300 nested at Site *C*.

b. Write this information in the form of a column matrix N_{2006} as indicated below. 1 mark

$$N_{2006} = \left[\begin{array}{c} \\ \\ \end{array}\right] \begin{array}{l} A \\ B \\ C \end{array}$$

c. **i.** Use *T* and N_{2006} to write a matrix product N_{2007} that can be used to determine the number of birds expected to nest at each of the sites in 2007. 1 mark

ii. Complete the matrix multiplication to determine N_{2007}. 1 mark

d. Determine the number of birds expected to nest at each of the sites in 2010. Write your answer in the form of a column matrix N_{2010} with elements written correct to the nearest whole number. 1 mark

e. If this pattern of nesting continues indefinitely, show that, in the long run, the number of birds nesting at each site, correct to one decimal place, is given by the matrix 2 marks

$$N = \begin{bmatrix} 3430.0 \\ 1960.0 \\ 4410.0 \end{bmatrix}$$

Note to teachers: The solution of part **e.** requires students to show that, for increasing n, the elements of two state matrices are equal within the specified level of accuracy.

Question 3 (3 marks)

There are four types of predators on the island that take chicks from the nest; cats, rats, lizards and gulls. The matrix P shows the proportion of chicks lost each day to each type of predator at each site.

$$P = \begin{matrix} \text{cats} & \text{rats} & \text{lizards} & \text{gulls} \end{matrix}$$
$$P = \begin{bmatrix} 0.015 & 0.01 & 0.005 & 0.018 \end{bmatrix}$$

The number of chicks at each nesting site in 2006 is given by the matrix

$$C = \begin{bmatrix} 10000 \\ 6500 \\ 9750 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

a. Which of the matrix products PC or CP is defined? Explain why. 1 mark

b. **i.** Form the matrix product that is defined and call it R. 1 mark

ii. Explain the meaning of the information that matrix R contains. 1 mark