Further Mathematics

Written Examination 1 – October/November

Introduction

Further Mathematics Examination 1 is designed to assess students knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways. Students are required to respond to multiple-choice questions covering the Data analysis core area of study and three selected modules from the Applications area of study in relation to Outcomes 1 and 3.

Structure and format

The examination will consist of multiple-choice questions on the core and each of the six application modules. Students will be required to answer all questions on the core and three selected modules. There will be 13 multiple-choice questions on the core and 9 questions for each application module. The exam will be out of a total of 40 marks.

A formula sheet will be provided with the examination.

Approved materials

The following materials are permitted in this examination.

- Normal stationery; this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- One approved graphics calculator or CAS and, if desired, one scientific calculator.
- One bound reference that may be annotated. The reference may be typed or handwritten. The reference may be a textbook.

The VCAA publishes details of approved technology for use in mathematics examinations annually in the October *VCAA Bulletin*. Details concerning VCAA approved reference material for use in Further Mathematics examinations are published in the *VCE and VCAL Administrative Handbook.*

Note: protractors, set squares, aids for curve sketching are not required for this examination and have been **removed** from the list of approved materials.

Other Resources

Teachers should refer to the Examination section of the *VCE and VCAL Administrative Handbook, VCE Mathematics Assessment Handbook*, the VCE Further Mathematics Study page on the VCAA website and to the *VCAA Bulletin VCE, VCAL and VET* for further advice during the year.

Sample questions

The sample examination questions for Further Mathematics Examination 1 address new content areas from the core and each of the six modules. A full set of questions has been provided for the new Application Module 6: Matrices.

Questions from previous Further Mathematics examinations (2000–2005) for the Data analysis core and application modules 1–5 continue to be relevant.

Sample questions

Core: Data analysis

Question 1

A student's standard mark on a class test is $z = -2.1$.

On the basis of this score, and assuming the test marks follow a normal distribution, it can be concluded that his test mark was

A. in the bottom 2.5% of marks in the class but not in the bottom 0.15%.

B. in the bottom 5% of marks in the class but not in the bottom 2.5%.

C. in the bottom 16% of marks in the class but not in the bottom 5%.

D. in the top 5% of marks in the class but not in the top 2.5%.

E. in the top 2.5% of marks in the class but not in the top 0.15%.

Question 2

A student's standardised mark on a class test was $z = -1.7$. The mean mark for the class is 55 and the standard deviation is 2.85.

Their mark on the test was closest to

A. 49

B. 50

C. 53

D. 58

E. 60

Question 3

A student wants to construct a box plot with outliers for the following set of data.

2	5	11	17	22	22	22	23	23	24	24
24	28	30	31	32	32	35	38	40	41	

She first obtains the following five number summary.

Minimum value = 2, $Q_1 = 22$ Median = 24, $Q_3 = 32$, Maximum value = 41

For this set of data, which of the following values from the data set would be displayed as outliers in the box plot?

A. 2 only

B. 2 and 5

C. 33 and 39

D. 39 and 41

E. 41 only

Question 4

Given that, for a set of bivariate data,

$$r = 0.5675,\ s_x = 8.67 \text{ and } s_y = 67.98,$$

the slope of the corresponding least squares regression line $y = a + bx$ is closest to

A. 0.07

B. 4.45

C. 4.92

D. 7.84

E. 38.57

Question 5

A trend line that can be used to forecast **deseasonalised** quarterly sales (in thousands of dollars) of a store is given by

$$sales = 230 + 45.6 \times quarter\ number$$

where Quarter 1 is summer 2006, Quarter 2 is autumn 2006 and so on.

The seasonal indices for summer, autumn, winter and spring are shown in the table below.

summer	autumn	winter	spring
1.20	0.95	0.85	1.00

Using the trend line and the table of seasonal indices, the **seasonalised** sales for **winter 2007** are predicted to be closest to

A. \$311 780

B. \$366 800

C. \$466 820

D. \$549 200

E. \$646 120

Module 1: Number patterns and their applications

Question 1

In the Fibonacci sequence, $t_{26} = 121\,393$ and $t_{28} = 317\,811$.

The value of the 27th term is

A. 27

B. 75 025

C. 121 394

D. 196 418

E. 439 204

Question 2

A difference equation is defined by

$$t_n = t_{n-2} + t_{n-1} \qquad \text{where} \quad t_1 = 1 \text{ and } t_2 = 4$$

The 6th term is

A. 6

B. 9

C. 14

D. 23

E. 37

Module 2: Geometry

Question 1

The angle of depression of a car sighted from the top of a tower is 23º. The tower is 34.6 metres tall and the base of the tower and the car are at the same level.

The horizontal distance of the car from the base of the tower is closest to

A. 14.7 m

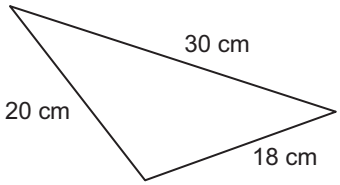
B. 15.7 m

C. 34.7 m

D. 81.5 m

E. 88.6 m

Question 2



The area of the triangle shown is closest to

A. 34 cm^2

B. 68 cm^2

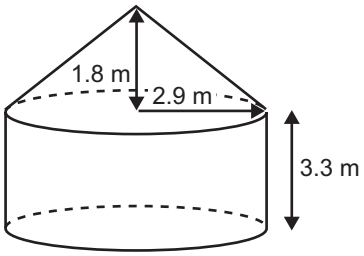
C. 175 cm^2

D. 180 cm^2

E. 30 450 cm^2

Question 3

A grain store has a cylindrical base and conical roof with dimensions as shown.



The volume of the grain store is closest to

A. 45 m3

B. 87 m3

C. 103 m3

D. 135 m3

E. 229 m3

Module 4 Business and related mathematics

Question 1

A capital gain of $9 650 is made selling shares. Capital gains tax is charged at the rate of 23.5% per dollar.

The amount of capital gains tax payable is

A. $23.50
B. $410.63
C. $965.00
D. $2 267.75
E. $11 917.00

Question 2

In the year 2005 a train fare costs $12.50.

It is planned to increase the cost of the train fare by 2.3% for 2006, 3.1% for 2007 and 3.5% for 2008.

In 2008, the cost of the train fare will be closest to

A. $12.50
B. $13.60
C. $13.65
D. $13.70
E. $14.50

Question 3

Jodie inherits $100 000 from a rich uncle. She invests the money in a perpetuity that pays a fixed amount monthly. The interest rate is 3.5 % per annum.

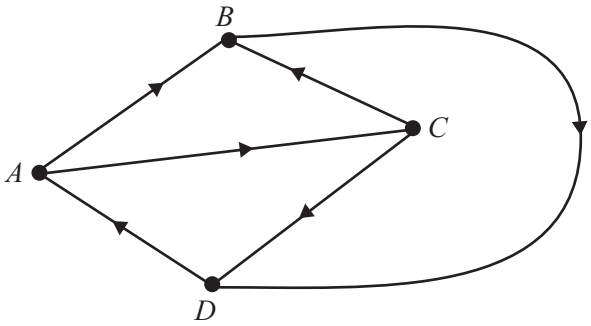
How much will she receive each month?

A. $35.00
B. $291.67
C. $350.00
D. $988.86
E. $1 686.05

Module 5: Networks and decision mathematics

The following information relates to Questions 1 and 2.

The directed graph below shows the results of a round robin competition between four sporting teams. An arrow from *A* to *B* indicates that team *A* defeated team *B*.



Question 1

A matrix that can be used to show both the one and two-step dominances between the four teams is

A.

	A	*B*	*C*	*D*
A	1	0	0	2
B	1	0	0	1
C	1	2	0	1
D	2	1	1	0

B.

	A	*B*	*C*	*D*
A	0	2	1	2
B	1	0	0	1
C	1	1	0	2
D	1	1	1	0

C.

	A	*B*	*C*	*D*
A	0	2	1	2
B	0	0	0	1
C	0	1	0	1
D	1	0	0	0

D.

	A	*B*	*C*	*D*
A	0	2	0	3
B	0	0	0	1
C	1	0	0	1
D	1	1	1	0

E.

	A	*B*	*C*	*D*
A	1	2	0	0
B	1	1	0	1
C	1	0	1	1
D	1	1	1	1

Question 2

Taking into account one-and two-step dominances, the team ranked first is

A. Team *A*

B. Team *B*

C. Team *C*

D. Team *D*

E. Team *A* and Team *C* tied

Module 6: Matrices

Question 1

In the matrix $Z = \begin{bmatrix} 8 & 8 & 0 & -2 \\ -4 & -4 & 0 & 1 \\ 12 & 12 & 0 & -3 \\ -8 & -8 & 0 & 2 \end{bmatrix}$, the element $z_{3,4}$ is

A. –3

B. 0

C. 2

D. 3

E. 12

Question 2

$\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} - 3\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ equals

A. [7]

B. $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 14 \\ 11 \end{bmatrix}$

D. $\begin{bmatrix} -2 & 2 \\ -7 & -8 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & -6 \\ 1 & -9 \end{bmatrix}$

Question 3

If $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$, the order of the matrix product AB is

A. (1×3)

B. (2×3)

C. (3×2)

D. (3×3)

E. not defined

Question 4

If $A = \begin{bmatrix} 1 & 4 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -1 \\ 3 \\ -2 \end{bmatrix}$, the matrix product AB is

A.

[0]

B.

[6]

C.

[8]

D.

$\begin{bmatrix} 8 & 8 & 0 & -2 \\ -4 & -4 & 0 & 1 \\ 12 & 12 & 0 & -3 \\ -8 & -8 & 0 & 2 \end{bmatrix}$

E.

not defined

Question 5

If $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$, the matrix product BA is

A.

$\begin{bmatrix} 6 & 22 \\ -3 & 5 \\ 10 & 19 \end{bmatrix}$

B.

$\begin{bmatrix} 6 & 22 \\ 3 & -5 \\ 10 & 19 \end{bmatrix}$

C.

$\begin{bmatrix} 3 & 9 & 12 \\ -1 & 0 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

D.

$\begin{bmatrix} 6 & 22 & -3 \\ -3 & 0 & 5 \\ 10 & 1 & 19 \end{bmatrix}$

E.

not defined

Question 6

Time of flight	Number of passengers in			Profit from flight ($)
	First class	Business class	Economy class	
morning	5	22	187	6100
lunchtime	4	2	208	4800
evening	2	27	167	5740

The table above shows the number of First class, Business class and Economy class passengers carried by an airline on its morning, lunchtime and evening flights to Canberra. It also shows the profit made on each flight.

The matrix that displays the number of business passengers carried on each of the three flights is

A.

$$\begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

B.

$$\begin{bmatrix} 6100 \\ 4800 \\ 5740 \end{bmatrix}$$

C.

$$\begin{bmatrix} 22 \\ 2 \\ 27 \end{bmatrix}$$

D.

$$\begin{bmatrix} 5 & 22 & 187 \\ 4 & 2 & 208 \\ 2 & 27 & 167 \end{bmatrix}$$

E.

$$\begin{bmatrix} 5 & 22 & 187 & 6100 \\ 4 & 2 & 208 & 4800 \\ 2 & 27 & 167 & 5740 \end{bmatrix}$$

Question 7

The solution of the matrix equation

$$\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$$

is

A.

$$\begin{bmatrix} 0.2 \\ 9 \end{bmatrix}$$

B.

$$\begin{bmatrix} 0.2 \\ 1 \end{bmatrix}$$

C.

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

D.

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

E.

$$\begin{bmatrix} 0.33 \\ 1.4 \end{bmatrix}$$

Question 8

Which of the following systems of simultaneous linear equations have a unique solution?

I $4x + 2y = 9$
$x - 5y = 4$

II $2y = 9$
$x - 2y = 5$

III $3x - 12y = 3$
$x - 3y = 4$

IV $2x = 3$
$x - 2y = 1$

A. **I** only
B. **I** and **II** only
C. **III** only
D. **I** and **IV** only
E. **I**, **II**, **III** and **IV**

Question 9

The matrix

$$P = \begin{bmatrix} 0.78 \\ 0.18 \\ 0.04 \end{bmatrix} \begin{matrix} \text{Economy} \\ \text{Business} \\ \text{First} \end{matrix}$$

lists the proportion of Global Air passengers who fly Economy, Business and First class.

The matrix N that lists the number of passengers flying Economy, Business and First class in a 380 seat Global Air plane is given by

A. $N = P$

B. $N = \frac{1}{380}P$

C. $N = \frac{100}{380}P$

D. $N = 380P$

E. $N = \frac{380}{100}P$