VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2013

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS (CAS)

Written examination 1

Wednesday 6 November 2013

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 14 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (5 marks)

a. If $y = x^2\log_e(x)$, find $\dfrac{dy}{dx}$. 2 marks

b. Let $f(x) = e^{x^2}$.

Find $f'(3)$. 3 marks

Question 2 (2 marks)

Find an anti-derivative of $(4 - 2x)^{-5}$ with respect to x.

Question 3 (2 marks)

The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.

Given that $g(1) = \dfrac{1}{\pi}$, find $g(x)$.

Question 4 (2 marks)

Solve the equation $\sin\left(\frac{x}{2}\right) = -\frac{1}{2}$ for $x \in [2\pi, 4\pi]$.

Question 5 (4 marks)

a. Solve the equation $2\log_3(5) - \log_3(2) + \log_3(x) = 2$ for x. 2 marks

b. Solve the equation $3^{-4x} = 9^{6-x}$ for x. 2 marks

Question 6 (3 marks)

Let $g: R \to R$, $g(x) = (a - x)^2$, where a is a real constant.

The average value of g on the interval $[-1, 1]$ is $\frac{31}{12}$.

Find all possible values of a.

Question 7 (6 marks)

The probability distribution of a discrete random variable, X, is given by the table below.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	$0.6p^2$	0.1	$1 - p$	0.1

a. Show that $p = \frac{2}{3}$ or $p = 1$. 3 marks

b. Let $p = \frac{2}{3}$.

i. Calculate $E(X)$. 2 marks

ii. Find $\Pr(X \geq E(X))$. 1 mark

Question 8 (3 marks)

A continuous random variable, X, has a probability density function

$$f(x)=\begin{cases}\dfrac{\pi}{4}\cos\left(\dfrac{\pi x}{4}\right) & \text{if } x\in[0,\ 2]\\ 0 & \text{otherwise}\end{cases}$$

Given that $\dfrac{d}{dx}\left(x\sin\left(\dfrac{\pi x}{4}\right)\right)=\dfrac{\pi x}{4}\cos\left(\dfrac{\pi x}{4}\right)+\sin\left(\dfrac{\pi x}{4}\right)$, find $E(X)$.

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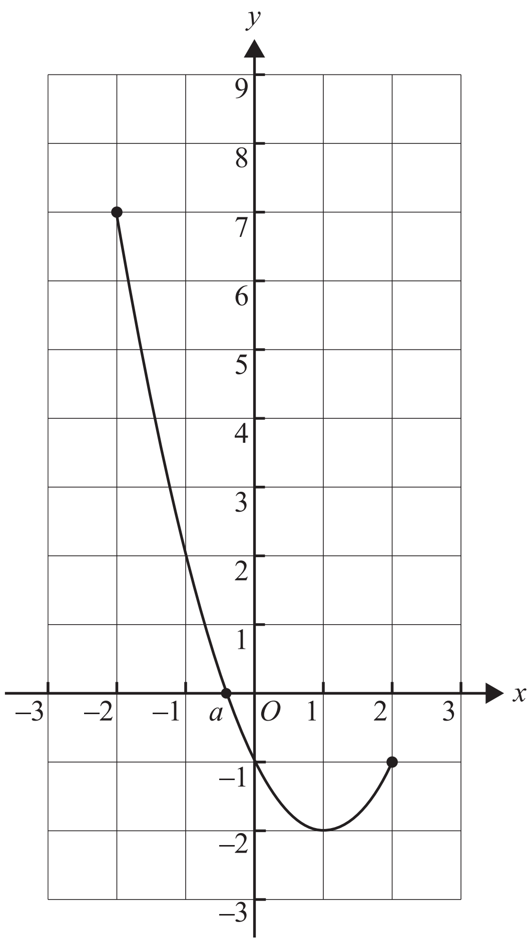
Question 9 (6 marks)

The graph of $f(x) = (x - 1)^2 - 2$, $x \in [-2, 2]$, is shown below. The graph intersects the x-axis where $x = a$.



a. Find the value of a. 1 mark

b. On the axes above, sketch the graph of $g(x) = |f(x)| + 1$, for $x \in [-2, 2]$. Label the end points with their coordinates. 2 marks

c. The following sequence of transformations is applied to the graph of the function $g: [-2, 2] \to R, g(x) = |f(x)| + 1$.

- a translation of one unit in the negative direction of the x-axis
- a translation of one unit in the negative direction of the y-axis
- a dilation from the x-axis of factor $\frac{1}{3}$

Find

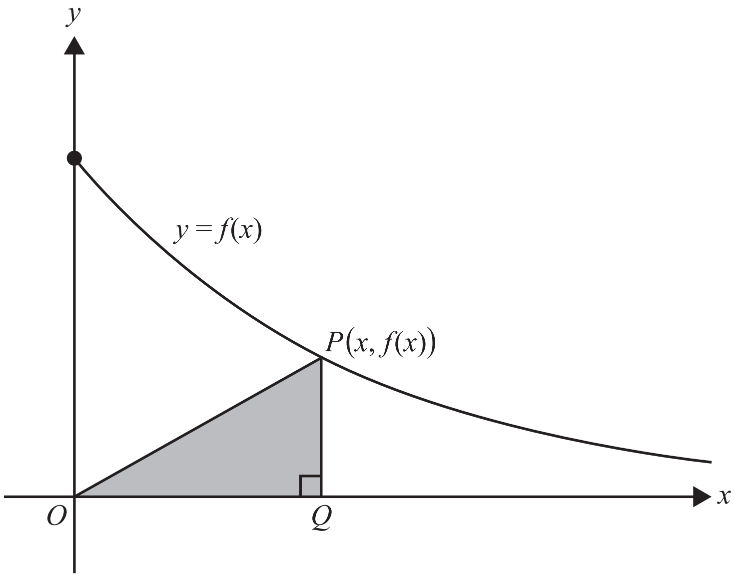
i. the rule of the image of g after the sequence of transformations has been applied 2 marks

ii. the domain of the image of g after the sequence of transformations has been applied. 1 mark

Question 10 (7 marks)

Let $f\colon [0, \infty) \to R$, $f(x) = 2e^{-\frac{x}{5}}$.

A right-angled triangle OQP has vertex O at the origin, vertex Q on the x-axis and vertex P on the graph of f, as shown. The coordinates of P are $(x, f(x))$.

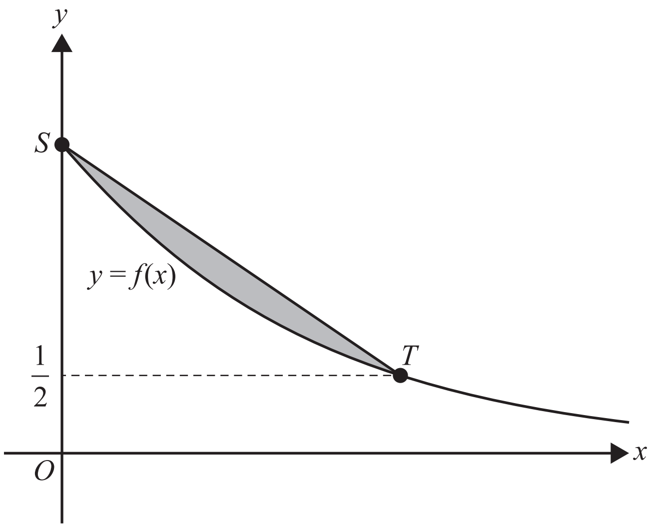


a. Find the area, A, of the triangle OQP in terms of x. 1 mark

b. Find the maximum area of triangle OQP and the value of x for which the maximum occurs. 3 marks

c. Let S be the point on the graph of f on the y-axis and let T be the point on the graph of f with the y-coordinate $\frac{1}{2}$.

Find the area of the region bounded by the graph of f and the line segment ST. 3 marks



END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

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Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad$ $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad$ $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ $\qquad$ $\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad$ $\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad$ $\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $\qquad$ quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ $\qquad$ approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$ $\qquad$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ $\qquad$ transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$ $\qquad$ variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET