



Victorian Certificate of Education 2009

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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MATHEMATICAL METHODS (CAS)

Written examination 1

Friday 6 November 2009

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Differentiate $x \log_e(x)$ with respect to x .

2 marks

- b. For $f(x) = \frac{\cos(x)}{2x+2}$ find $f'(\pi)$.

3 marks

TURN OVER

Question 2

- a. Find an anti-derivative of $\frac{1}{1-2x}$ with respect to x .

2 marks

- b. Evaluate $\int_1^4 (\sqrt{x} + 1) dx$.

3 marks

Question 3

- Let $f: R \setminus \{0\} \rightarrow R$ where $f(x) = \frac{3}{x} - 4$. Find f^{-1} , the inverse function of f .

3 marks

Question 4

Solve the equation $\tan(2x) = \sqrt{3}$ for $x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$.

3 marks

Question 5

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

- a. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?

1 mark

- b. What is the probability that the sum of the numbers on the two balls is 5?

1 mark

- c. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

2 marks

TURN OVER

Question 6

Oil is leaking at a constant rate to form a circular puddle on the floor. The oil is being added to the puddle at the rate of 10 mm^3 per minute causing the puddle to spread out evenly, with constant depth of 2 mm.

When the radius of the puddle is r mm, the volume, $V \text{ mm}^3$, of oil in the puddle is given by $V = 2\pi r^2$.

Find the rate of change of the radius of the puddle when the radius is 30 mm. Give an exact answer, with units of mm per minute.

3 marks

Question 7

The random variable X has this probability distribution.

X	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

Find

a. $\Pr(X > 1 | X \leq 3)$

2 marks

b. $\text{Var}(X)$, the variance of X .

3 marks

TURN OVER

Question 8

Let $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = e^x + k$, where k is a real number. The tangent to the graph of f at the point where $x = a$ passes through the point $(0, 0)$. Find the value of k in terms of a .

3 marks

Question 9

Solve the equation $2 \log_e(x) - \log_e(x+3) = \log_e\left(\frac{1}{2}\right)$ for x .

4 marks

Question 10

- a. Use the relationship $f(x+h) \approx f(x) + hf'(x)$ for a small positive value of h , to find an approximate value for $\sqrt[3]{8.06}$.

4 marks

- b. Explain why this approximate value is greater than the exact value for $\sqrt[3]{8.06}$.

1 mark

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

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Mathematical Methods and Mathematical Methods (CAS)

Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$