VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2008

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures									
Words									

SPECIALIST MATHEMATICS

Written examination 2

Tuesday 11 November 2008

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 24 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

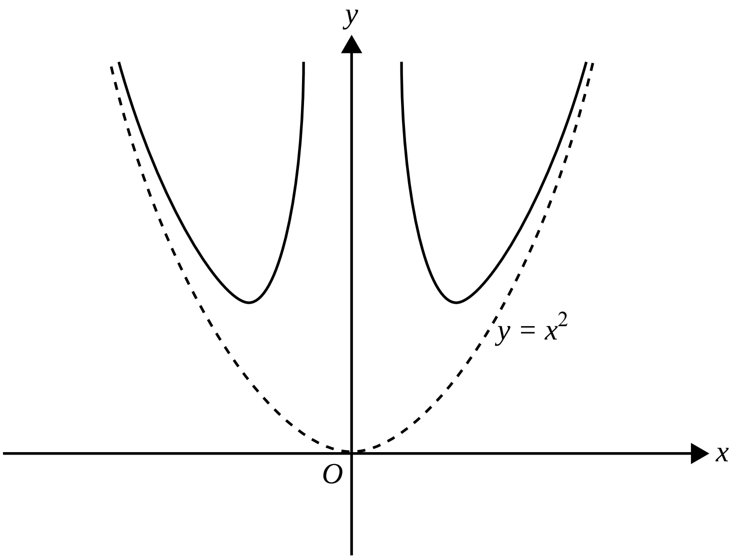
Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1



A possible equation for the graph of the curve shown above is

A. $y = \dfrac{x^3 + a}{x}, \quad a > 0$

B. $y = \dfrac{x^3 + a}{x}, \quad a < 0$

C. $y = \dfrac{2x^4 + a}{x^2}, \quad a > 0$

D. $y = \dfrac{x^4 + a}{x^2}, \quad a > 0$

E. $y = \dfrac{x^4 + a}{x^2}, \quad a < 0$

Question 2

The equation $x^2 + ax + y^2 + 1 = 0$, where a is a real constant, will represent a circle if

A. $a < -2$ only

B. $a > -2$ only

C. $a = \pm 2$ only

D. $-2 < a < 2$

E. $a < -2$ or $a > 2$

Question 3

The maximal domain and range of the function with rule $f(x) = 3\sin^{-1}(4x-1) + \frac{\pi}{2}$ are respectively

A. $[-\pi, 2\pi]$ and $\left[0, \frac{1}{2}\right]$

B. $\left[0, \frac{1}{2}\right]$ and $[-\pi, 2\pi]$

C. $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$ and $\left[-\frac{1}{2}, 0\right]$

D. $\left[0, \frac{1}{2}\right]$ and $[0, 3\pi]$

E. $\left[-\frac{1}{2}, 0\right]$ and $[-\pi, 2\pi]$

Question 4

P is any point on the hyperbola with equation $x^2 - \frac{y^2}{4} = 1$.

If m is the gradient of the hyperbola at P, then m could be

A. any real number.

B. any real number in the interval $(-2, 2)$

C. any real number in the interval $[-2, 2]$

D. any real number in the interval $R\backslash(-2, 2)$

E. any real number in the interval $R\backslash[-2, 2]$

Question 5

For a certain complex number z where $\text{Arg}(z) = \frac{\pi}{5}$, $\text{Arg}(z^7)$ is

A. $-\frac{7\pi}{5}$

B. $-\frac{3\pi}{5}$

C. $\frac{2\pi}{5}$

D. $\frac{3\pi}{5}$

E. $\frac{7\pi}{5}$

Question 6

If $z = \frac{3+4i}{1+2i}$, the imaginary part of z is

A. -2

B. $-\frac{2}{5}i$

C. $-\frac{2}{5}$

D. $-2i$

E. 2

Question 7

The relation $(z+2)\ (\bar{z}+2) = 4$, when graphed on an argand diagram, would be

A. a circle of radius 4 with centre at (–2, 0)

B. a circle of radius 2 with centre at (2, 0)

C. a circle of radius 4 with centre (2, 0)

D. a circle of radius 2 with centre at (–2, 0)

E. a circle of radius 2 with centre at (0, –2)

Question 8

In polar form, the complex number $i - 1$ is

A. $\sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right)$

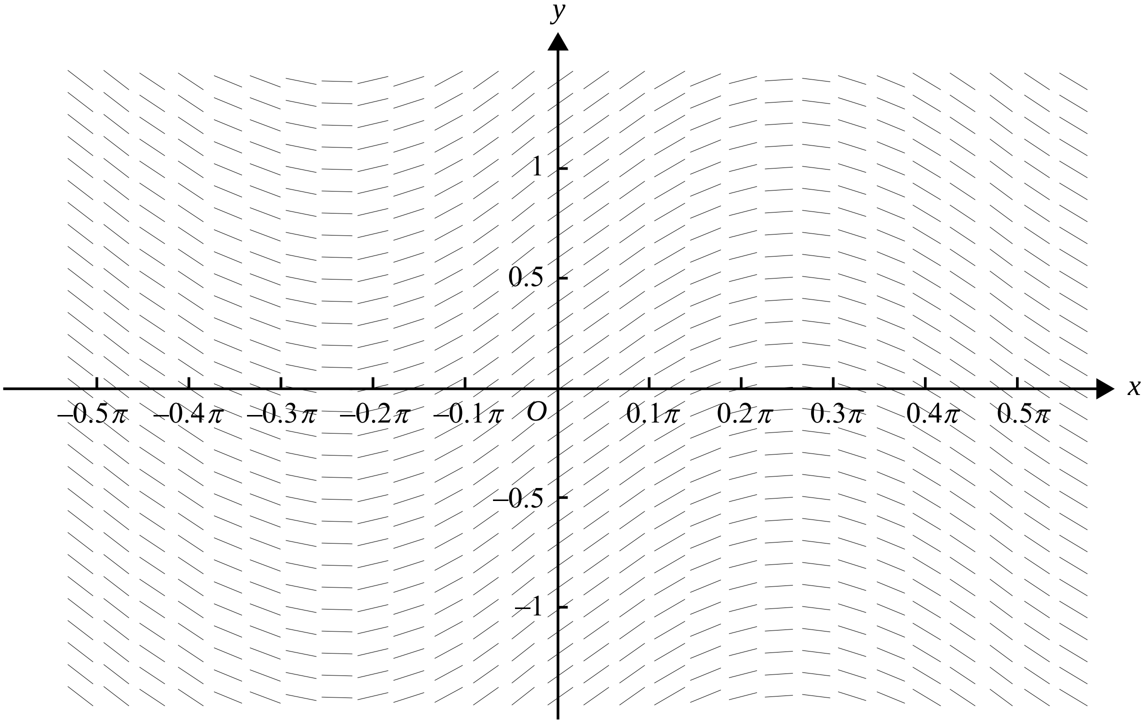
B. $2\text{cis}\left(\frac{3\pi}{4}\right)$

C. $\text{cis}\left(-\frac{5\pi}{4}\right)$

D. $\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$

E. $2\text{cis}\left(-\frac{5\pi}{4}\right)$

Question 9



The direction (slope) field for a certain first order differential equation is shown above.
The differential equation could be

A. $\frac{dy}{dx} = \sin(2x)$

B. $\frac{dy}{dx} = \cos(2x)$

C. $\frac{dy}{dx} = \cos\left(\frac{1}{2}y\right)$

D. $\frac{dy}{dx} = \sin\left(\frac{1}{2}y\right)$

E. $\frac{dy}{dx} = \cos\left(\frac{1}{2}x\right)$

Question 10

The volume of water V m^3 in a cylindrical tank when it is filled to a depth of h metres is given by $V = 4h$.
Water flows into the tank at a rate of 0.2 m^3 per minute and leaks out at a rate of $0.01\sqrt{h}$ m^3 per minute.
The differential equation, which when solved would enable h to be expressed in terms of t, is

A. $\frac{dh}{dt} = 0.2 - 0.01\sqrt{h}$

B. $\frac{dh}{dt} = 4\left(0.2 - 0.01\sqrt{h}\right)$

C. $\frac{dh}{dt} = \frac{20 - \sqrt{h}}{400}$

D. $\frac{dh}{dt} = \frac{400}{20 - \sqrt{h}}$

E. $\frac{dh}{dt} = 20 - \frac{400}{\sqrt{h}}$

Question 11

When Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = 2\tan^{-1}(x+1)$ with $x_0 = 0$ and $y_0 = 1$, the value of y_2 would be given by

A. $1 + 0.1\pi$

B. $1 + 0.4\tan^{-1}(1.2)$

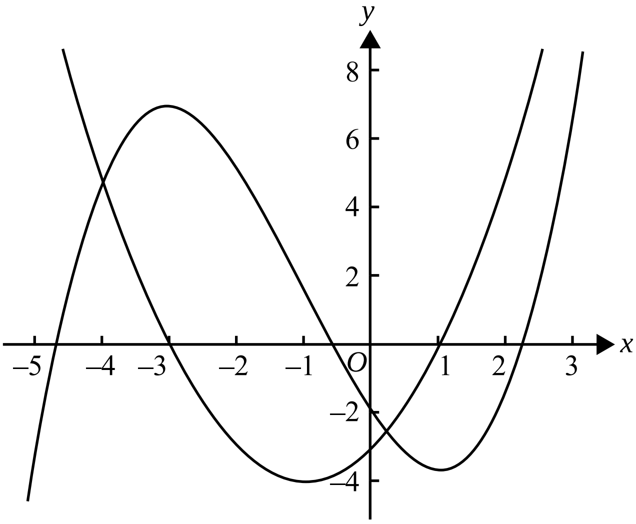
C. $0.1\pi + 0.4\tan^{-1}(1.2)$

D. $1 + 0.1\pi + 0.4\tan^{-1}(1.2)$

E. $1 + 0.4\tan^{-1}(1.2) + 0.4\tan^{-1}(1.4)$

Question 12

The graph of a function f together with the graph of one of its antiderivative functions is shown below.



The value of $\int_{-3}^{0} f(x)\,dx$ is closest to

A. –9

B. –7

C. –5

D. 5

E. 9

Question 13

A cricket ball is hit from an origin at ground level so that its position vector at time t is given by $\underset{\sim}{r}(t) = 15t\underset{\sim}{i} + \left(20t - 5t^2\right)\underset{\sim}{j}$ for $t \geq 0$, where $\underset{\sim}{i}$ is a unit vector in the forward direction and $\underset{\sim}{j}$ is a unit vector vertically up.

When the cricket ball reaches its maximum height, its position vector is

A. $\underset{\sim}{r} = 20\underset{\sim}{i} + 30\underset{\sim}{j}$

B. $\underset{\sim}{r} = 15\underset{\sim}{i} + 20\underset{\sim}{j}$

C. $\underset{\sim}{r} = 60\underset{\sim}{i}$

D. $\underset{\sim}{r} = 30\underset{\sim}{i} + 10\underset{\sim}{j}$

E. $\underset{\sim}{r} = 30\underset{\sim}{i} + 20\underset{\sim}{j}$

Question 14

If the vectors $\underset{\sim}{a} = m\underset{\sim}{i} + 4\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{b} = m\underset{\sim}{i} + m\underset{\sim}{j} - 4\underset{\sim}{k}$ are perpendicular, then

A. $m = 0$

B. $m = -6$ or $m = 2$

C. $m = -2$ or $m = 6$

D. $m = -2$ or $m = 0$

E. $m = -1$ or $m = 1$

Question 15

Two forces $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ act on a body. $\underset{\sim}{P}$ acts in the direction of $\underset{\sim}{i}$ with magnitude one newton and $\underset{\sim}{Q}$ acts in the direction of $\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$ with magnitude four newtons.

The magnitude of the total force acting on the body, in newtons, is

A. 3

B. $\sqrt{15}$

C. $\sqrt{17}$

D. $\sqrt{21}$

E. 5

Question 16

Using a suitable substitution $\int_0^{\sqrt{3}} \frac{\log_e(\arctan(x))}{1+x^2}\,dx$ can be expressed completely in terms of u as

A. $\int_0^{\sqrt{3}} \log_e(u)\,du$

B. $\int_0^{\frac{\pi}{6}} \frac{\log_e(u)}{1+\tan^2(u)}\,du$

C. $\int_0^{\frac{\pi}{3}} \log_e(u)\,du$

D. $\int_0^{\frac{\pi}{6}} \log_e(u)\,du$

E. $\int_0^{\frac{\pi}{3}} \frac{\log_e(u)}{1+\tan^2(u)}\,du$

Question 17

P, Q and R are three collinear points with position vectors $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ respectively, where Q lies between P and R.

If $|\overrightarrow{QR}| = \frac{1}{2}|\overrightarrow{PQ}|$, then $\underset{\sim}{r}$ is equal to

A. $\frac{3}{2}\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}$

B. $\frac{3}{2}\underset{\sim}{p} - \frac{1}{2}\underset{\sim}{q}$

C. $\frac{3}{2}\underset{\sim}{q} - \frac{3}{2}\underset{\sim}{p}$

D. $\frac{1}{2}\underset{\sim}{p} - \frac{3}{2}\underset{\sim}{q}$

E. $\frac{3}{2}\underset{\sim}{p} - \frac{3}{2}\underset{\sim}{q}$

Question 18

A force $\underset{\sim}{F}$ is applied to a body causing it to accelerate in the direction of vector $\underset{\sim}{d}$.

The magnitude of the force which causes the body to accelerate in this direction is given by

A. $\underset{\sim}{F}.\underset{\sim}{d}$

B. $\dfrac{\underset{\sim}{d}}{\underset{\sim}{F}}$

C. $\dfrac{\underset{\sim}{F}.\underset{\sim}{d}}{|\underset{\sim}{F}|}$

D. $\dfrac{\underset{\sim}{F}}{\underset{\sim}{d}}$

E. $\dfrac{\underset{\sim}{F}.\underset{\sim}{d}}{|\underset{\sim}{d}|}$

Question 19

The momentum of a 5 kg mass, which is travelling in a straight line, has magnitude 30 kg ms^{-1}. Six seconds later the magnitude of the momentum of the mass is 40 kg ms^{-1}.

Assuming that the mass is accelerating at a constant rate, the distance covered by the mass over the 6 seconds is

A. 28 m

B. 10 m

C. 42 m

D. 45 m

E. 12 m

Question 20

The velocity v ms^{-1} of a body which is moving in a straight line, when it is x m from the origin, is given by $v = \sin^{-1}(x)$.

The acceleration of the body in ms^{-2} is given by

A. $-\cos^{-1}(x)$

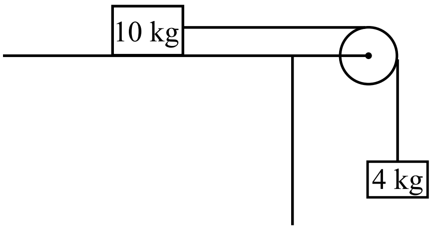
B. $-\dfrac{\cos(x)}{\sin^2(x)}$

C. $-\cot(x)\text{cosec}^2(x)$

D. $\dfrac{1}{\sqrt{1-x^2}\,\sin(x)}$

E. $\dfrac{\sin^{-1}(x)}{\sqrt{1-x^2}}$

Question 21



A 10 kg mass on a rough horizontal table is connected to a 4 kg mass by a light inextensible string which remains horizontal until it passes over a smooth pulley. The 10 kg mass moves along the table while the 4 kg mass falls toward the ground. Given that the coefficient of friction between the 10 kg mass and the table is 0.1, the acceleration of the 10 kg mass in ms^{-2} is

A. $\frac{3g}{14}$

B. $\frac{3g}{10}$

C. $\frac{3}{14}$

D. $\frac{3}{10}$

E. $\frac{5g}{14}$

Question 22

A body moves in a straight line so that at time t its velocity is v and its acceleration is a where $a = f(v)$.
Given that $v = v_0$ when $t = t_0$, and $v = v_1$ when $t = t_1$, it follows that

A. $v_1 = \int_{t_0}^{t_1} f(v)\,dv + v_0$

B. $t_1 = \int_{v_0}^{v_1} \frac{1}{f(v)}\,dv + t_0$

C. $t_1 = \int_{v_0}^{v_1} \left(\frac{1}{f(v)} + t_0\right) dv$

D. $t_1 = \int_{v_0}^{v_1} f(v)\,dv + t_0$

E. $v_1 = \int_{t_0}^{t_1} \frac{1}{f(v)}\,dv + v_0$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The function $f:[0,\infty) \to R$ where $f(x) = \dfrac{6x\sqrt{x}}{3x^2+1}$ has first and second derivatives with rules given by

$$f'(x) = \frac{9\sqrt{x}(1-x^2)}{(3x^2+1)^2} \text{ and } f''(x) = \frac{9(9x^4-26x^2+1)}{2\sqrt{x}(3x^2+1)^3}.$$

a. Find the coordinates of the maximum turning point of the graph of f and use an appropriate test to verify its nature.

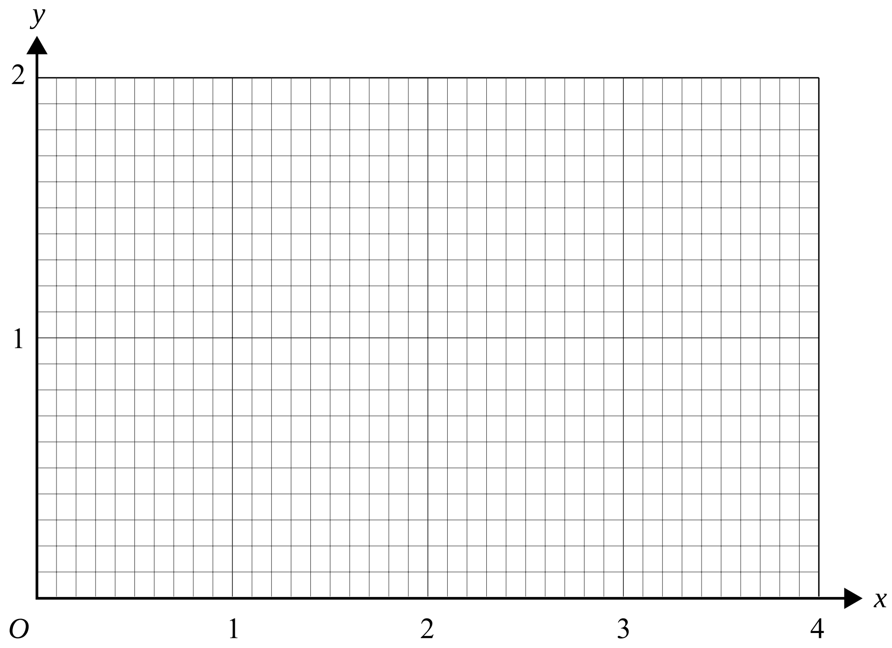
2 marks

b. **i.** Write down a polynomial equation, which when solved will give the x coordinates of the points of inflection of the graph of f.

ii. Find the coordinates of the two points of inflection of the graph of f. Give your answers correct to one decimal place.

1 + 2 = 3 marks

c. Sketch the graph of f on the axes below, clearly indicating the location of any intercepts with the axes, the maximum turning point and the two points of inflection.



2 marks

The graph of f is rotated about the x-axis between $x = 0$ and $x = \frac{1}{\sqrt{3}}$ to form a solid of revolution with volume V.

d. **i.** Show that $V = 2\pi \int_0^{\frac{1}{\sqrt{3}}} \frac{18x^3}{\left(3x^2+1\right)^2}\, dx$.

ii. Use the substitution $u = 3x^2 + 1$ to express V in the form

$$2\pi \int_a^b \left(\frac{c}{u} + \frac{d}{u^2} \right) du.$$

iii. **Hence**, by using an appropriate antiderivative, find V in exact form.

1 + 2 + 2 = 5 marks

Total 12 marks

Question 2

A 'parasailing' water-skier (a water-skier with a parachute attached) of mass 80 kg is towed by a boat in a straight line from rest. The boat exerts a constant force of 390 N acting horizontally on the skier. At this stage the resistance to the motion of the skier is a constant 30 N which acts horizontally.

a. Show that the acceleration of the skier is 4.5 m/s^2.

1 mark

b. Show that the speed of the skier, having been towed a distance of 16 m, is 12 m/s.

1 mark

After the skier has been towed 16 m across the water, the drag of the parachute becomes significant. The drag of the parachute produces an **additional** resistance of $6v$ N to the horizontal motion of the skier, where v m/s is the velocity of the skier.

c. If a m/s^2 is the acceleration of the skier, write down the equation of motion of the skier and hence express a in terms of v.

2 marks

d. Find the time required for the skier to reach a speed of 18 m/s from a speed of 12 m/s. Give your answer in seconds correct to one decimal place.

2 marks

After some time, the parasailing skier is being towed horizontally at a constant speed and at a fixed distance above the water. The tow rope from the boat makes an angle of 30° to the horizontal, and the parachute cord makes an angle of θ to the horizontal.

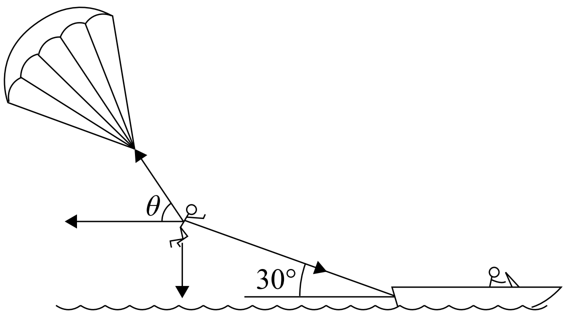
e. **i.** On the diagram below, label the following forces which are now acting on the parasailing water-skier.

The weight force 80g newtons of the skier.

The tension T newtons in the parachute cord.

A horizontal resistance force of 100 newtons.

The force of 390 newtons exerted by the tow rope.



ii. By resolving forces in the horizontal and vertical directions, write down a pair of equations which would enable θ and T to be found.

iii. By expressing sin (θ) and cos (θ) in terms of T, find the value of tan (θ) correct to three decimal places.

iv. Find the value of T correct to the nearest integer.

1 + 2 + 2 + 1 = 6 marks

Total 12 marks

Question 3

The position vector $\underset{\sim}{r}(t)$ of the front of a toy train at time t seconds on a closed track is given by

$$\underset{\sim}{r}(t) = \sin\left(\frac{t}{3}\right)\underset{\sim}{i} + \frac{1}{2}\sin\left(\frac{2t}{3}\right)\underset{\sim}{j},\ t \geq 0$$

where displacement components are measured in metres.

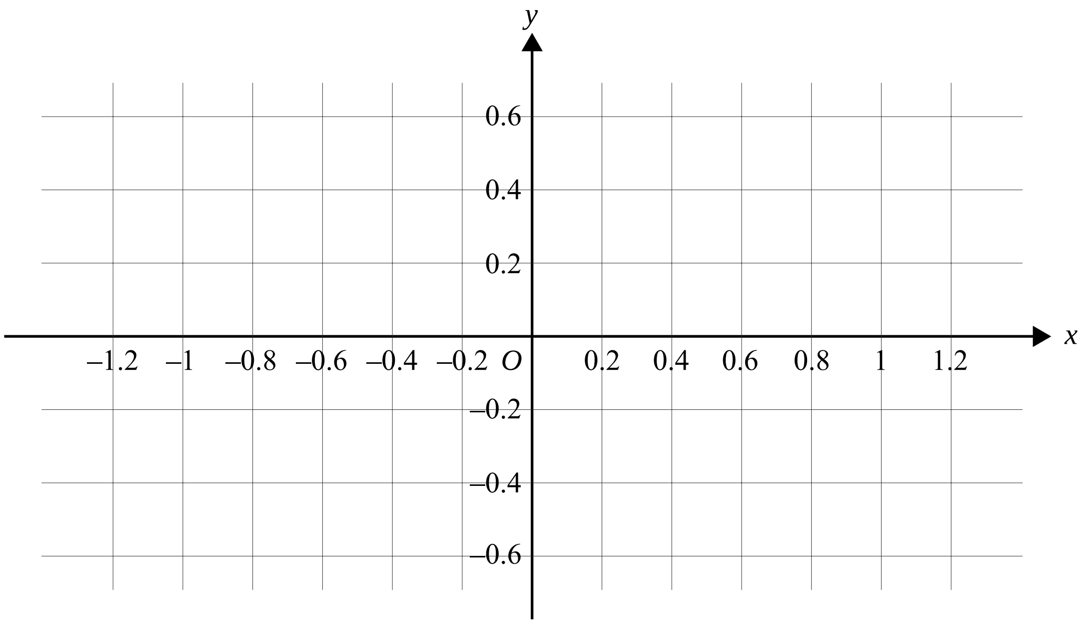
a. **i.** If the front of the train is at the point $P(x, y)$ at time t, show that

$$y^2 = \sin^2\left(\frac{t}{3}\right)\cos^2\left(\frac{t}{3}\right).$$

ii. **Hence** find the cartesian equation of the path of the train.

1 + 1 = 2 marks

b. Sketch the path of the train on the axes below.



2 marks

c. Find the exact time, in seconds, that it takes the train to complete one circuit of the track.

2 marks

d. Find the exact speed, in m/s, of the train as the front of the train passes through the origin.

2 marks

e. The distance travelled by the train between times $t = t_0$ and $t = t_1$ is given by

$$\int_{t_0}^{t_1} \left| \underset{\sim}{v}(t) \right| dt$$

where $\left| \underset{\sim}{v}(t) \right|$ is the speed of the train at time t.

i. Write down a definite integral, involving only scalar quantities, which gives the distance travelled by the train when it completes exactly one circuit of the track.

ii. Find the distance in metres, for one circuit of the track, correct to one decimal place.

1 + 1 = 2 marks

Total 10 marks

Question 4

An island has a population of rabbits and a population of foxes. The foxes eat rabbits as their food source and if the rabbit population decreases, then after some time, so will the fox population. Also, if the rabbit population increases, then after some time, so too will the fox population.

At time t months from the start of the year there are x thousand rabbits and y hundred foxes. A model for the two populations is given by the parametric equations

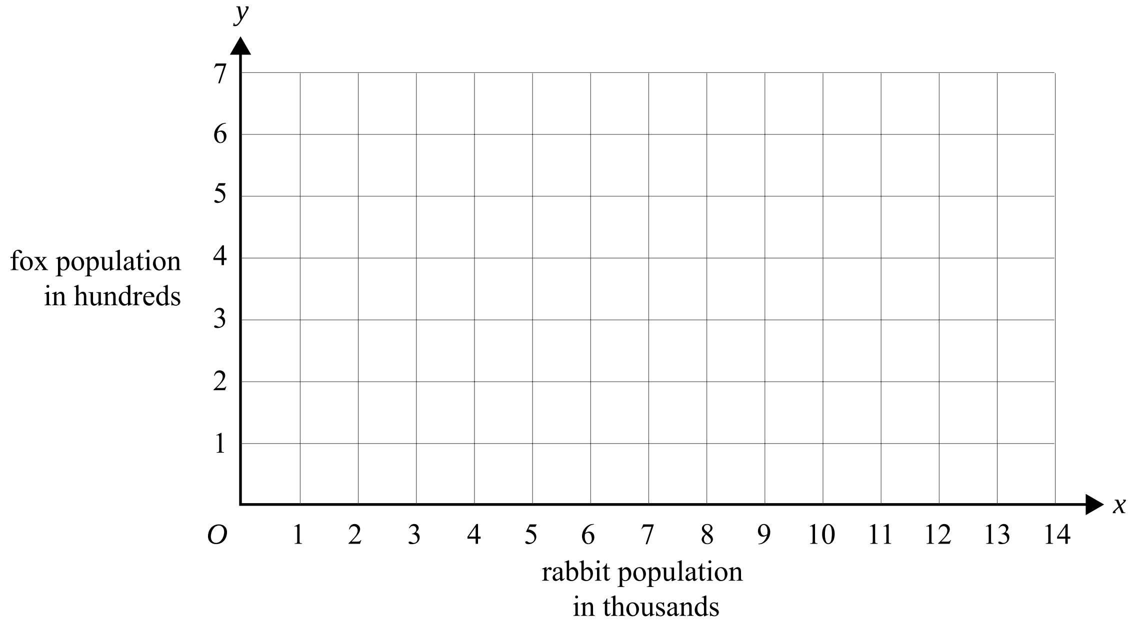
Rabbits $\quad x = 10 + 3\cos\left(\frac{\pi t}{6}\right), t \geq 0$

Foxes $\quad y = 5 + \sin\left(\frac{\pi t}{6}\right), t \geq 0.$

a. Find the cartesian equation relating x and y according to this model.

2 marks

b. Sketch the relationship between x and y on the axes below.



2 marks

c. **i.** After how many months from the start of the year is the population of rabbits, x thousand, a minimum?

ii. How many foxes are on the island at this time?

1 + 1 = 2 marks

An alternative model for the interaction of the two populations, which more accurately allows for the dependency of the foxes on the rabbits as a food source, is given by the pair of differential equations

Rabbits $\frac{dx}{dt} = 0.5x - 0.1xy,\ t \geq 0$

Foxes $\frac{dy}{dt} = -0.2y + 0.02xy,\ t \geq 0$

d. **i.** Show that $\frac{dy}{dx} = \frac{xy - 10y}{25x - 5xy}$.

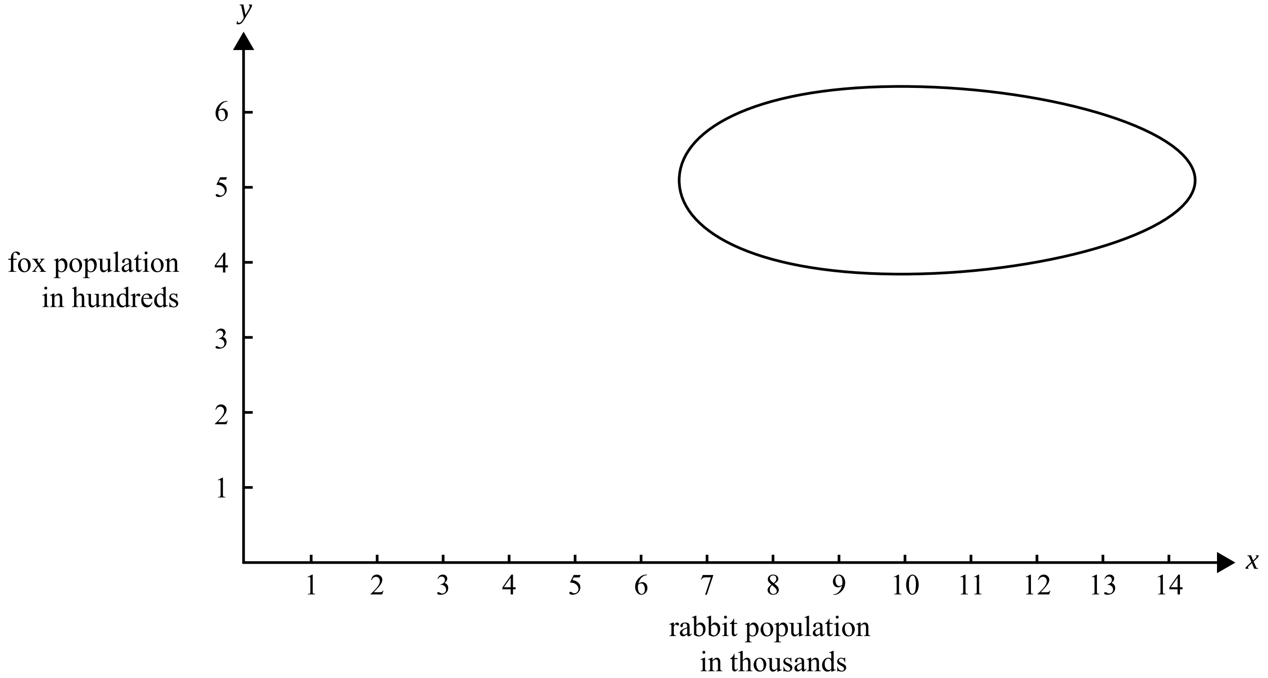
ii. Use calculus to verify that the curve with equation

$$25\log_e(y) - 5y - x + 10\log_e(x) = c$$

where c is a constant of integration, satisfies the differential equation given in **part d. i.**

1 + 2 = 3 marks

For certain populations of foxes and rabbits $c = 27.5$. The graph of the solution curve from **part d. ii.** for this value of c is shown below.



e. Determine the **minimum** and **maximum** numbers of rabbits possible according to the alternative model, correct to the nearest ten rabbits.

3 marks

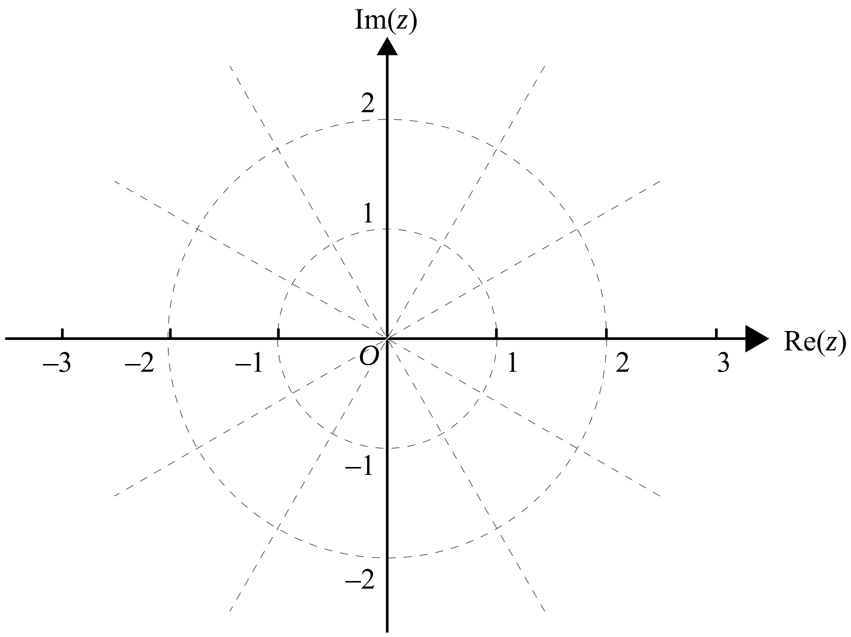
Total 12 marks

Question 5

a. Verify that $\frac{\sqrt{3}}{2}+\frac{1}{2}i$ is one root of the equation $z^3 = i$.

1 mark

b. Plot the three roots of i on the argand diagram below.



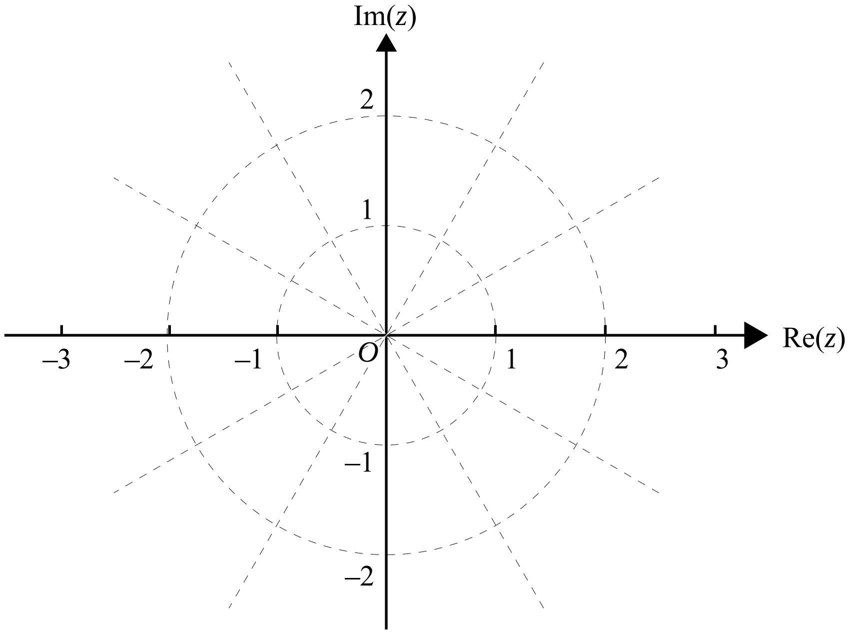
2 marks

c. Find the points of intersection of the curves given by

$$|z-i|=1 \text{ and } \mathrm{Re}(z)=-\frac{1}{\sqrt{3}}\mathrm{Im}(z).$$

3 marks

d. Sketch the curves given by the relations $|z-i|=1$ and $\text{Re}(z) = -\frac{1}{\sqrt{3}}\text{Im}(z)$ on the argand diagram below.



2 marks

e. On the argand diagram above shade the region given by

$$\{z:|z-i|\le 1\}\cap\left\{z:0\le \text{Arg}(z)\le \frac{2\pi}{3}\right\}.$$

2 marks

f. Find the area of the shaded region in **part e.**, correct to two decimal places.

2 marks

Total 12 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax} dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x} dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET