



Victorian Certificate of Education 2007

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STUDENT NUMBER

Letter

Figures

Words

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SPECIALIST MATHEMATICS

Written examination 2

Friday 16 November 2007

Reading time: 3.00 pm to 3.15 pm (15 minutes)

Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 24 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1**Instructions for Section I**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

A hyperbola has equation $\frac{(x-2)^2}{a^2} - \frac{4(y+3)^2}{a^2} = 1$, where a is a non-zero real constant. The product of the gradients of the asymptotes is

- A. -4
- B. $-\frac{1}{4}$
- C. $-\frac{1}{16}$
- D. $\frac{1}{2}$
- E. 0

Question 2

An ellipse has a horizontal semi-axis length of 3 and a vertical semi-axis length of 2.

Given that the centre of the ellipse has coordinates $(1, 3)$, a possible parametric form for the ellipse is

- A. $x = 1 + 9\cos(t)$ and $y = 3 + 4\sin(t)$
- B. $x = 3 + 2\cos(t)$ and $y = 1 + 3\sin(t)$
- C. $x = -1 + 3\cos(t)$ and $y = -3 + 2\cos(t)$
- D. $x = -1 + 3\sin(t)$ and $y = 3 + 2\cos(t)$
- E. $x = 1 + 3\cos(t)$ and $y = 3 + 2\sin(t)$

Question 3

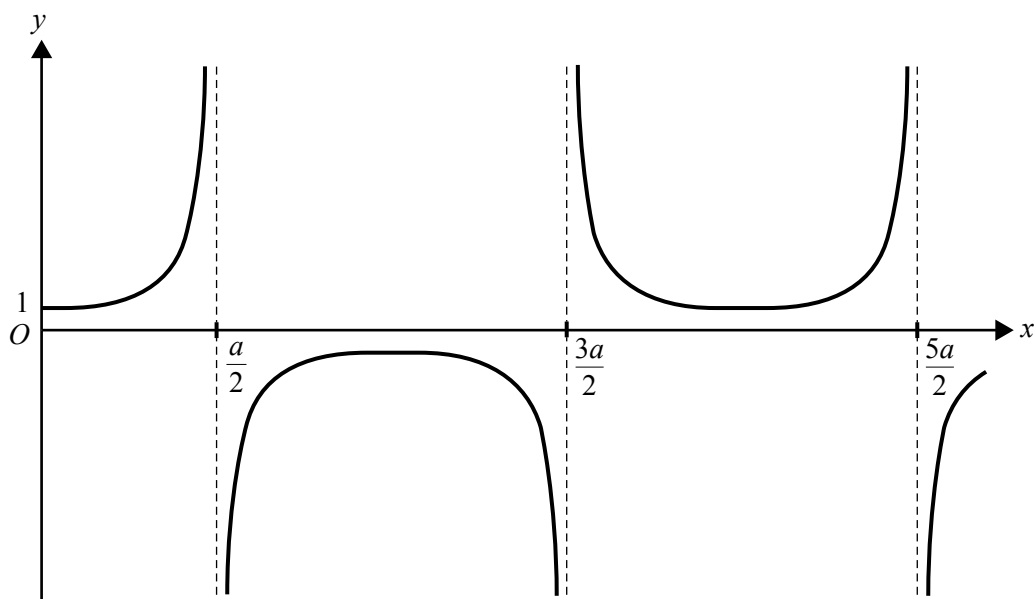
The range of the function f with rule $f(x) = a + b \arctan(x - c)$ where a , b and c are positive real constants is

- A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- B. $\left(a - \frac{c\pi}{2}, a + \frac{c\pi}{2}\right)$
- C. $\left(c - \frac{b\pi}{2}, c + \frac{b\pi}{2}\right)$
- D. $\left(a - \frac{b\pi}{2}, a + \frac{b\pi}{2}\right)$
- E. $\left(a - \frac{(b-c)\pi}{2}, a + \frac{(b-c)\pi}{2}\right)$

Question 4

Which one of the following is **not** true?

- A. $\sec^2(4\theta) - 1 = \tan^2(4\theta)$
- B. $\cot^2(4\theta) - \operatorname{cosec}^2(4\theta) = 1$
- C. $\cos(4\theta) = \cos^2(2\theta) - \sin^2(2\theta)$
- D. $\sin(4\theta) = 2\sin(2\theta) \cos(2\theta)$
- E. $\cos(4\theta) + 4\sin^2(\theta) \cos^2(\theta) = \cos^2(2\theta)$

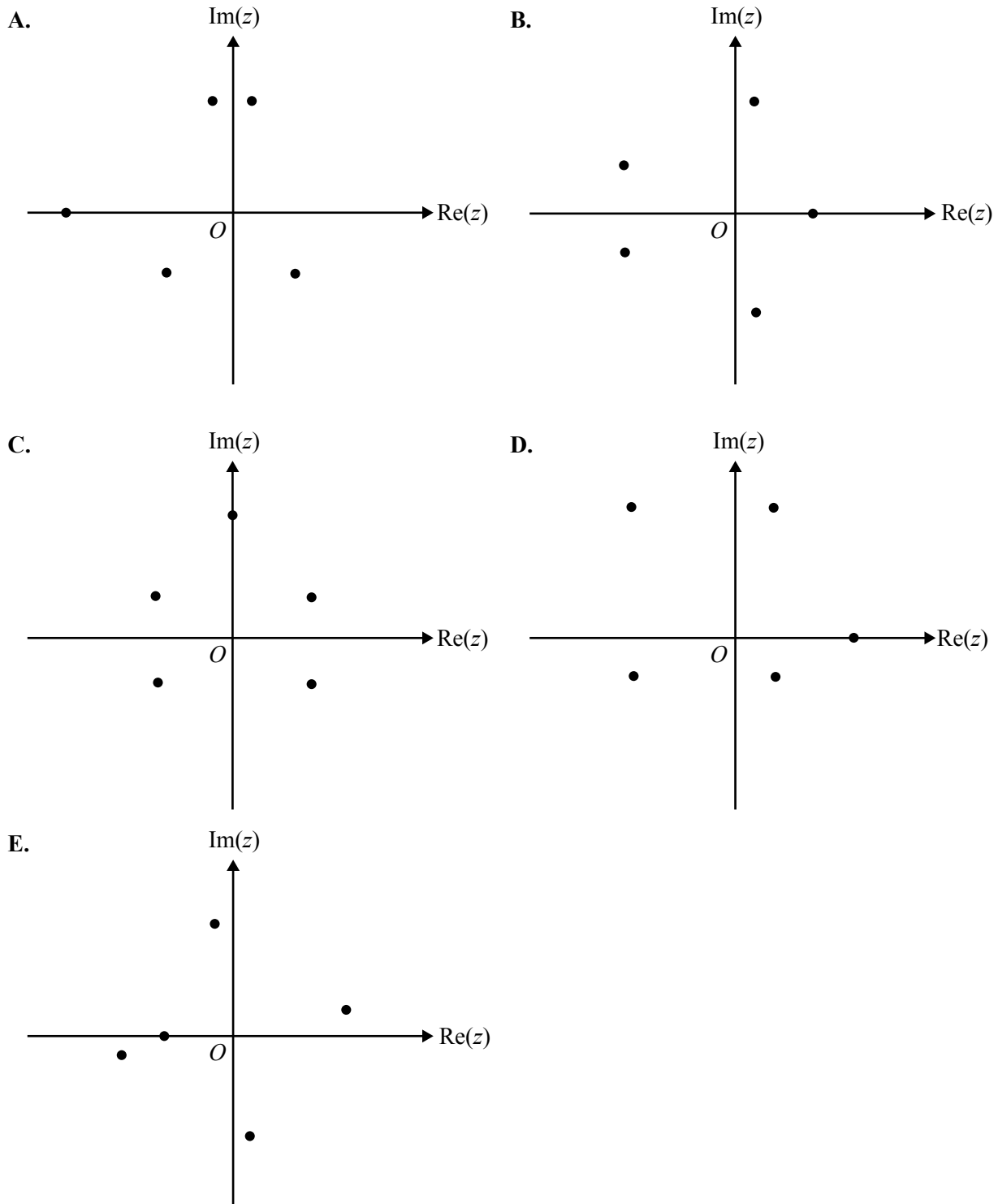
Question 5

A rule for the function whose graph is shown above, where a is a positive constant, could be

- A. $y = \operatorname{cosec}\left(\frac{2\pi}{a}\left(x - \frac{a}{2}\right)\right)$
- B. $y = \operatorname{cosec}\left(\frac{2\pi}{a}\left(x + \frac{a}{4}\right)\right)$
- C. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x - \frac{a}{2}\right)\right)$
- D. $y = \operatorname{cosec}\left(\frac{\pi}{a}x + \frac{a}{2}\right)$
- E. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x + \frac{a}{2}\right)\right)$

Question 6

Which one of the following diagrams could represent the location of the roots of $z^5 + z^2 - z + c = 0$ in the complex plane, where $c \in \mathbb{R}$?



Question 7

If $z = 3 - 4i$, then $\frac{1}{1-z}$ is equal to

- A. $\frac{-1-2i}{10}$
- B. $\frac{-1+2i}{10}$
- C. $\frac{-1-i}{6}$
- D. $\frac{-1+i}{6}$
- E. $\frac{1-i}{6}$

Question 8

If $z = 2 \operatorname{cis}\left(-\frac{\pi}{3}\right)$, then z^4 is equal to

- A. $8 \operatorname{cis}\left(-\frac{4\pi}{3}\right)$
- B. $8 \operatorname{cis}\left(-\frac{\pi}{12}\right)$
- C. $16 \operatorname{cis}\left(\frac{2\pi}{3}\right)$
- D. $8 \operatorname{cis}\left(\frac{2\pi}{3}\right)$
- E. $-16 \operatorname{cis}\left(-\frac{4\pi}{3}\right)$

Question 9

The algebraic fraction $\frac{x}{3(x+c)^2}$, where c is a non-zero real number, can be written in partial fraction form, where A and B are real numbers, as

- A. $\frac{A}{x+c} + \frac{B}{x+c}$
- B. $\frac{A}{3x+c} + \frac{B}{(x+c)^2}$
- C. $\frac{A}{3x+c} + \frac{B}{x+c}$
- D. $\frac{A}{x+c} + \frac{B}{(x+c)^2}$
- E. $\frac{A}{3(x+c)} + \frac{B}{x+c}$

Question 10

The curve given by $y = \sin^{-1}(2x)$, where $0 \leq x \leq \frac{1}{2}$, is rotated about the y -axis to form a solid of revolution.

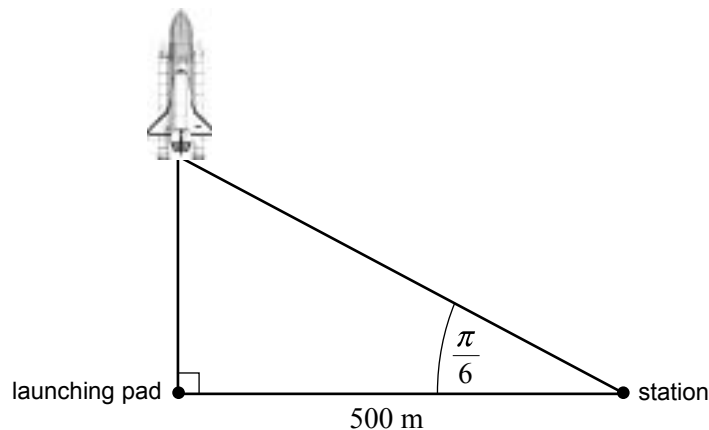
The volume of the solid may be found by evaluating

- A. $\frac{\pi}{4} \int_0^{\frac{\pi}{2}} (1 - \cos(2y)) dy$
- B. $\frac{\pi}{8} \int_0^{\frac{1}{2}} (1 - \cos(2y)) dy$
- C. $\frac{\pi}{8} \int_0^{\frac{\pi}{2}} (1 - \cos(2y)) dy$
- D. $\frac{1}{8} \int_0^{\frac{\pi}{2}} (1 - \cos(2y)) dy$
- E. $\frac{\pi}{8} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos(2y)) dy$

Question 11

If $\frac{dy}{dx} = \sqrt{\sin(x)}$ and $y = 1$ when $x = 0$, then the value of y when $x = \frac{\pi}{3}$ can be found by evaluating

- A. $\int_0^{\frac{\pi}{3}} \sqrt{\sin(t)} dt$
- B. $\int_0^{\frac{\pi}{3}} \sqrt{\sin(t)} dt - 1$
- C. $\int_0^{\frac{\pi}{3}} (\sqrt{\sin(t)} + 1) dt$
- D. $\int_0^{\frac{\pi}{3}} \sqrt{\sin(t)} dt + 1$
- E. $\int_0^{\frac{\pi}{3}} (\sqrt{\sin(t)} - 1) dt$

Question 12

An ascending space shuttle rises vertically from a launching pad. As it rises the shuttle is tracked from a station at ground level 500 metres away. When the angle of elevation of the shuttle is $\frac{\pi}{6}$ radians from the horizontal direction, and is increasing at a rate of 0.5 radians per second, the speed of the shuttle is closest to

- A. 333 ms^{-1}
- B. 144 ms^{-1}
- C. 289 ms^{-1}
- D. 577 ms^{-1}
- E. 500 ms^{-1}

Question 13

Using a suitable substitution, $\int_1^{e^3} \left(\frac{(\log_e(x))^3}{x} \right) dx$ may be expressed completely in terms of u as

- A. $\int_0^3 \left(\frac{u^3}{e^u} \right) du$
- B. $\int_0^{e^3} u^3 du$
- C. $\int_0^{\log_e(3)} u^3 du$
- D. $\int_1^{\log_e(3)} u^3 du$
- E. $\int_0^3 u^3 du$

Question 14

The rate at which a type of bird flu spreads throughout a population of 1000 birds in a certain area is proportional to the product of the number N of infected birds and the number of birds still **not** infected after t days. Initially two birds in the population are found to be infected.

A differential equation, the solution of which models the number of infected birds after t days, is

- A. $\frac{dN}{dt} = k \frac{(1000 - N)}{1000}$
- B. $\frac{dN}{dt} = k(N - 2)(1000 - N)$
- C. $\frac{dN}{dt} = kN(1000 - N)$
- D. $\frac{dN}{dt} = kN(1000 - (N + 2))$
- E. $\frac{dN}{dt} = k(N + 2)(1000 - N)$

Question 15

In the cartesian plane, a vector perpendicular to the line $3x + 2y + 1 = 0$ is

- A. $3\mathbf{i} + 2\mathbf{j}$
- B. $-\frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j}$
- C. $2\mathbf{i} - 3\mathbf{j}$
- D. $\frac{1}{2}\mathbf{i} - \frac{1}{3}\mathbf{j}$
- E. $2\mathbf{i} + 3\mathbf{j}$

Question 16

A force of magnitude 18 newtons acts on a body at an angle of 150° in the anticlockwise direction to the vector $\hat{i}$.

A vector representation of this force could be

- A. $9\sqrt{3}\hat{i} + 9\hat{j}$
- B. $-9\hat{i} + 9\sqrt{3}\hat{j}$
- C. $-9\sqrt{3}\hat{i} + 9\hat{j}$
- D. $-9\hat{i} - 9\sqrt{3}\hat{j}$
- E. $9\sqrt{3}\hat{i} - 9\hat{j}$

Question 17

The angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$ is best represented by

- A. $-\frac{4}{9}$
- B. $-\cos^{-1}\left(\frac{4}{9}\right)$
- C. $\pi + \cos^{-1}\left(-\frac{4}{9}\right)$
- D. $\pi - \cos^{-1}\left(\frac{4}{9}\right)$
- E. $\cos^{-1}\left(\pi - \frac{4}{9}\right)$

Question 18

Let $\vec{u} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{v} = a\hat{i} + 2\hat{j} - \hat{k}$.

If the scalar resolute of $\vec{v}$ in the direction of $\vec{u}$ is 1, then the value of a is

- A. $-\frac{3}{2}$
- B. $-\frac{2}{3}$
- C. 3
- D. $\frac{2}{3}$
- E. $\frac{3}{2}$

Question 19

A particle is initially travelling at 5 m/s. A constant force of 18 newtons is applied to the particle, in the direction of motion, for 4 seconds. The particle accelerates at 1.5 m/s^2 .

The momentum of the particle after 4 seconds is

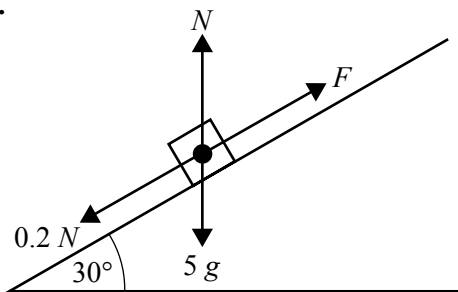
- A. 72 kg m/s
- B. 132 kg m/s
- C. 120 kg m/s
- D. 108 kg m/s
- E. 198 kg m/s

Question 20

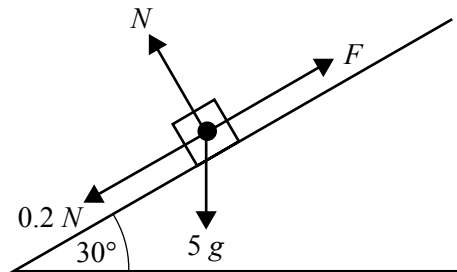
A 5 kg box is given an initial push by a force F , and then slides up a rough plane inclined at 30° to the horizontal. The coefficient of friction μ between the box and the plane is 0.2 and N is the normal reaction force of the plane on the box.

A suitable diagram showing the forces on the box as it slides up the plane after the push by F is

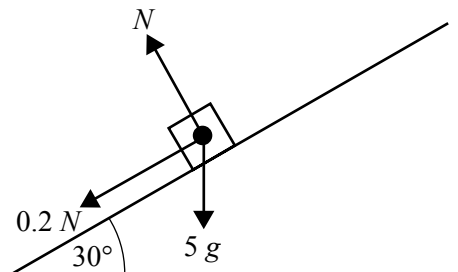
A.



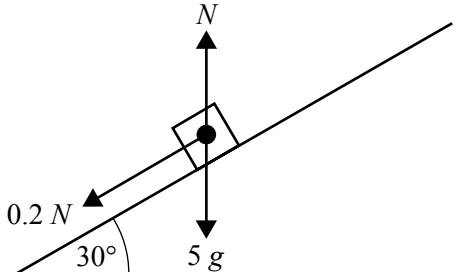
B.



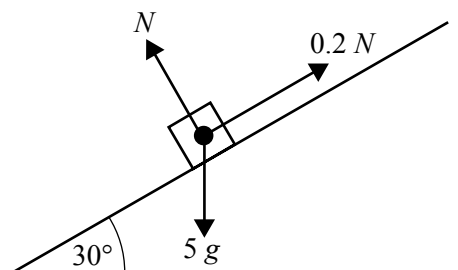
C.



D.



E.



Question 21

A 12 kg mass moves in a straight line under the action of a variable force F , so that its velocity $v \text{ ms}^{-1}$ when it is x metres from the origin is given by $v = \sqrt{3x^2 - x^3 + 16}$.

The force F acting on the mass is given by

- A. $12\left(3x - \frac{3x^2}{2}\right)$
- B. $12(3x^2 - x^3 + 16)$
- C. $12(6x - 3x^2)$
- D. $12\sqrt{3x^2 - x^3 + 16}$
- E. $12(3 - 3x)$

Question 22

A horizontal force of P newtons causes a jet of mass m kg to accelerate along a runway. The total resistance to the jet's motion is kv^2 newtons per kilogram of its mass, where $v \text{ ms}^{-1}$ is the speed of the jet and k is a real positive constant.

The equation of motion of the jet can be expressed as

- A. $m \frac{dv}{dt} = P - kv^2$
- B. $m \frac{dv}{dt} = P - mkv^2$
- C. $\frac{dv}{dt} = P - kv^2$
- D. $m \frac{dv}{dt} = P - mgkv^2$
- E. $\frac{dv}{dt} = P - mkv^2$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

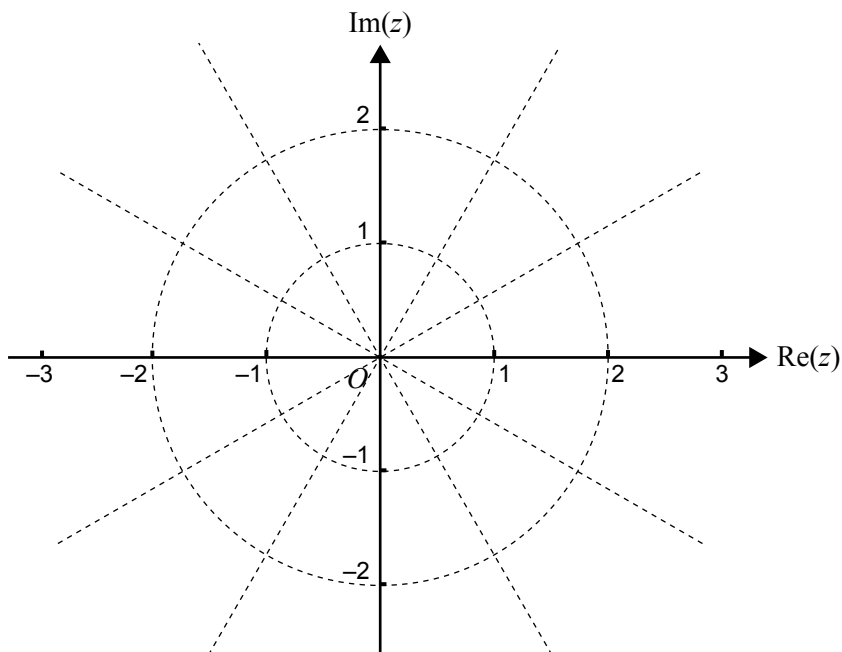
Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

- a. Express $z_1 = -\sqrt{3} + i$ in polar form.

2 marks

- b. Plot and label z_1 on the Argand diagram below.



1 mark

- c. By solving $z^2 - 2\sqrt{3}z + 4 = 0$ algebraically, show that the roots of this equation are $z = \sqrt{3} + i$ and $z = \sqrt{3} - i$.

2 marks

- d. Express the roots of $z^2 - 2\sqrt{3}z + 4 = 0$ in terms of z_1 , where $z_1 = -\sqrt{3} + i$.

2 marks

- e. Show that the cartesian equation for the relation $|z| = |z - z_1|$ is given by $y = \sqrt{3}x + 2$.

2 marks

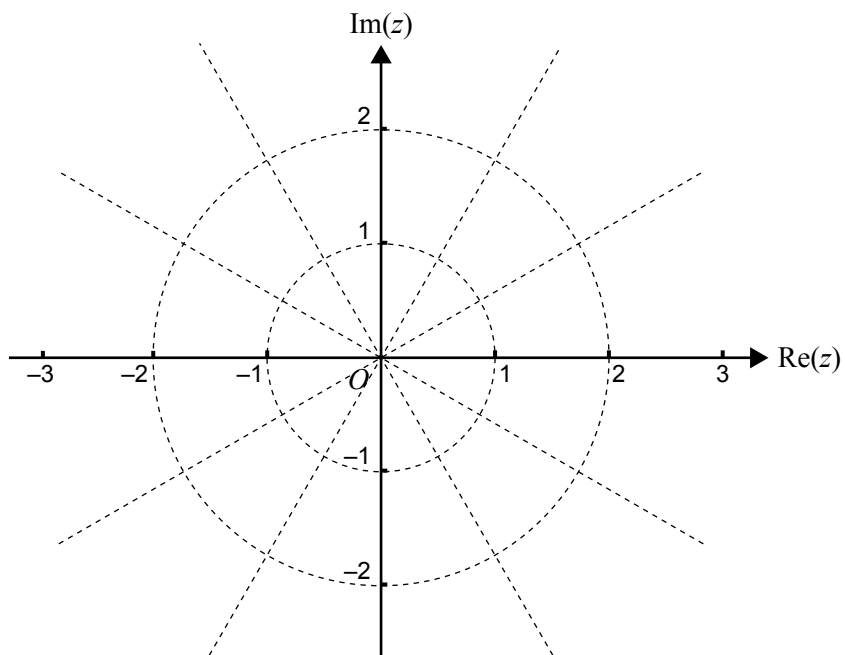
- f. Show that $\bar{z}_1$ satisfies the relation $|z| = |z - z_1|$.

1 mark

- g. Shade the region specified by

$$\{z : |z| \leq 2\} \cap \{z : |z| \geq |z - z_1|, \text{ where } z \in C\}$$

on the Argand diagram below.



2 marks

Total 12 marks

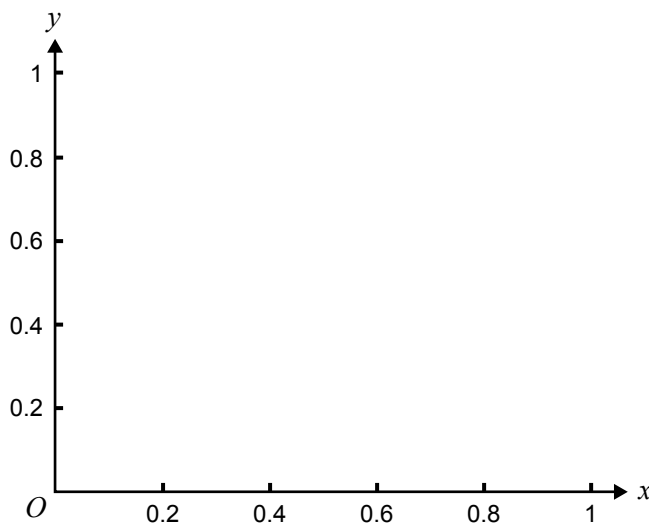
Question 2

Let $f(x) = x \arctan(x)$.

- a. Find $f'(x)$, and calculate the slope of the graph of f at $x = 0$.

2 marks

- b. On the axes below, sketch the curves $y = x \arctan(x)$ and $y = \arctan(x)$ over the domain $[0, 1]$, clearly labelling each graph.



2 marks

- c. i. Write down a definite integral which gives the area enclosed by the graphs of $y = x \arctan(x)$, $y = 0$ and $x = 1$.

- ii. Find the area defined in **part i.** correct to three decimal places.

2 marks

- d.** Use the result for $f'(x)$ in **part a.** to show that an antiderivative of $\arctan(x)$ is

$$x \arctan(x) - \frac{1}{2} \log_e(1 + x^2).$$

2 marks

- e.** Use the antiderivative given in **part d.** to find the exact area enclosed by the graphs of $y = \arctan(x)$, $y = 0$ and $x = 1$.

2 marks

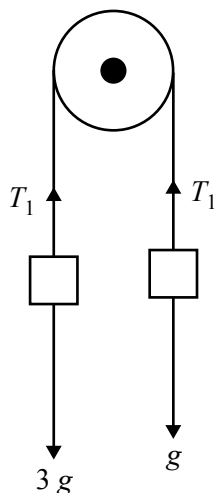
- f.** Find the area enclosed by the curves $y = x \arctan(x)$ and $y = \arctan(x)$. Give the answer correct to two decimal places.

2 marks

Total 12 marks

Question 3

The diagram shows particles of mass 1 kg and 3 kg connected by a light inextensible string passing over a smooth pulley. The tension in the string is T_1 newtons.



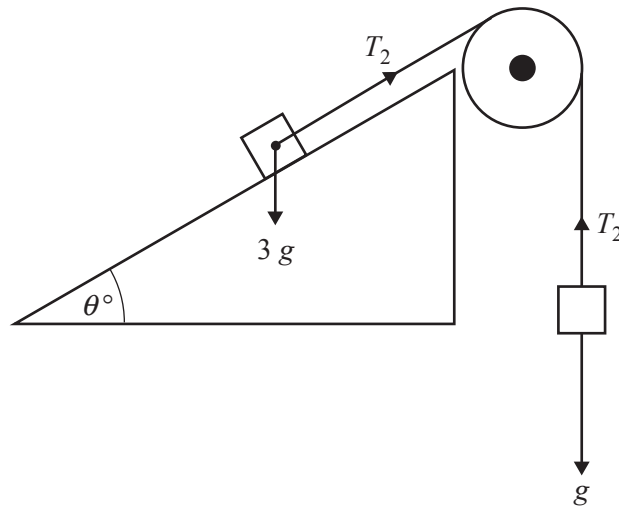
- a. Let $a \text{ ms}^{-2}$ be the acceleration of the 3 kg mass downwards.
Find the value of a .

2 marks

- b. Find the value of T_1 .

1 mark

The 3 kg mass is placed on a smooth plane inclined at an angle of θ° to the horizontal. The tension in the string is now T_2 newtons.



- c. When $\theta^\circ = 30^\circ$, the acceleration of the 1 kg mass upwards is $b \text{ ms}^{-2}$.
Find the value of b .

3 marks

- d. For what angle θ° will the 3 kg mass be at rest on the plane?
Give your answer correct to one decimal place.

2 marks

Question 4

An aircraft approaching an airport with velocity $\underline{v} = 30\underline{i} - 40\underline{j} - 4\underline{k}$ is observed on the control tower radar screen at time $t = 0$ seconds. Ten seconds later it passes over a navigation beacon with position vector $-500\underline{i} + 2500\underline{j}$ relative to the base of the control tower, at an altitude of 200 metres.

Let $\underline{i}$ and $\underline{j}$ be horizontal orthogonal unit vectors and let $\underline{k}$ be a unit vector in the vertical direction. Displacement components are measured in metres.

- a. Show that the position vector of the aircraft relative to the base of the control tower at time t is given by

$$\underline{r}(t) = (30t - 800)\underline{i} + (2900 - 40t)\underline{j} + (240 - 4t)\underline{k}.$$

3 marks

- b. When does the aircraft land and how far (correct to the nearest metre) from the base of the control tower is the point of landing?

3 marks

- c. At what angle from the runway, correct to the nearest tenth of a degree, does the aircraft land?

2 marks

- d. At what time, correct to the nearest second, is the aircraft closest to the base of the control tower?

2 marks

- e. What distance does the aircraft travel from the time it is observed on the radar screen to the time it lands?
Give your answer correct to the nearest metre.

2 marks

Total 12 marks

SECTION 2 – continued

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Question 5

A car travelling at 20 ms^{-1} passes a stationary police car, and then decelerates so that its velocity $v \text{ ms}^{-1}$, at time t seconds after passing the police car, is given by $v = 20 - 2 \tan^{-1}(t)$.

- a. After how many seconds will the car's speed be 17 ms^{-1} ? Give your answer correct to one decimal place.

1 mark

- b. Explain why v will never equal 16.

1 mark

- c. Write down a definite integral which gives the distance, x metres, travelled by the car after T seconds.

1 mark

Three seconds later the police car starts to chase the passing car which has a polluting exhaust pipe. The police car accelerates so that its velocity $v \text{ ms}^{-1}$ at time t seconds after the polluting car passed it is given by

$$v = 13 \cos^{-1} \left(\frac{13 - 2t}{7} \right) \text{ for } t \in [3, 8].$$

- d. Write down an expression which gives how far the polluting car is ahead of the police car when $t = 8$ seconds.

Find this distance in metres correct to one decimal place.

3 marks

After accelerating for five seconds the police car continues at a constant velocity.

- e. At time $t = T_c$ the police car catches the polluting car. Write an equation which, when solved, gives the value of T_c .

3 marks

- f. Find T_c correct to the nearest second.

1 mark

Total 10 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$