VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2006

# FURTHER MATHEMATICS
# Written examination 1

**Monday 30 October 2006**

**Reading time: 11.45 am to 12.00 noon (15 minutes)**
**Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)**

## MULTIPLE-CHOICE QUESTION BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
|---|---|---|---|---|---|
| A | 13 | 13 | | | 13 |
| B | 54 | 27 | 6 | 3 | 27 |
| | | | | | Total 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**
- Question book of 37 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

**Instructions**
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**At the end of the examination**
- You may keep this question book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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Working space

# SECTION A

## Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

## Core – Data analysis

*The following information relates to Questions 1, 2 and 3.*

The back-to-back **ordered** stemplot below shows the distribution of maximum temperatures (in °Celsius) of two towns, Beachside and Flattown, over 21 days in January.

| **Beachside** | | **Flattown** |
|---:|:---:|:---|
| 9 8 7 5 | 1 | 8 9 |
| 4 3 2 2 1 1 0 0 | 2 | |
| 9 9 8 7 6 5 | 2 | 8 9 |
| 3 2 | 3 | 3 3 4 |
| 8 | 3 | 5 5 6 7 7 7 8 8 |
| | 4 | 0 0 1 2 |
| | 4 | 5 6 |

**Question 1**

The variables

temperature (°Celsius)

and

town (Beachside or Flattown)

are

**A.** both categorical variables.

**B.** both numerical variables.

**C.** categorical and numerical variables respectively.

**D.** numerical and categorical variables respectively.

**E.** neither categorical nor numerical variables.

**Question 2**

For **Beachside**, the range of maximum temperatures is

**A.** 3°C

**B.** 23°C

**C.** 32°C

**D.** 33°C

**E.** 38°C

**Question 3**

The distribution of maximum temperatures for **Flattown** is best described as

**A.** negatively skewed.

**B.** positively skewed.

**C.** positively skewed with outliers.

**D.** approximately symmetric.

**E.** approximately symmetric with outliers.

**Question 4**

The head circumference (in cm) of a population of infant boys is normally distributed with a mean of 49.5 cm and a standard deviation of 1.5 cm.

Four hundred of these boys are selected at random and each boy's head circumference is measured.

The number of these boys with a head circumference of less than 48.0 cm is closest to

**A.** 3

**B.** 10

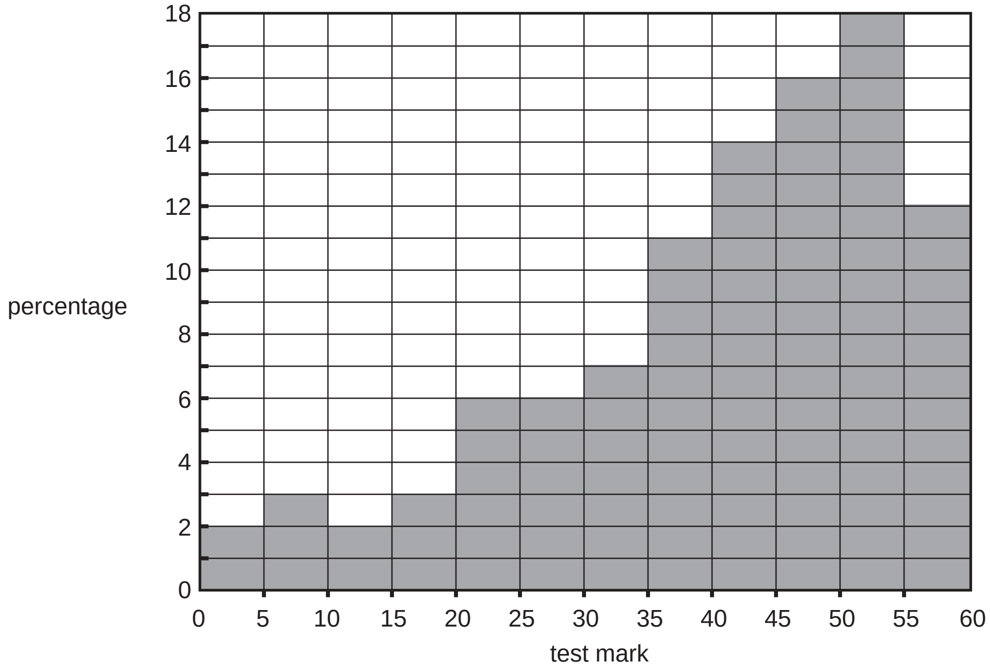
**C.** 64

**D.** 272

**E.** 336

*The following information relates to Questions 5 and 6.*

The distribution of test marks obtained by a large group of students is displayed in the percentage frequency histogram below.




**Question 5**

The pass mark on the test was 30 marks.

The percentage of students who passed the test is

**A.** 7%

**B.** 22%

**C.** 50%

**D.** 78%

**E.** 87%

**Question 6**

The median mark lies between

**A.** 35 and 40

**B.** 40 and 45

**C.** 45 and 50

**D.** 50 and 55

**E.** 55 and 60

**Question 7**

For a set of bivariate data, involving the variables $x$ and $y$,

$$r = -0.5675,\ \bar{x} = 4.56,\ s_x = 2.61,\ \bar{y} = 23.93 \text{ and } s_y = 6.98$$

The equation of the least squares regression line $y = a + bx$ is closest to

**A.** $y = 30.9 - 1.52x$

**B.** $y = 17.0 - 1.52x$

**C.** $y = -17.0 + 1.52x$

**D.** $y = 30.9 - 0.2x$

**E.** $y = 24.9 - 0.2x$

**Question 8**

The waist measurement (cm) and weight (kg) of 12 men are displayed in the table below.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **waist (cm)** | 84 | 74 | 89 | 75 | 106 | 114 | 80 | 101 | 101 | 94 | 126 | 82 |
| **weight (kg)** | 84 | 72 | 67 | 59 | 97 | 112 | 67 | 91 | 98 | 89 | 117 | 62 |

Using this data, the equation of the least squares regression line that enables weight to be predicted from waist measurement is

$$\text{weight} = -20 + 1.11 \times \text{waist}$$

When this equation is used to predict the weight of the man with a waist measurement of 80 cm, the residual value is closest to

**A.** –11 kg

**B.** 11 kg

**C.** –2 kg

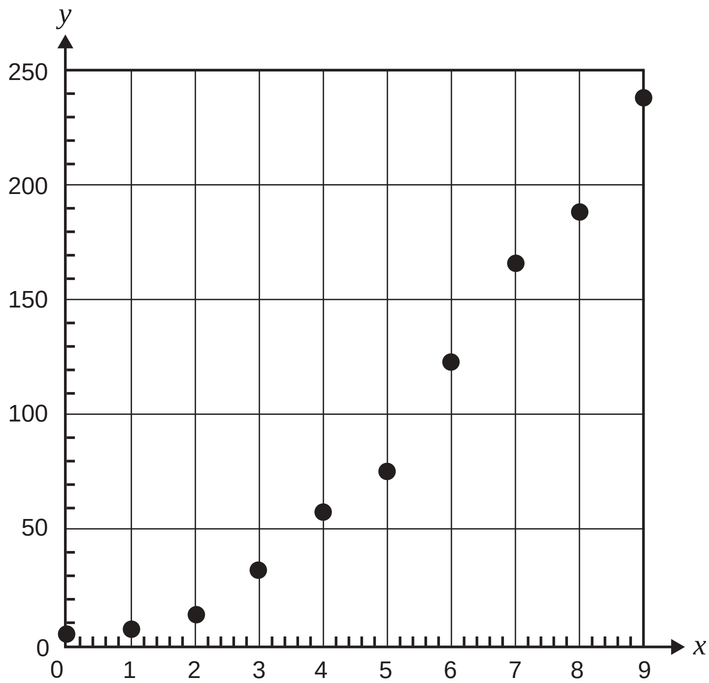
**D.** 2 kg

**E.** 69 kg

**Question 9**

A student uses the following data to construct the scatterplot shown below.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ***x*** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ***y*** | 5 | 7 | 14 | 33 | 58 | 76 | 124 | 166 | 188 | 238 |




To linearise the scatterplot, she applies an $x$-squared transformation.

She then fits a least squares regression line to the **transformed data** with $y$ as the dependent variable.

The equation of this least squares regression line is closest to

**A.** $y = 7.1 + 2.9x^2$

**B.** $y = -29.5 + 26.8x^2$

**C.** $y = 26.8 - 29.5x^2$

**D.** $y = 1.3 + 0.04x^2$

**E.** $y = -2.2 + 0.3x^2$

**Question 10**

The table below displays the total monthly rainfall (in mm) in a reservoir catchment area over a one-year period.

| **month** | Jan | Feb | Mar | April | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **rainfall** | 9 | 65 | 35 | 99 | 75 | 90 | 133 | 196 | 106 | 56 | 76 | 76 |

Using three **mean** moving average smoothing, the smoothed value for the total rainfall in April is closest to

**A.** 65

**B.** 66

**C.** 70

**D.** 75

**E.** 88

*The following information relates to Questions 11, 12 and 13.*

| **month** | Jan | Feb | Mar | April | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **seasonal index** | 1.30 | 1.21 | 1.00 | 0.95 | 0.95 | 0.86 | 0.86 | 0.89 | 0.94 | | 0.99 | 1.07 |

The table shows the seasonal indices for the monthly unemployment numbers for workers in a regional town.

**Question 11**

The seasonal index for October is missing from the table.

The value of the missing seasonal index for October is

**A.** 0.93

**B.** 0.95

**C.** 0.96

**D.** 0.98

**E.** 1.03

**Question 12**

The actual number of unemployed in the regional town in September is 330.

The **deseasonalised** number of unemployed in September is closest to

**A.** 310

**B.** 344

**C.** 351

**D.** 371

**E.** 640

**Question 13**

A trend line that can be used to forecast the **deseasonalised** number of unemployed workers in the regional town for the first nine months of the year is given by

$$\text{deseasonalised number of unemployed} = 373.3 - 3.38 \times \text{month number}$$

where month 1 is January, month 2 is February, and so on.

The **actual** number of unemployed for June is predicted to be closest to

**A.** 304

**B.** 353

**C.** 376

**D.** 393

**E.** 410

**END OF SECTION A**

## SECTION B

### Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

# Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

**Question 1**

Which one of the following sequences shows the first five terms of an arithmetic sequence?

**A.** 1, 3, 9, 27, 81 …

**B.** 1, 3, 7, 15, 31 …

**C.** –10, –5, 5, 10, 15 …

**D.** –4, –1, 2, 5, 8 …

**E.** 1, 3, 8, 15, 24 …

**Question 2**

The first three terms of a geometric sequence are 6, $x$, 54.

A possible value of $x$ is

**A.** 9

**B.** 15

**C.** 18

**D.** 24

**E.** 30

*The following information relates to Questions 3 and 4.*

A farmer plans to breed sheep to sell.

In the first year she starts with 50 breeding sheep.

During the first year, the sheep numbers increase by 84%.

At the end of the first year, the farmer sells 40 sheep.

**Question 3**

How many sheep does she have at the start of the second year?

**A.** 2

**B.** 42

**C.** 52

**D.** 84

**E.** 92

**Question 4**

If $S_n$ is the number of sheep at the start of year $n$, a difference equation that can be used to model the growth in sheep numbers over time is

**A.** $S_{n+1} = 1.84S_n - 40$ where $S_1 = 50$

**B.** $S_{n+1} = 0.84S_n - 50$ where $S_1 = 40$

**C.** $S_{n+1} = 0.84S_n - 40$ where $S_1 = 50$

**D.** $S_{n+1} = 0.16S_n - 50$ where $S_1 = 40$

**E.** $S_{n+1} = 0.16S_n - 40$ where $S_1 = 50$

**Question 5**

A difference equation is defined by

$$f_{n+1} - f_n = 5 \quad \text{where} \quad f_1 = -1$$

The sequence $f_1,\ f_2,\ f_3, \ldots$ is

**A.** 5, 4, 3 …

**B.** 4, 9, 14 …

**C.** –1, –6, –11 …

**D.** –1, 4, 9 …

**E.** –1, 6, 11 …

**Question 6**

A crystal measured 12.0 cm in length at the beginning of a chemistry experiment.

Each day it increased in length by 3%.

The length of the crystal after 14 days growth is closest to

**A.** 12.4 cm

**B.** 16.7 cm

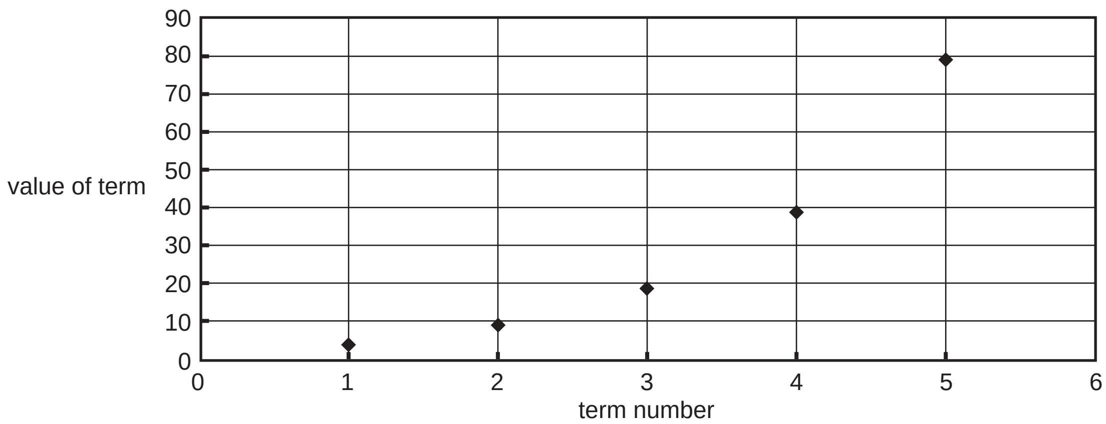
**C.** 17.0 cm

**D.** 17.6 cm

**E.** 18.2 cm

**Question 7**

The values of the first five terms of a sequence are plotted on the graph shown below.




The first order difference equation that could describe the sequence is

**A.** $t_{n+1} = t_n + 5, \quad t_1 = 4$

**B.** $t_{n+1} = 2t_n + 1, \quad t_1 = 4$

**C.** $t_{n+1} = t_n - 3, \quad t_1 = 4$

**D.** $t_{n+1} = t_n + 3, \quad t_1 = 4$

**E.** $t_{n+1} = 3t_n, \quad t_1 = 4$

**Question 8**

Paula started a stamp collection. She decided to buy a number of new stamps every week.

The number of stamps bought in the $n$th week, $t_n$, is defined by the difference equation

$$t_n = t_{n-1} + t_{n-2} \quad \text{where} \quad t_1 = 1 \text{ and } t_2 = 2$$

The **total** number of stamps in her collection after five weeks is

**A.** 8

**B.** 12

**C.** 15

**D.** 19

**E.** 24

**Question 9**

A healthy eating and gym program is designed to help football recruits build body weight over an extended period of time.

Roh, a new recruit who initially weighs 73.4 kg, decides to follow the program.

In the first week he gains 400 g in body weight.

In the second week he gains 380 g in body weight.

In the third week he gains 361 g in body weight.

If Roh continues to follow this program indefinitely, and this pattern of weight gain remains the same, his eventual body weight will be closest to

**A.** 74.5 kg

**B.** 77.1 kg

**C.** 77.3 kg

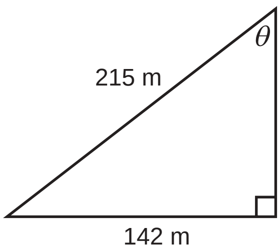
**D.** 80.0 kg

**E.** 81.4 kg

## Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

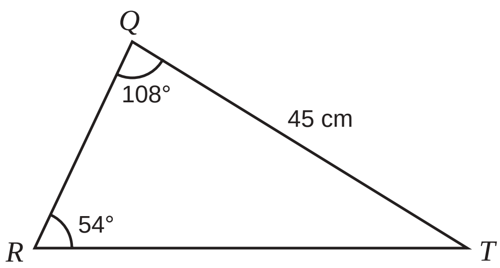
**Question 1**




For the triangle shown, the size of angle $\theta$ is closest to

**A.** 33°
**B.** 41°
**C.** 45°
**D.** 49°
**E.** 57°

**Question 2**

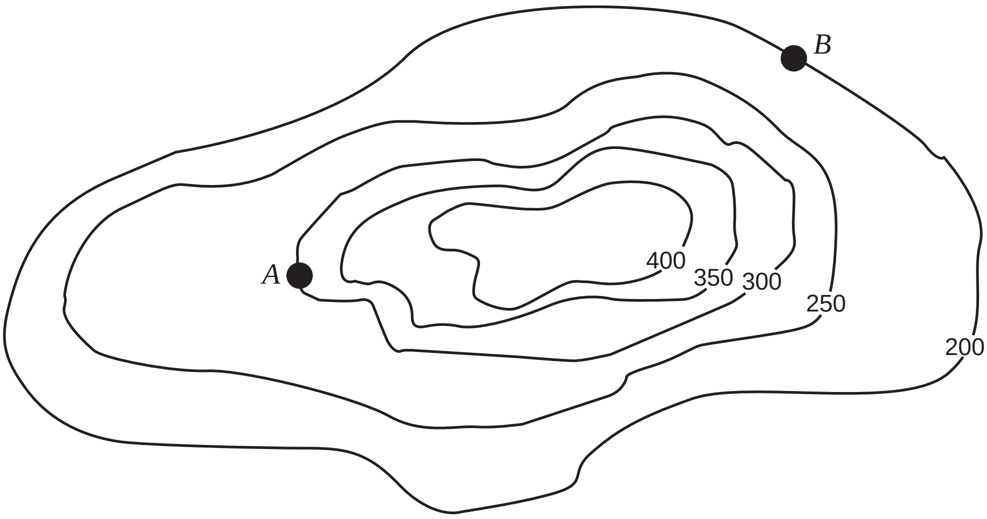



The length of $RT$ in the triangle shown is closest to

**A.** 17 cm
**B.** 33 cm
**C.** 45 cm
**D.** 53 cm
**E.** 57 cm

**Question 3**

The contour map below uses 50-metre intervals.




The difference in height between the locations represented by points *A* and *B* is

**A.** 100 m

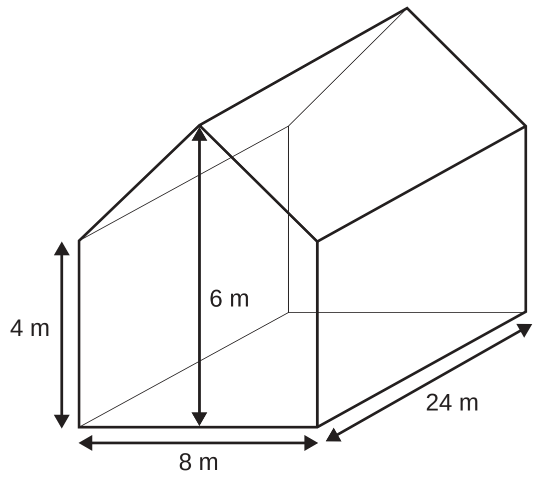
**B.** 150 m

**C.** 200 m

**D.** 250 m

**E.** 300 m

**Question 4**




The building shown in the diagram is 8 m wide and 24 m long.
The side walls are 4 m high.
The peak of the roof is 6 m vertically above the ground.
In cubic metres, the volume of this building is

**A.** 384
**B.** 576
**C.** 960
**D.** 1152
**E.** 4608

**Question 5**

A block of land is triangular in shape.
The three sides measure 36 m, 58 m and 42 m.
To calculate the area, Heron's formula is used.
The correct application of Heron's formula for this triangle is

**A.** Area = $\sqrt{136\,(136-36)\,(136-58)\,(136-42)}$

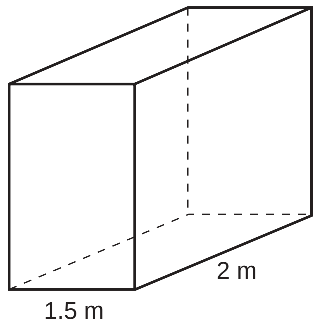
**B.** Area = $\sqrt{136\,(136-18)\,(136-29)\,(136-21)}$

**C.** Area = $\sqrt{68\,(68-36)\,(68-58)\,(68-42)}$

**D.** Area = $\sqrt{68\,(68-18)\,(68-29)\,(68-21)}$

**E.** Area = $\sqrt{68\,(136-36)\,(136-58)\,(136-42)}$

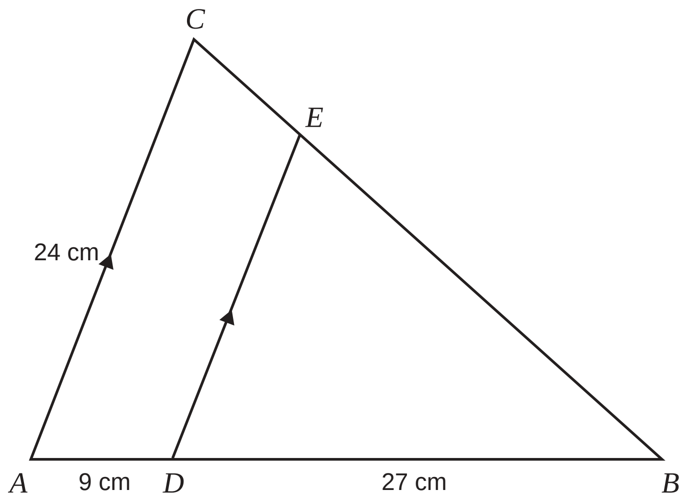
**Question 6**




The rectangular box shown in this diagram is closed at the top and at the bottom.
It has a volume of 6 $m^3$.
The base dimensions are 1.5 m × 2 m.
The total surface area of this box is

**A.** 10 $m^2$
**B.** 13 $m^2$
**C.** 13.5 $m^2$
**D.** 20 $m^2$
**E.** 27 $m^2$

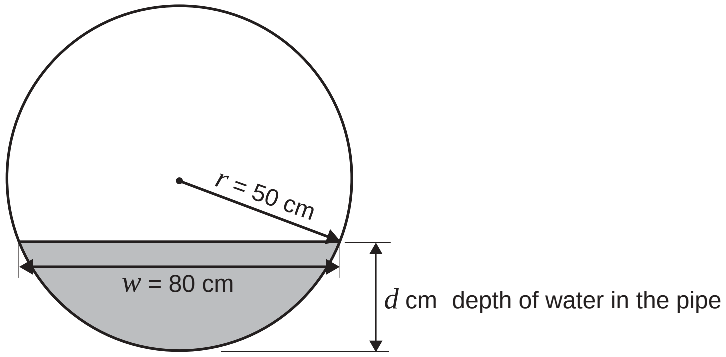
**Question 7**




In the diagram, *AD* = 9 cm, *AC* = 24 cm and *DB* = 27 cm.
Line segments *AC* and *DE* are parallel.
The length of *DE* is

**A.** 6 cm
**B.** 8 cm
**C.** 12 cm
**D.** 16 cm
**E.** 18 cm

**Question 8**




The cross-section of a water pipe is circular with a radius, *r*, of 50 cm, as shown above.
The surface of the water has a width, *w*, of 80 cm.
The depth of water in the pipe, *d*, could be

**A.** 20 cm
**B.** 25 cm
**C.** 30 cm
**D.** 40 cm
**E.** 50 cm

**Question 9**

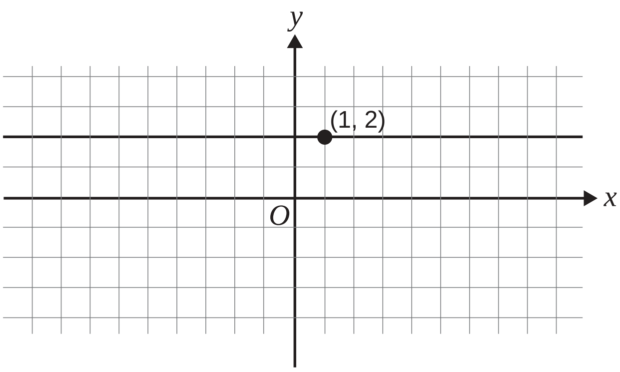
Points *M* and *P* are the same distance from a third point *O*.
The bearing of *M* from *O* is 038° and the bearing of *P* from *O* is 152°.
The bearing of *P* from *M* is

**A.** between 000° and 090°
**B.** between 090° and 180°
**C.** exactly 180°
**D.** between 180° and 270°
**E.** between 270° and 360°

## Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

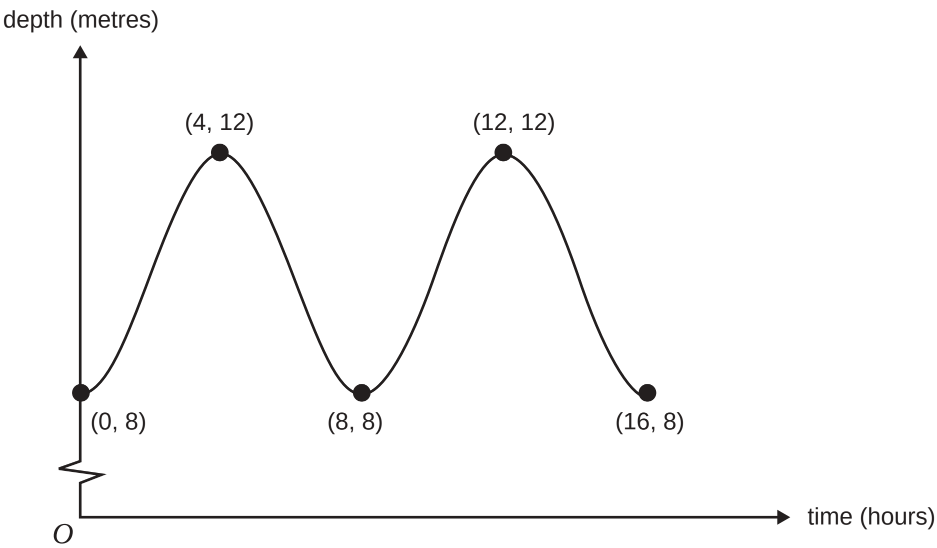
**Question 1**




On the graph above the equation of the line passing through the point (1, 2) is

**A.** $x = 1$
**B.** $y = 1$
**C.** $x = 2$
**D.** $y = 2$
**E.** $y = x + 1$

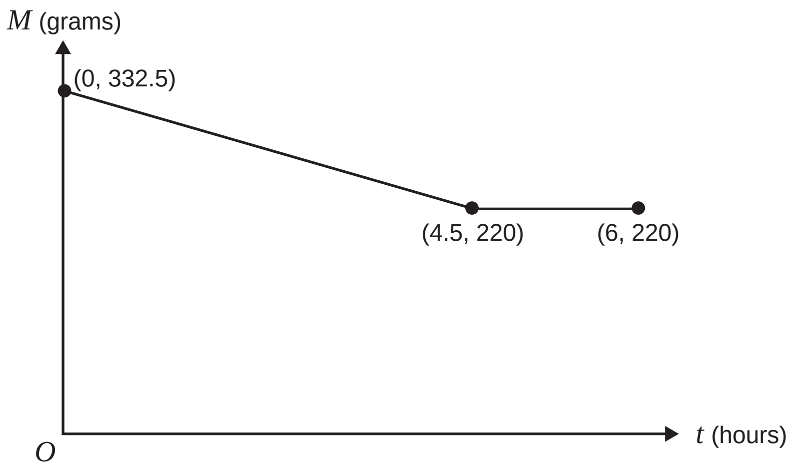
**Question 2**




The graph above represents the depth of water in a channel (in metres) as it changes over time (in hours). During the time interval shown, the number of times the depth of the water in the channel is 10 metres is

**A.** 0
**B.** 1
**C.** 2
**D.** 3
**E.** 4

*The following graph relates to Questions 3 and 4.*




A gas-powered camping lamp is lit and the gas is left on for six hours. During this time the lamp runs out of gas.

The graph shows how the mass, $M$, of the gas container (in grams) changes with time, $t$ (in hours), over this period.

**Question 3**

Assume that the loss in weight of the gas container is due only to the gas being burnt.

From the graph it can be seen that the lamp runs out of gas after

**A.** 1.5 hours.

**B.** 3 hours.

**C.** 4.5 hours.

**D.** 6 hours.

**E.** 220 hours.

**Question 4**

Which one of the following rules could be used to describe the graph above?

**A.** $M = \begin{cases} 332.5 - 25t & \text{for } 0 \le t \le 4.5 \\ 220 & \text{for } 4.5 < t \le 6 \end{cases}$

**B.** $M = \begin{cases} 332.5 - 25t & \text{for } 0 \le t \le 4.5 \\ 220t & \text{for } 4.5 < t \le 6 \end{cases}$

**C.** $M = \begin{cases} 332.5 + 25t & \text{for } 0 \le t \le 4.5 \\ 220t & \text{for } 4.5 < t \le 6 \end{cases}$

**D.** $M = \begin{cases} 332.5 - 12.5t & \text{for } 0 \le t \le 4.5 \\ 220t & \text{for } 4.5 < t \le 6 \end{cases}$

**E.** $M = \begin{cases} 332.5 - 12.5t & \text{for } 0 \le t \le 4.5 \\ 220 & \text{for } 4.5 < t \le 6 \end{cases}$

**Question 5**

Which one of the following statements about the line with equation $12x - 4y = 0$ is **not** true?

**A.** the line passes through the origin

**B.** the line has a slope of 12

**C.** the line has the same slope as the line with the equation $12x - 4y = 12$

**D.** the point (1, 3) lies on the line

**E.** for this line, as $x$ increases $y$ increases

**Question 6**

The point of intersection of two lines is (2, –2).

One of these two lines could be

**A.** $x - y = 0$

**B.** $2x + 2y = 8$

**C.** $2x + 2y = 0$

**D.** $2x - 2y = 4$

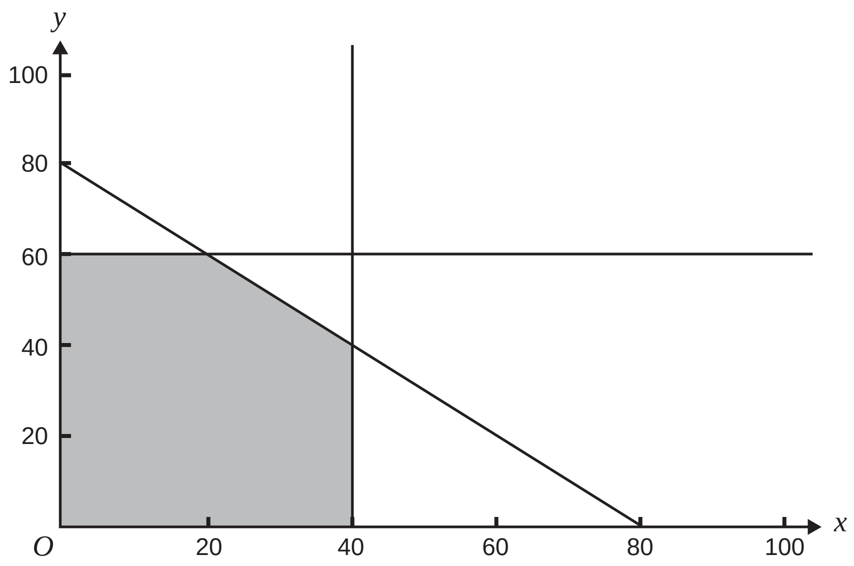
**E.** $2x - 2y = 0$

**Question 7**

In a linear programming problem involving animal management on a farm

- $x$ represents the number of cows on the farm
- $y$ represents the number of sheep on the farm.

The feasible region (with boundaries included) for the problem is indicated by the shaded region on the diagram below.




One of the constraints defining the feasible region indicates that

**A.** there must be 20 cows and 60 sheep.

**B.** there must be 40 cows and 40 sheep.

**C.** the number of sheep cannot exceed 40.

**D.** the number of cows must be at least 60.

**E.** the total number of cows and sheep cannot exceed 80.

**Question 8**

The cost of manufacturing a number of frying pans consists of a fixed cost of \$400 plus a cost of \$50 per frying pan.

The manufacturer could break even by selling

**A.** 10 frying pans at \$90 each.

**B.** 10 frying pans at \$45 each.

**C.** 15 frying pans at \$60 each.

**D.** 15 frying pans at \$30 each.

**E.** 20 frying pans at \$50 each.

**Question 9**

The four inequalities below were used to construct the feasible region for a linear programming problem.

$$x \geq 0$$

$$y \geq 0$$

$$x + y \leq 9$$

$$y \leq \frac{1}{2}x$$

A point that lies within this feasible region is

**A.** (4, 4)

**B.** (5, 3)

**C.** (6, 2)

**D.** (6, 4)

**E.** (7, 3)

## Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

**Question 1**

$4000 is invested at a simple interest rate of 5% per annum.

The amount of interest earned in the first year is

**A.** $20

**B.** $200

**C.** $220

**D.** $420

**E.** $2000

**Question 2**

A bank statement for the month of October is shown below.

| Date | Description of transaction | Debit | Credit | Balance |
|---|---|---|---|---|
| **01 Oct** | **Opening balance** | | | **853.92** |
| 01 Oct | Withdrawal – Internet banking | 380.00 | | 473.92 |
| 16 Oct | Deposit – Cheque | | 518.15 | 992.07 |
| 18 Oct | Credit card payment | 125.56 | | 866.51 |
| 23 Oct | Withdrawal – Internet banking | 250.00 | | 616.51 |
| **31 Oct** | **Closing balance** | | | **616.51** |

Interest on this account is calculated at a rate of 0.15% per month on the minimum monthly balance.

The interest payment for the month of October will be

**A.** $0.19

**B.** $0.57

**C.** $0.71

**D.** $0.92

**E.** $1.28

**Question 3**

Grandpa invested in an ordinary perpetuity from which he receives a monthly pension of $584.

The interest rate for the investment is 6.2% per annum.

The amount Grandpa has invested in the perpetuity is closest to

**A.** $3 600

**B.** $9 420

**C.** $94 200

**D.** $43 400

**E.** $113 000

### Question 4

An item was purchased for a price of $825.

The price included 10% GST (Goods and Services Tax).

The amount of GST included in the price is

**A.** $8.25

**B.** $75.00

**C.** $82.50

**D.** $90.75

**E.** $125.00

### Question 5

A photocopier is depreciated by $0.04 for each copy it makes.

Three years ago the photocopier was purchased for $48 000.

Its depreciated value now is $21 000.

The total number of copies made by the photocopier in the three years is

**A.** 108 000

**B.** 192 000

**C.** 276 000

**D.** 525 000

**E.** 675 000

### Question 6

A $2000 lounge suite was sold under a hire-purchase agreement.

A deposit of $200 was paid.

The balance was to be paid in 36 equal monthly instalments of $68.

The annual flat rate of interest applied to this agreement is

**A.** 10.0%

**B.** 11.4%

**C.** 12.0%

**D.** 22.4%

**E.** 36.0%

### Question 7

Mervyn bought a new lawn mower at a sale.

First, there was a 20% discount from the original price.

Then, an $80 trade-in for his old mower was subtracted from this reduced price.

This left Mervyn with $368 to pay for the new lawn mower.

The original price of the new lawn mower was

**A.** $468.00

**B.** $537.50

**C.** $540.00

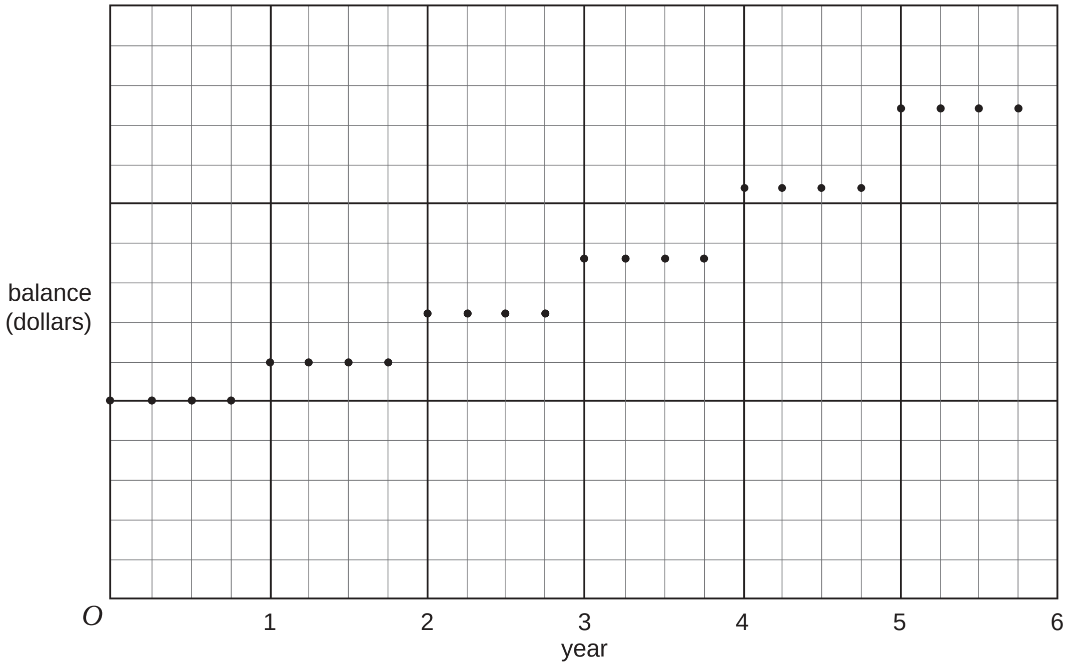
**D.** $560.00

**E.** $580.00

**Question 8**

The points on the graph below show the balance of an investment at the start of each quarter for a period of six years.

The same rate of interest applied for these six years.




In relation to this investment, which one of the following statements is **true**?

**A.** interest is compounding annually and is credited annually

**B.** interest is compounding annually and is credited quarterly

**C.** interest is compounding quarterly and is credited quarterly

**D.** simple interest is paid on the opening balance and is credited annually

**E.** simple interest is paid on the opening balance and is credited quarterly

**Question 9**

Jenny borrowed \$18 000. She will fully repay the loan in five years with equal monthly payments.

Interest is charged at the rate of 9.2% per annum, calculated monthly, on the reducing balance.

The amount Jenny has paid off the principal immediately following the **tenth** repayment is

**A.** \$1876.77

**B.** \$2457.60

**C.** \$3276.00

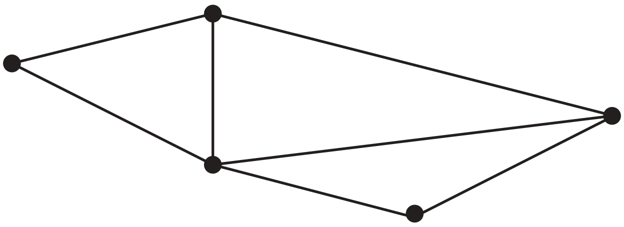
**D.** \$3600.44

**E.** \$3754.00

## Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions.

### Question 1

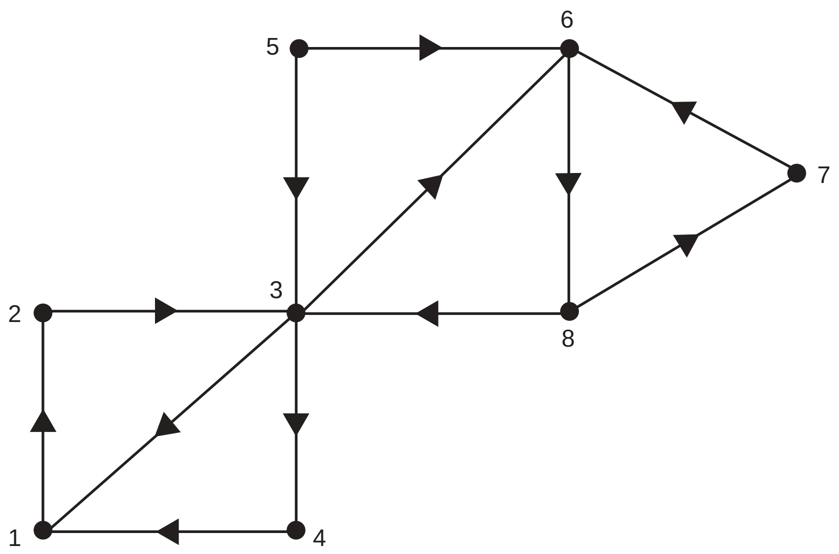


The number of vertices with an odd degree in the network above is

**A.** 1
**B.** 2
**C.** 3
**D.** 4
**E.** 5

### Question 2

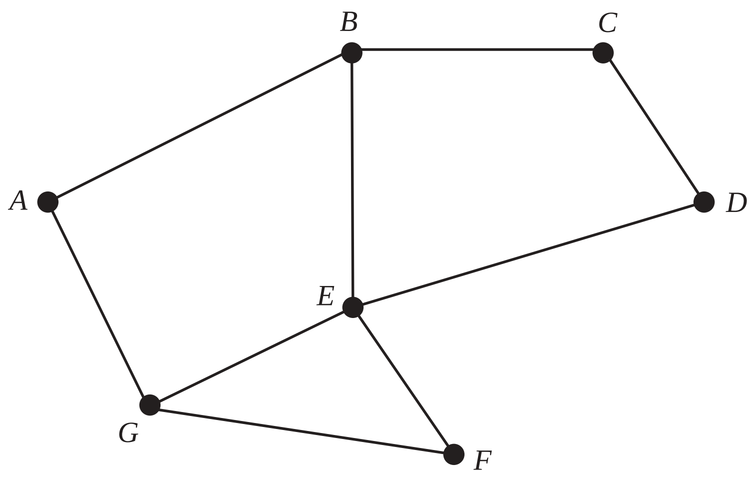
The following directed graph represents a series of one-way streets with intersections numbered as nodes 1 to 8.




All intersections can be reached from

**A.** intersection 4
**B.** intersection 5
**C.** intersection 6
**D.** intersection 7
**E.** intersection 8

**Question 3**




Which one of the following statements is **true** regarding the network above?

**A.** *ABCDEFG* is a Hamiltonian circuit.

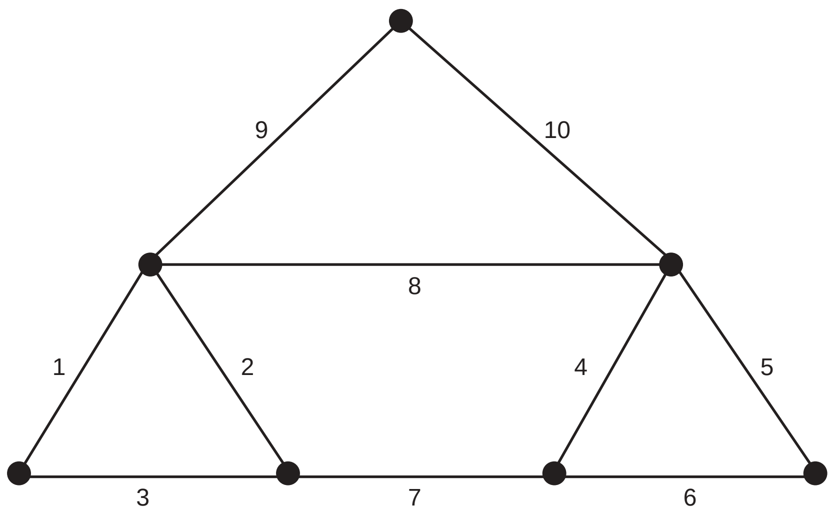
**B.** Only one Hamiltonian path exists.

**C.** *CBAGFEDC* is an Eulerian circuit.

**D.** At least two Eulerian paths exist.

**E.** There are no circuits.

**Question 4**




The minimal spanning tree for the network above will include the edge that has a weight of

**A.** 3

**B.** 6

**C.** 8

**D.** 9

**E.** 10

**Question 5**

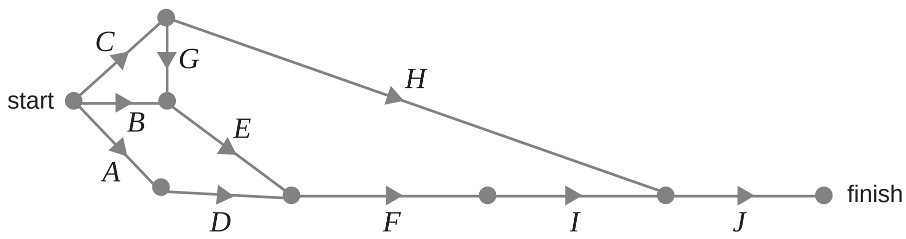
For a particular project there are ten activities that must be completed.

These activities and their immediate predecessors are given in the following table.

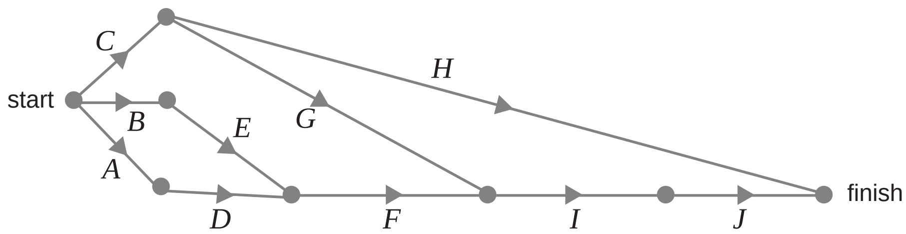
| Activity | Immediate predecessors |
|---|---|
| $A$ | – |
| $B$ | – |
| $C$ | – |
| $D$ | $A$ |
| $E$ | $B$ |
| $F$ | $D, E$ |
| $G$ | $C$ |
| $H$ | $C$ |
| $I$ | $F, G$ |
| $J$ | $H, I$ |

A directed graph that could represent this project is

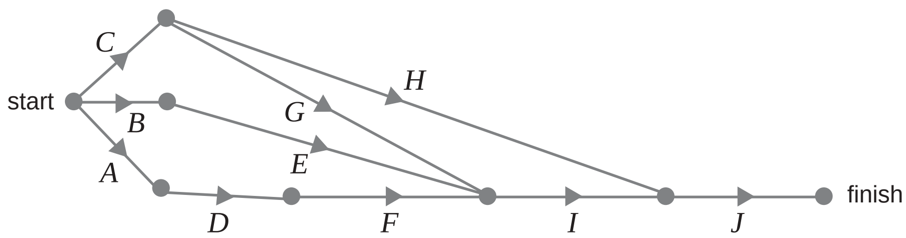
**A.**



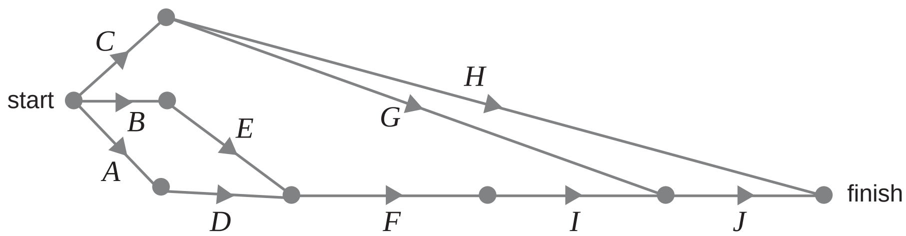

**B.**



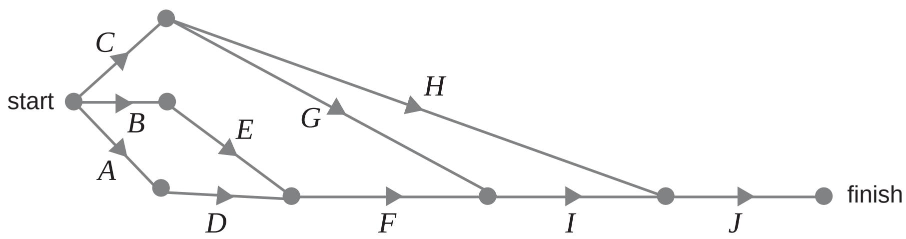

**C.**



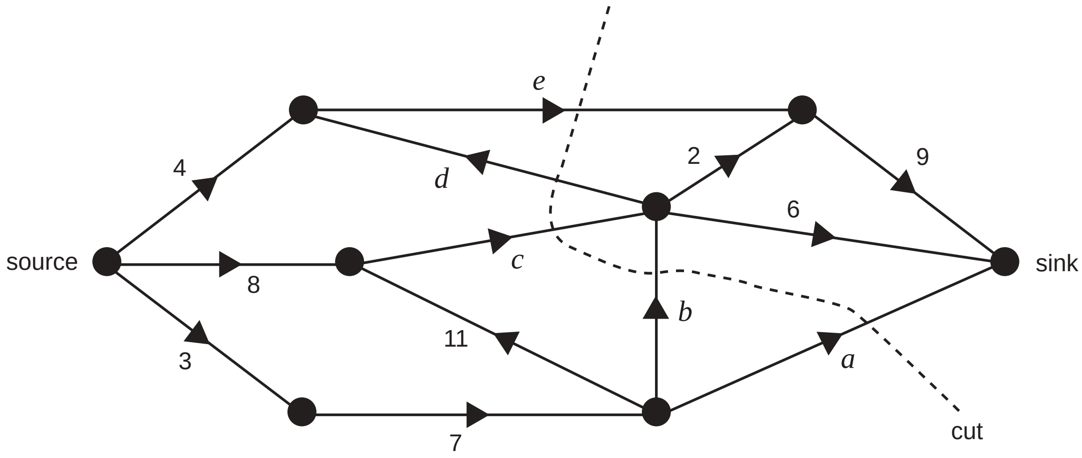

**D.**




**E.**



**Question 6**




In the directed graph above the weight of each edge is non-zero.

The capacity of the cut shown is

**A.** $a + b + c + d + e$

**B.** $a + c + d + e$

**C.** $a + b + c + e$

**D.** $a + b + c - d + e$

**E.** $a - b + c - d + e$

**Question 7**

A **complete** graph with six vertices is drawn.

This network would best represent

**A.** the journey of a paper boy who delivers to six homes covering the minimum distance.

**B.** the cables required to connect six houses to pay television that minimises the length of cables needed.

**C.** a six-team basketball competition where all teams play each other once.

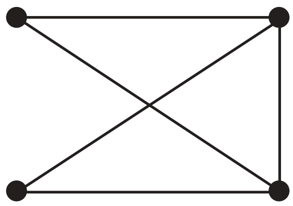
**D.** a project where six tasks must be performed between the start and finish.

**E.** the allocation of different assignments to a group of six students.

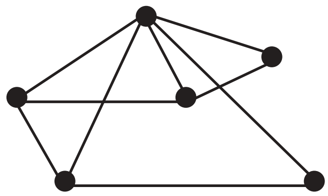
**Question 8**

Euler's formula, relating vertices, faces and edges, does **not** apply to which one of the following graphs?

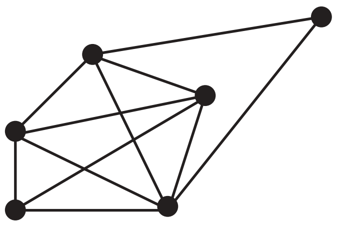
**A.**



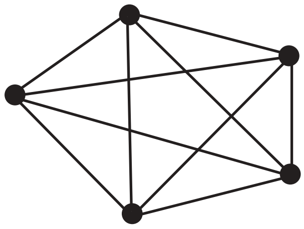
**B.**



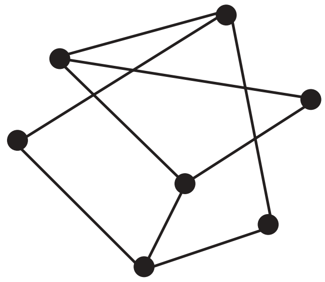
**C.**



**D.**

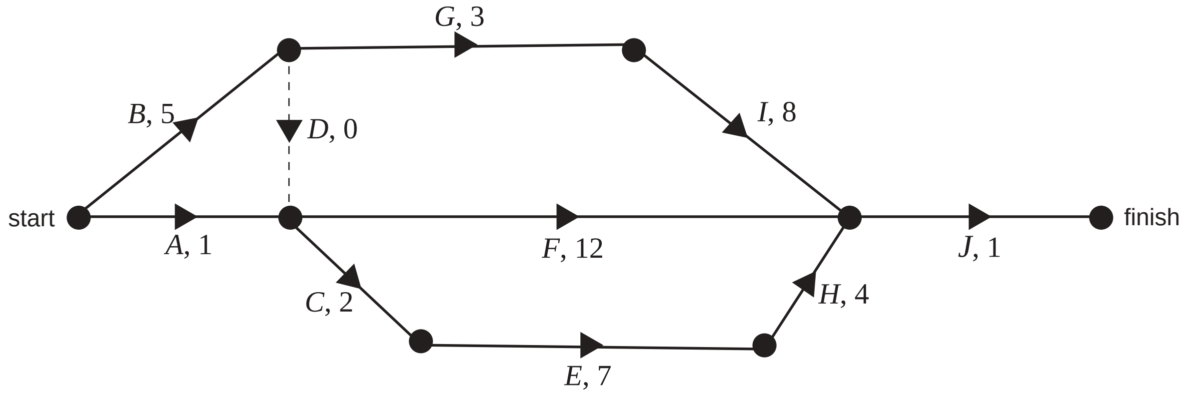


**E.**



**Question 9**

The network below shows the activities and their completion times (in hours) that are needed to complete a project.




The project is to be crashed by reducing the completion time of **one** activity only.

This will reduce the completion time of the project by a maximum of

**A.** 1 hour

**B.** 2 hours

**C.** 3 hours

**D.** 4 hours

**E.** 5 hours

## Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions.

**Question 1**

The matrix $\begin{bmatrix} 12 & 36 \\ 0 & 24 \end{bmatrix}$ is equal to

**A.** $12\begin{bmatrix} 0 & 3 \\ 0 & 2 \end{bmatrix}$

**B.** $12\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

**C.** $12\begin{bmatrix} 0 & 24 \\ -12 & 12 \end{bmatrix}$

**D.** $12\begin{bmatrix} 0 & 24 \\ 0 & 12 \end{bmatrix}$

**E.** $12\begin{bmatrix} 1 & 3 \\ -12 & 2 \end{bmatrix}$

**Question 2**

Let $A = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 9 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \end{bmatrix}$

Using these matrices, the matrix product that is **not** defined is

**A.** $AB$

**B.** $AC$

**C.** $BA$

**D.** $BC$

**E.** $CB$

**Question 3**

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Then $A^3(B - C)$ equals

**A.** $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

**B.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

**C.** $\begin{bmatrix} 3 & 6 \\ 6 & -3 \end{bmatrix}$

**D.** $\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$

**E.** $\begin{bmatrix} 5 & 10 \\ 10 & -5 \end{bmatrix}$

**Question 4**

Three teams, Blue ($B$), Green ($G$) and Red ($R$), compete for three different sporting competitions.

The table shows the competition winners for the past three years.

| | **Athletics** | **Cross country** | **Swimming** |
|---|---|---|---|
| **2004** | Green | Green | Blue |
| **2005** | Green | Red | Blue |
| **2006** | Blue | Green | Blue |

A matrix that shows the **total number** of competitions won by each of the three teams in **each** of these three years could be

**A.**
$$\begin{array}{c c} & \begin{array}{ccc} B & G & R \end{array} \\ \begin{array}{c} 2004 \\ 2005 \\ 2006 \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

**B.**
$$\begin{array}{c c} & \begin{array}{ccc} B & G & R \end{array} \\ \begin{array}{c} 2004 \\ 2005 \\ 2006 \end{array} & \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$

**C.**
$$\begin{array}{c c} & \begin{array}{ccc} B & G & R \end{array} \\ \begin{array}{c} 2004 \\ 2005 \\ 2006 \end{array} & \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

**D.**
$$\begin{array}{c c} & \begin{array}{ccc} B & G & R \end{array} \\ \begin{array}{c} 2004 \\ 2005 \\ 2006 \end{array} & \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

**E.**
$$\begin{array}{c c} & \begin{array}{ccc} B & G & R \end{array} \\ \begin{array}{c} 2004 \\ 2005 \\ 2006 \end{array} & \begin{bmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix} \end{array}$$

**Question 5**

A company makes Regular ($R$), Queen ($Q$) and King ($K$) size beds. Each bed comes in either the Classic style or the more expensive Deluxe style.

The price of each style of bed, in dollars, is listed in a price matrix $P$, where

$$\begin{array}{cc} & \begin{array}{ccc} R & Q & K \end{array} \\ P = & \begin{bmatrix} 145 & 210 & 350 \\ 185 & 270 & 410 \end{bmatrix} \end{array} \begin{array}{l} \text{Classic} \\ \text{Deluxe} \end{array}$$

The company wants to increase the price of all beds.

A new price matrix, listing the increased prices of the beds, can be generated from $P$ by forming a **matrix product** with the matrix, $M$, where

$$M = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.35 \end{bmatrix}$$

This new price matrix is

**A.**

$$\begin{bmatrix} 145 & 210 & 350 \\ 185 & 270 & 410 \end{bmatrix}$$

**B.**

$$\begin{bmatrix} 234.90 & 340.20 & 567 \\ 299.70 & 437.40 & 664.20 \end{bmatrix}$$

**C.**

$$\begin{bmatrix} 174 & 252 & 420 \\ 222 & 324 & 492 \end{bmatrix}$$

**D.**

$$\begin{bmatrix} 174 & 252 & 420 \\ 249.75 & 364.50 & 553.50 \end{bmatrix}$$

**E.**

$$\begin{bmatrix} 195.75 & 283.50 & 472.50 \\ 249.75 & 364.50 & 553.50 \end{bmatrix}$$

**Question 6**

If $A = \begin{bmatrix} 1 & 3 \\ 6 & 4 \\ 0 & 0 \end{bmatrix}$ and the matrix product $XA = \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 3 & 5 \end{bmatrix}$, then the order of matrix $X$ is

**A.** $(2 \times 2)$
**B.** $(2 \times 3)$
**C.** $(3 \times 1)$
**D.** $(3 \times 2)$
**E.** $(3 \times 3)$

**Question 7**

How many of the following five sets of simultaneous linear equations have a unique solution?

| | | | | |
|---|---|---|---|---|
| $4x + 2y = 10$<br>$2x + y = 5$ | $x = 0$<br>$x + y = 6$ | $x - y = 3$<br>$x + y = 3$ | $2x + y = 5$<br>$2x + y = 10$ | $x = 8$<br>$y = 2$ |

**A.** 1
**B.** 2
**C.** 3
**D.** 4
**E.** 5

**Question 8**

Australians go on holidays either within Australia or overseas.

Market research shows that

- 95% of those who had their last holiday in Australia said that their next holiday would be in Australia
- 20% of those who had their last holiday overseas said that their next holiday would also be overseas.

A transition matrix that could be used to describe this situation is

**A.** $\begin{bmatrix} 0.95 \\ 0.20 \end{bmatrix}$

**B.** $\begin{bmatrix} 0.95 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.20 \\ 0.80 \end{bmatrix}$

**C.** $\begin{bmatrix} 0.95 & 0.95 \\ 0.20 & 0.20 \end{bmatrix}$

**D.** $\begin{bmatrix} 0.95 & 0.20 \\ 0.05 & 0.80 \end{bmatrix}$

**E.** $\begin{bmatrix} 0.95 & 0.80 \\ 0.05 & 0.20 \end{bmatrix}$

**Question 9**

A large population of birds lives on a remote island. Every night each bird settles at either location *A* or location *B*.

It was found on the first night that the number of birds at each location was the same.

On each subsequent night, a percentage of birds changed the location at which they settled.

This movement of birds is described by the transition matrix

$$\begin{array}{cc} & \begin{array}{cc} A & \;\;B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.8 & 0 \\ 0.2 & 1 \end{bmatrix} \end{array}$$

Assume this pattern of movement continues.

In the long term, the number of birds that settle at location *A* will

**A.** not change.

**B.** gradually decrease to zero.

**C.** eventually settle at around 20% of the island's bird population.

**D.** eventually settle at around 80% of the island's bird population.

**E.** gradually increase.

**END OF QUESTION BOOK**

# FURTHER MATHEMATICS

# Written examinations 1 and 2

## FORMULA SHEET

**Directions to students**

| |
|---|
| Detach this formula sheet during reading time.<br>This formula sheet is provided for your reference. |

# Further Mathematics Formulas

## Core: Data analysis

| | |
|---|---|
| standardised score: | $z = \dfrac{x - \bar{x}}{s_x}$ |
| least squares line: | $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$ |
| residual value: | residual value = actual value – predicted value |
| seasonal index: | seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$ |

## Module 1: Number patterns

| | |
|---|---|
| arithmetic series: | $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$ |
| geometric series: | $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$ |
| infinite geometric series: | $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, \lvert r \rvert < 1$ |

## Module 2: Geometry and trigonometry

| | |
|---|---|
| area of a triangle: | $\dfrac{1}{2}bc\sin A$ |
| Heron's formula: | $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$ |
| circumference of a circle: | $2\pi r$ |
| area of a circle: | $\pi r^2$ |
| volume of a sphere: | $\dfrac{4}{3}\pi r^3$ |
| surface area of a sphere: | $4\pi r^2$ |
| volume of a cone: | $\dfrac{1}{3}\pi r^2 h$ |
| volume of a cylinder: | $\pi r^2 h$ |
| volume of a prism: | area of base × height |
| volume of a pyramid: | $\dfrac{1}{3}$ area of base × height |

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

## Module 3: Graphs and relations

### Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

## Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

## Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

## Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

**END OF FORMULA SHEET**