



Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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SPECIALIST MATHEMATICS

Written examination 2

(Analysis task)

Wednesday 5 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 15 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- Where an **exact** answer is required to a question, appropriate working must be shown.
- In questions where more than one mark is available, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Working space

TURN OVER

Later, the sled and snow tractor are parked on the slope with the snow tractor's brakes applied.

Let T newtons be the magnitude of the tension in the tow bar and F newtons be the magnitude of the frictional force between the sled and the ski slope.

- b.** Express T in terms of F .

2 marks

- c.** Find T , correct to the nearest integer, if the coefficient of friction between the sled and the ski slope is

- i.** 0.09

1 mark

- ii.** 0.15

1 mark

Total 8 marks

TURN OVER

Question 2

- a.** Using the fact that $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$, show that

$$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{and} \quad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

3 marks

Let $u = \operatorname{cis}\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4} + \frac{\sqrt{6} - \sqrt{2}}{4}i$, and let v be the complex conjugate of u .

- b.** Find the **exact** value of $\text{Arg}(v - u)$.

3 marks

- c. Find the **exact** value of $\text{Arg}\left(\frac{v}{u}\right)$.

2 marks

- d. Let u and v satisfy the quadratic equation $z^2 + az + b = 0$.
Show that $b = 1$ and find the **exact** value of a in surd form.

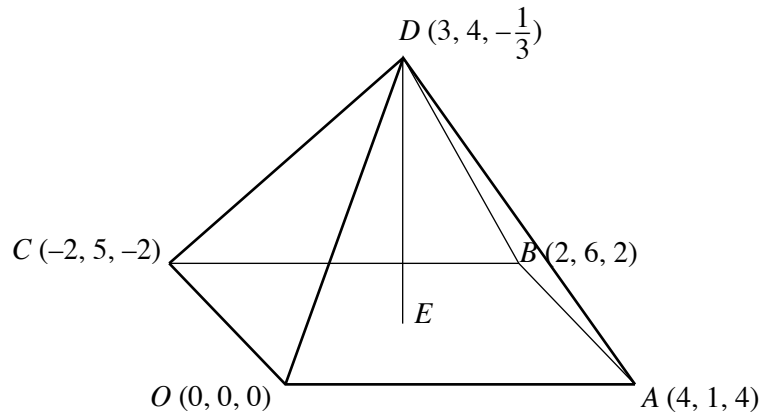
3 marks

Total 11 marks

TURN OVER

Question 3

$OABCD$ is a pyramid. The height of the pyramid is the length of DE where E is the point on the base $OABC$ such that DE is perpendicular to the base.



- a. Show that the base $OABC$ is a rhombus.

3 marks

- b. Use a vector method to find $\angle AOC$ correct to the nearest tenth of a degree.

3 marks

Question 3 – continued

c. i. Show that $q = 0$ and find the **exact** values of p and r .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

ii. Hence find the **exact** height of the pyramid.

[illegible]

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Question 4

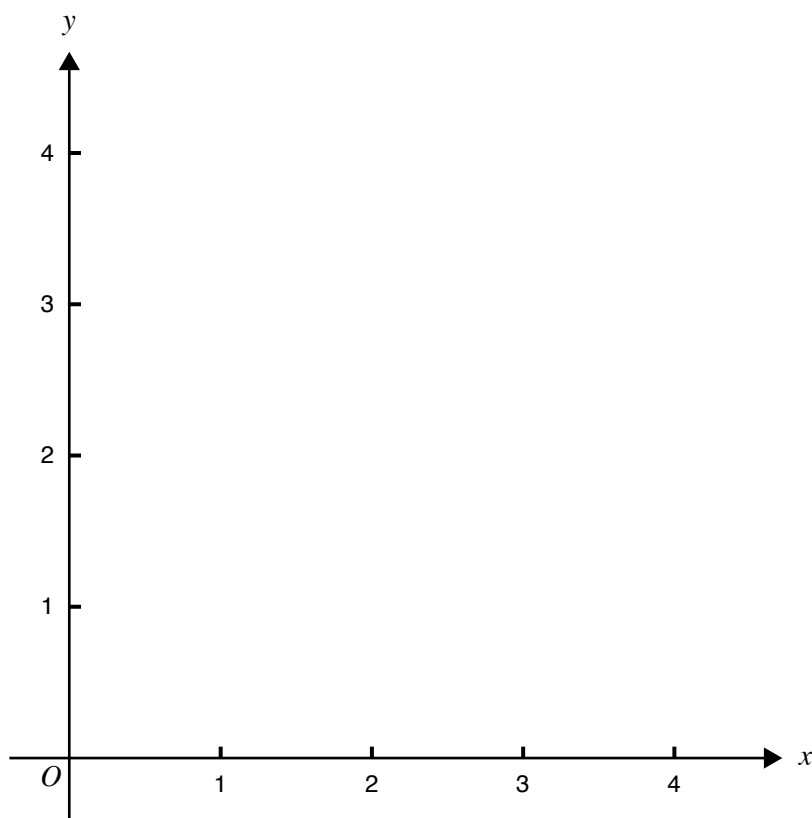
Consider the function $f: [0, 3) \rightarrow \mathbb{R}$ where $f(x) = -2 + 2 \sec\left(\frac{\pi x}{6}\right)$.

- a. Evaluate $f(2)$.

1 mark

Let f^{-1} be the inverse function of f .

- b. On the axes below, sketch the graphs of f and f^{-1} , showing their points of intersection.



2 marks

- c. The rule for f^{-1} can be written as $f^{-1}(x) = a \cos^{-1} \left(\frac{2}{x+2} \right)$.
Find the **exact** value of a .

2 marks

Let A be the magnitude of the area enclosed by the graphs of f and f^{-1} .

- d. Write a definite integral expression for A and evaluate it correct to three decimal places.

2 marks

- e. i. Given that $0 < kx < \frac{\pi}{2}$, show that the derivative of $\log_e \left(\frac{1 + \sin(kx)}{\cos(kx)} \right)$ is $k \sec(kx)$.

3 marks

- ii. Hence find the **exact** value of A , the magnitude of the area enclosed by the graphs of f and f^{-1} .

4 marks

Total 14 marks

Working space

TURN OVER

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c. Sketch a graph of y in terms of t from when the baby's bottle is put into the boiling water to when the milk is ready to give to baby Alex.



d. Find the ratio $\frac{a}{b}$ correct to three significant figures.

[illegible]

Total 14 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a + b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$
hyperbola:	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a)f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + h f(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$$

Mechanics

momentum:

$$\underline{p} = m\underline{v}$$

equation of motion:

$$\underline{R} = m\underline{a}$$

friction:

$$F \leq \mu N$$