



Victorian Certificate of Education 2003

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 3 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 17 pages, with a detachable sheet of miscellaneous formulas in the centrefold and two blank pages for rough working.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

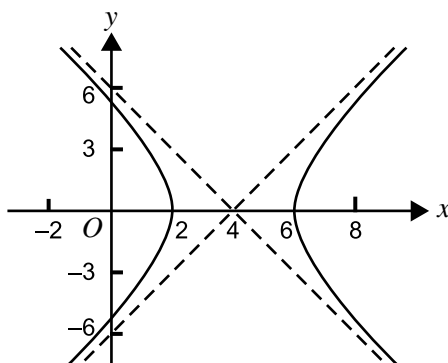
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1



The graph shown above could have the equation

- A. $\frac{(x-4)^2}{16} - \frac{y^2}{9} = 1$
- B. $\frac{(x-4)^2}{4} - \frac{y^2}{36} = 1$
- C. $\frac{(x-4)^2}{16} - \frac{y^2}{36} = 1$
- D. $\frac{(x-4)^2}{4} - \frac{y^2}{9} = 1$
- E. $\frac{(x-4)^2}{9} - \frac{y^2}{4} = 1$

Question 2

If $\sin(x) = -\frac{1}{5}$, $\pi \leq x \leq \frac{3\pi}{2}$, then $\cot(x)$ is equal to

- A. $-2\sqrt{6}$
- B. $2\sqrt{6}$
- C. $-\frac{1}{2\sqrt{6}}$
- D. $\frac{1}{2\sqrt{6}}$
- E. $-\frac{5}{2\sqrt{6}}$

Question 3

The number of solutions of $\sin^2(2x) = \frac{3}{4}$, for $0 \leq x \leq 2\pi$, is

- A. 1
- B. 2
- C. 4
- D. 6
- E. 8

Question 4

If $y = \sin^{-1}\left(\frac{4}{x}\right)$, $x > 4$, then $\frac{dy}{dx}$ is equal to

- A. $-\frac{4}{x\sqrt{x^2-16}}$
- B. $\frac{x}{\sqrt{x^2-16}}$
- C. $-\frac{4}{x\sqrt{x^2-4}}$
- D. $-\frac{4}{\sqrt{x^2-16}}$
- E. $\frac{4}{x\sqrt{x^2-16}}$

Question 5

Which one of the following is a polar form of $-\sqrt{3} - i$?

- A. $2\text{cis}\left(-\frac{2\pi}{3}\right)$
- B. $2\text{cis}\left(\frac{\pi}{6}\right)$
- C. $2\text{cis}\left(\frac{\pi}{3}\right)$
- D. $2\text{cis}\left(\frac{5\pi}{6}\right)$
- E. $2\text{cis}\left(\frac{7\pi}{6}\right)$

Question 6

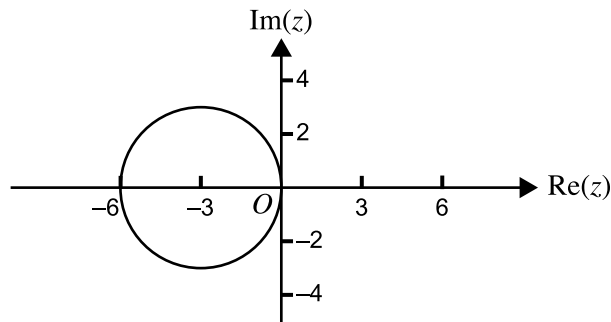
If $z^2 = 4\text{cis}\left(\frac{4\pi}{3}\right)$, then z is equal to

- A. $1 - \sqrt{3}i$ or $-1 + \sqrt{3}i$
- B. $\sqrt{3} + i$ or $-\sqrt{3} - i$
- C. $-1 + \sqrt{3}i$ or $-1 - \sqrt{3}i$
- D. $\sqrt{3} - i$ or $-\sqrt{3} + i$
- E. $1 - \sqrt{3}i$ or $1 + \sqrt{3}i$

Question 7

If $P(z) = z^3 - 2z^2 + 4z - 8$, $z \in \mathbb{C}$, then a linear factor of $P(z)$ is

- A. 2
- B. $2i$
- C. $z + 2$
- D. $z + 2i$
- E. $z^2 + 4$

Question 8

Which one of the following, where $z \in \mathbb{C}$, is the equation of the circle in the diagram above?

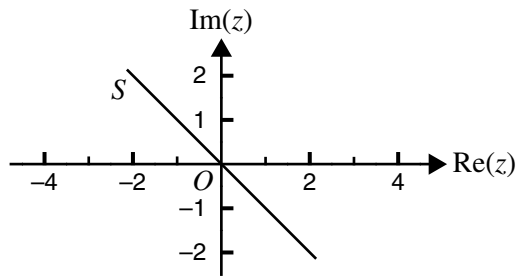
- A. $(z + 3)(\bar{z} + 3) = 9$
- B. $(z - 3)(\bar{z} + 3) = 9$
- C. $(z + 3)(\bar{z} - 3) = 9$
- D. $(z - 3)(\bar{z} - 3) = 9$
- E. $(z + 3i)(\bar{z} - 3i) = 9$

Question 9

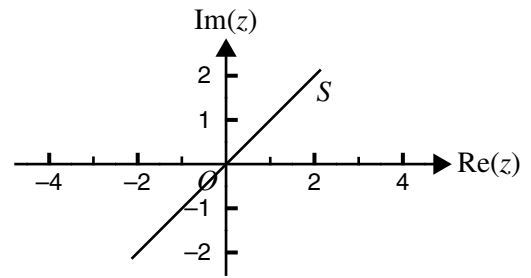
Which one of the following shows the region of the complex plane specified by

$$S = \{z: |z-1| - |z+i| = 0, z \in \mathbb{C}\}?$$

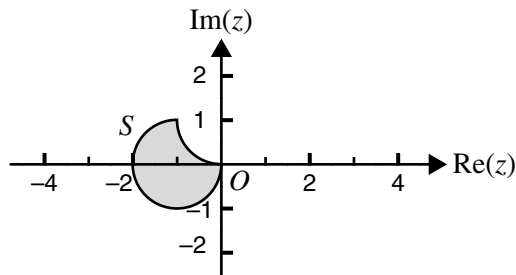
A.



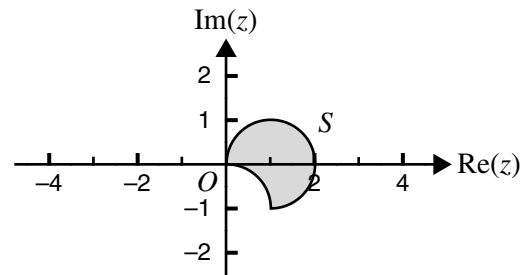
B.



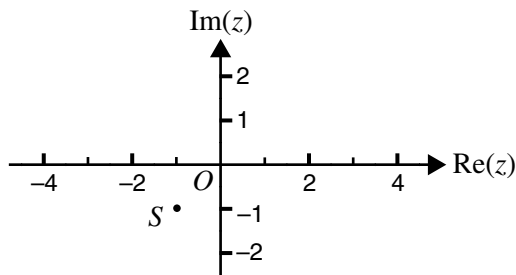
C.



D.



E.



Question 10

With a suitable substitution, $\int_0^{\frac{\pi}{6}} \cos^3(2x) dx$ can be expressed as

A. $\frac{1}{2} \int_0^{\frac{1}{2}} (1-u^2) du$

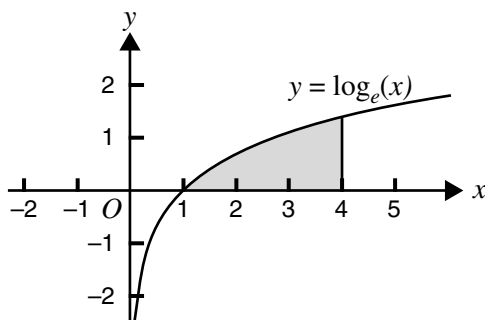
B. $\frac{1}{2} \int_0^{\frac{1}{2}} (u^2-1) du$

C. $\frac{1}{2} \int_0^{\frac{\sqrt{3}}{2}} (1-u^2) du$

D. $2 \int_0^{\frac{1}{2}} (u^2-1) du$

E. $2 \int_0^{\frac{\sqrt{3}}{2}} (1-u^2) du$

Question 11

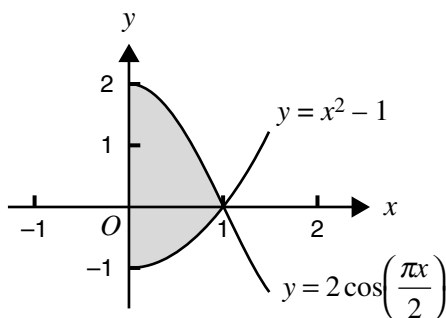


The shaded region in the diagram above is bounded by the graph of $y = \log_e(x)$, the x -axis, and the line with equation $x = 4$. The trapezium rule with three equal intervals is used to estimate the area of the shaded region.

The value obtained is $\log_e(a)$, where a is

- A. $\sqrt{17}$
- B. $\sqrt{24}$
- C. $\sqrt{96}$
- D. 12
- E. 13

Question 12



The shaded region in the diagram above is bounded by the y -axis, and the curves with equations $y = x^2 - 1$ and $y = 2 \cos\left(\frac{\pi x}{2}\right)$.

The **exact** value of the area of the shaded region is

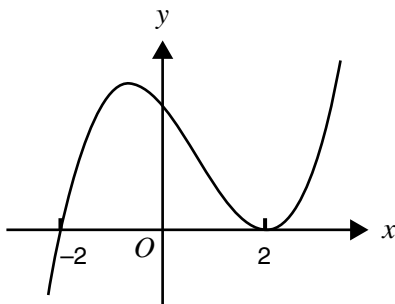
- A. $\frac{10}{3}$
- B. $\frac{14}{3}$
- C. $\frac{4}{\pi} - \frac{4}{3}$
- D. $\frac{4}{\pi} - \frac{2}{3}$
- E. $\frac{4}{\pi} + \frac{2}{3}$

Question 13

Which one of the following is an antiderivative of $\frac{3}{x(3-x)}$ for $0 < x < 3$?

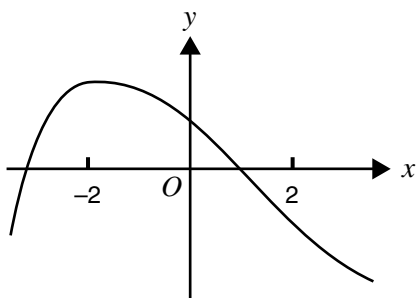
- A. $3(\log_e(x) - \log_e(3-x))$
- B. $\log_e(x) - \log_e(3-x)$
- C. $\log_e(x-3) - \log_e(x)$
- D. $3(\log_e(x) + \log_e(3-x))$
- E. $\log_e(x) + \log_e(3-x)$

Question 14

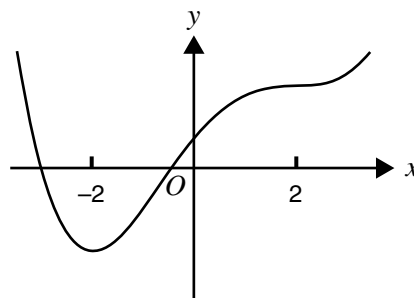


Part of the graph of the derivative of a function f is shown above. Which one of the following could be the graph of the function f ?

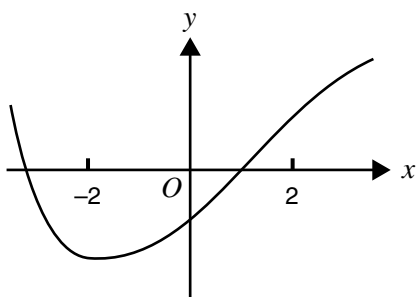
A.



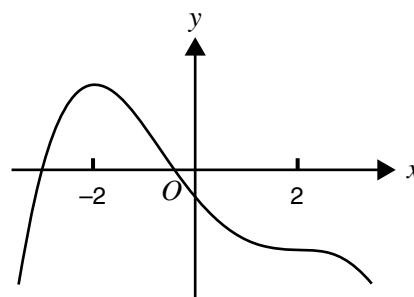
B.



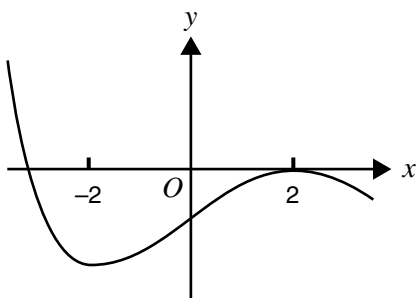
C.



D.



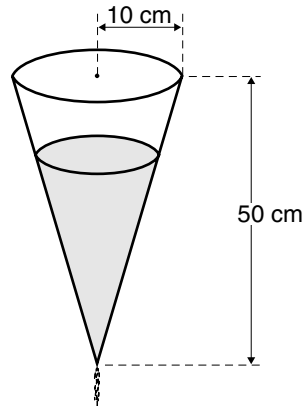
E.



Question 15

If $f'(x) = 2 \sin^2\left(\frac{x}{2}\right) - 1$ and $f\left(\frac{\pi}{2}\right) = 0$, then $f(x)$ is equal to

- A. $-\sin(x)$
- B. $1 - \sin(x)$
- C. $\sin(x) + 1$
- D. $\sin(x) - 1$
- E. $-1 - \sin(x)$

Question 16

Water is draining from a cone-shaped funnel at the constant rate of $600 \text{ cm}^3/\text{min}$.

The cone has height 50 cm and base radius 10 cm.

Let h cm be the depth of water in the funnel at time t min.

The rate of **decrease** of h , in cm/min, is given by

- A. 12
- B. $\frac{100\pi}{3}$
- C. $\frac{15000}{\pi h^2}$
- D. $24\pi h^2$
- E. $\frac{18}{\pi}$

Question 17

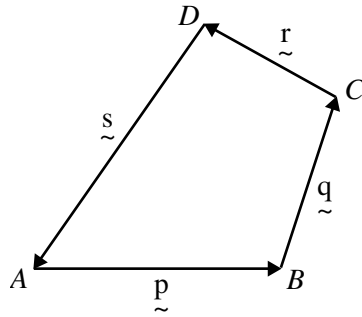
Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = \cos\left(\frac{x}{2}\right)$, with initial condition $y = 2$ when $x = 0$.

The approximation obtained for y when $x = 0.4$ is given by

- A. $2 + 0.4 \cos(0.1)$
- B. $2 + 0.4 \cos(0.2)$
- C. $2.2 + 0.2 \cos(0.1)$
- D. $2.2 + 0.2 \cos(0.2)$
- E. $2 + 0.2 \cos(0.1) + 0.2 \cos(0.2)$

Question 18

In the quadrilateral $ABCD$, $\overrightarrow{AB} = \underline{\underline{p}}$, $\overrightarrow{BC} = \underline{\underline{q}}$, $\overrightarrow{CD} = \underline{\underline{r}}$, and $\overrightarrow{DA} = \underline{\underline{s}}$ as shown below.



Which one of the following statements is true?

- A. $\underline{\underline{p}} + \underline{\underline{q}} = \underline{\underline{r}} + \underline{\underline{s}}$
- B. $\underline{\underline{p}} + \underline{\underline{r}} = \underline{\underline{q}} + \underline{\underline{s}}$
- C. $\underline{\underline{p}} - \underline{\underline{q}} = \underline{\underline{r}} - \underline{\underline{s}}$
- D. $\underline{\underline{p}} - \underline{\underline{r}} = \underline{\underline{q}} + \underline{\underline{s}}$
- E. $\underline{\underline{p}} + \underline{\underline{q}} = -\underline{\underline{r}} - \underline{\underline{s}}$

Question 19

Which one of the following is a unit vector opposite in direction to $\underline{\underline{i}} - 2\underline{\underline{j}} + 3\underline{\underline{k}}$?

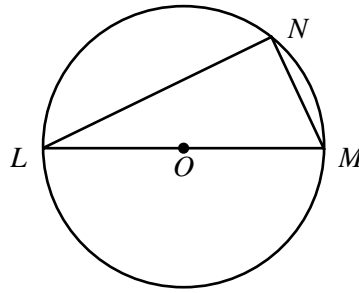
- A. $\frac{1}{\sqrt{14}}(-\underline{\underline{i}} + 2\underline{\underline{j}} - 3\underline{\underline{k}})$
- B. $\frac{1}{\sqrt{14}}(-\underline{\underline{i}} - 2\underline{\underline{j}} + 3\underline{\underline{k}})$
- C. $\frac{1}{\sqrt{6}}(-\underline{\underline{i}} + 2\underline{\underline{j}} - 3\underline{\underline{k}})$
- D. $\frac{1}{\sqrt{6}}(-\underline{\underline{i}} - 2\underline{\underline{j}} + 3\underline{\underline{k}})$
- E. $\frac{1}{\sqrt{2}}(-\underline{\underline{i}} + 2\underline{\underline{j}} - 3\underline{\underline{k}})$

Question 20

Let O denote the point $(0, 0)$, R the point $(-2, 1)$, and S the point $(-c, 2c)$ where $c \in R$.

The scalar product $\overrightarrow{OS} \cdot \overrightarrow{RS}$ is equal to

- A. $4c - 5$
- B. $5 - 4c$
- C. $4c - 5c^2$
- D. $5c^2 - 4c$
- E. $(c^2 - 2c)\underline{\underline{i}} + (4c^2 - 2c)\underline{\underline{j}}$

Question 21

In the diagram above, LOM is a diameter of the circle with centre O .

N is a point on the circumference of the circle.

Let $\overrightarrow{OM} = \underline{r}$ and $\overrightarrow{LN} = \underline{q}$.

Which one of the following must be true?

- A. $2\underline{r} + \underline{q} = \underline{0}$
- B. $2\underline{r} - \underline{q} = \underline{0}$
- C. $\underline{r} \cdot \underline{q} = \underline{q} \cdot \underline{q}$
- D. $2\underline{r} \cdot \underline{q} = \underline{q} \cdot \underline{q}$
- E. $2\underline{r} \cdot \underline{q} = -\underline{q} \cdot \underline{q}$

Question 22

Let $\underline{i}$ and $\underline{j}$ be unit vectors in the east and north directions respectively.

At time t , $t \geq 0$, the position vector of particle L is given by $\underline{r} = (5t - 8)\underline{i} + (t^2 - 5t + 6)\underline{j}$, and the position vector of particle M is given by $\underline{s} = (t^2 - t)\underline{i} + (3 - t)\underline{j}$.

Particle L is directly north of particle M at time

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 23

The position vector of a particle at time t is given by $\underline{r} = 4t\underline{i} - e^{2t}\underline{j} + 5\underline{k}$, $t \geq 0$.

The initial speed of the particle is

- A. $\sqrt{12}$
- B. $\sqrt{17}$
- C. $\sqrt{20}$
- D. $\sqrt{26}$
- E. $\sqrt{45}$

Question 24

The acceleration, in m/s^2 , of a particle at time t s, $t \geq 0$, is given by $\ddot{\mathbf{r}} = \cos(t) \mathbf{i} - \sin(t) \mathbf{j}$.

The initial velocity of the particle was $(\mathbf{i} + \mathbf{j})$ m/s.

The velocity of the particle, in m/s, at time t s is given by

- A. $-\sin(t) \mathbf{i} - \cos(t) \mathbf{j}$
- B. $\sin(t) \mathbf{i} + \cos(t) \mathbf{j}$
- C. $(\sin(t) + 1) \mathbf{i} + \cos(t) \mathbf{j}$
- D. $\sin(t) \mathbf{i} + (\cos(t) + 1) \mathbf{j}$
- E. $(1 - \sin(t)) \mathbf{i} + (2 - \cos(t)) \mathbf{j}$

Question 25

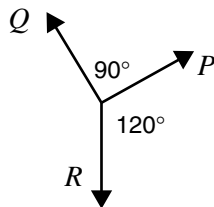
A body of mass 12 kg is pulled along rough, horizontal ground by a horizontal force of 66 newtons. The body is moving with an acceleration of 0.5 m/s^2 .

The coefficient of sliding friction between the body and the surface, correct to two decimal places, is

- A. 0.06
- B. 0.51
- C. 0.56
- D. 0.61
- E. 1.96

Question 26

The following diagram shows a particle in equilibrium in a plane under the action of three forces of magnitudes P , Q and R .

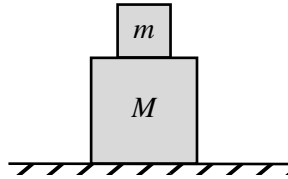


Which one of the following statements is **not** correct?

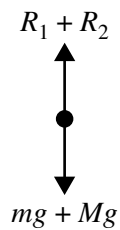
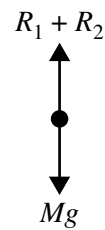
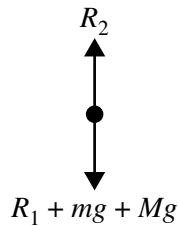
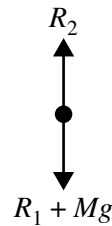
- A. $P = R \cos(60^\circ)$
- B. $Q = R \cos(30^\circ)$
- C. $R = Q \cos(30^\circ)$
- D. $Q \cos(60^\circ) = P \cos(30^\circ)$
- E. $P \cos(60^\circ) + Q \cos(30^\circ) = R$

Question 27

A small mass of m kg sits on top of a larger mass of M kg on rough, level ground. The two masses are at rest. There is a normal reaction force of magnitude R_1 on the smaller mass due to its contact with the larger mass. There is a normal reaction force of magnitude R_2 on the larger mass due to its contact with the ground. All forces are in newtons.



Which one of the following diagrams correctly shows the forces acting on the **larger** mass?

A.**B.****C.****D.****E.****Question 28**

A particle moves with constant acceleration in a straight line so that at time t , $t \geq 0$, its velocity is v and its displacement from a fixed point on the line is x .

Which one of the following equations could **not** be true?

- A.** $t = v - 1$
- B.** $t = x^2 - 1$
- C.** $x = t^2 - 1$
- D.** $x = v^2 - 1$
- E.** $v = t - 1$

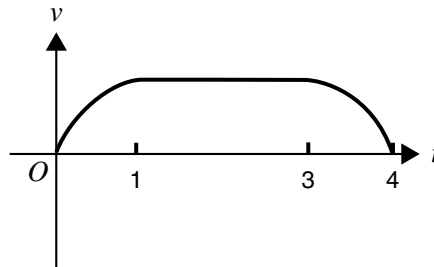
Question 29

At time t s, $t \geq 0$, the velocity, v m/s, of a particle moving in a straight line is given by $v = \cos(t) + \sqrt{3} \sin(t) - 1$.
For what value of t does the particle first attain its maximum **speed** of 3 m/s?

- A. $\frac{\pi}{6}$
 B. $\frac{\pi}{3}$
 C. $\frac{7\pi}{6}$
 D. $\frac{4\pi}{3}$
 E. The particle never attains a speed of 3 m/s.

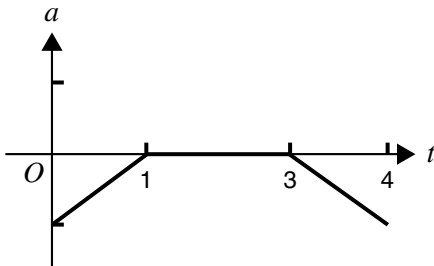
Question 30

The following is the velocity-time graph of a racing car over a short course.

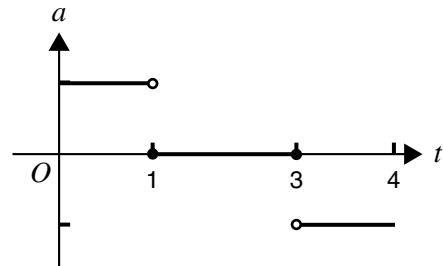


Which one of the following could be the acceleration-time graph of the car's motion?

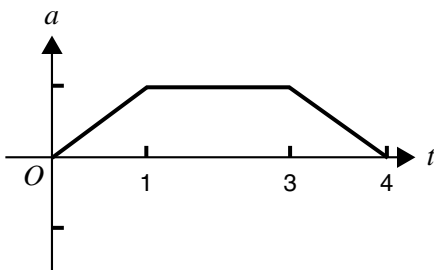
A.



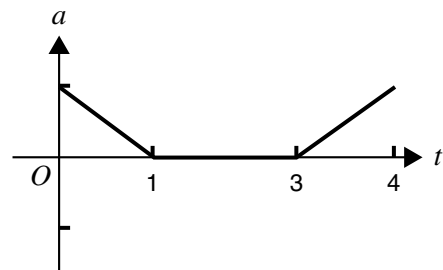
B.



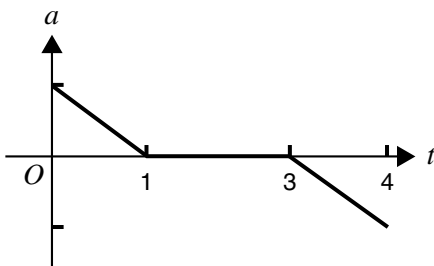
C.



D.



E.

**TURN OVER**

Working space

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK



Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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SPECIALIST MATHEMATICS

Written examination 1

(Facts, skills and applications)

Monday 3 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II

QUESTION AND ANSWER BOOK

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At the end of the examination

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Instructions for Part II

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an **exact** answer is required to a question, appropriate working must be shown.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

A particle of mass 5 kg moves in a straight line.

At time t s, $t \geq 0$, it has velocity v m/s, where $v = 4.5t + \cos(2t)$.

- a. Find an expression for the acceleration of the particle at time t .

1 mark

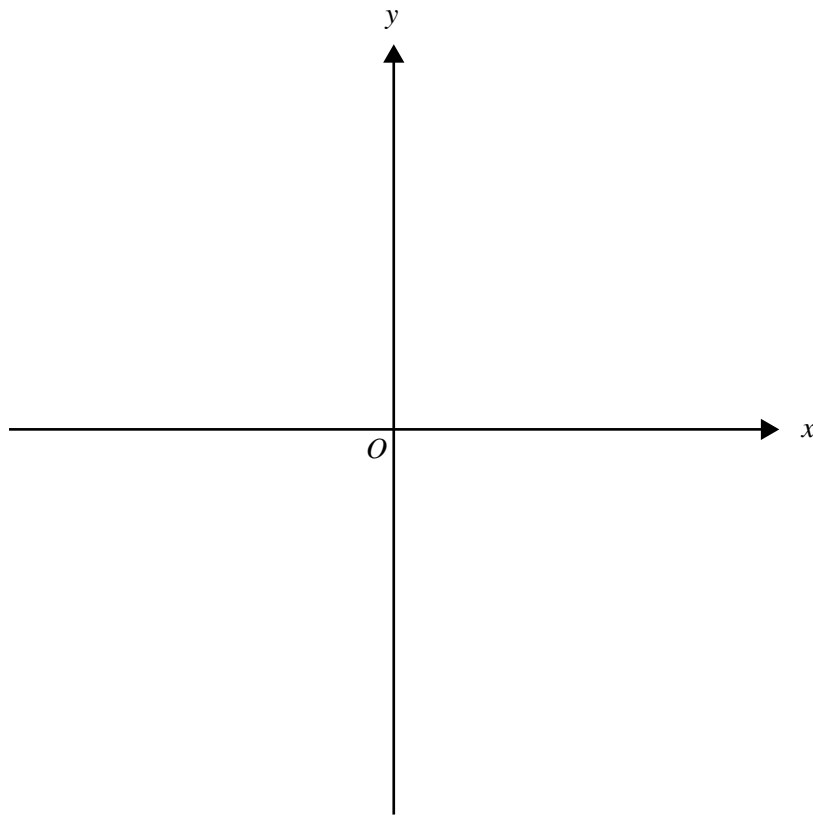
- b. Find the minimum resultant force acting on the particle during its motion.

2 marks

Question 2

On the axes below, sketch the graph of $f: \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = \frac{x^2 - 6}{2x}$.

Give the equations of any asymptotes, and the coordinates of any intercepts and any turning points.



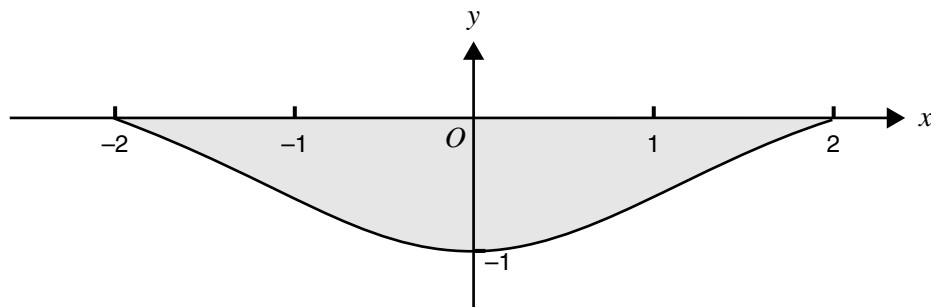
4 marks

$y = xe^{3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0$, where $m, n \in R$. Find the values of m and n .

3 marks

Question 4

The graph of $y = 1 - \frac{8}{x^2 + 4}$ is shown below for $-2 \leq x \leq 2$.



- a.** Find the **exact** value of the area of the shaded region.

3 marks

- b.** The shaded region is rotated about the y-axis to form a solid of revolution. Express the volume of this solid of revolution as a definite integral and hence find the volume correct to three significant figures.

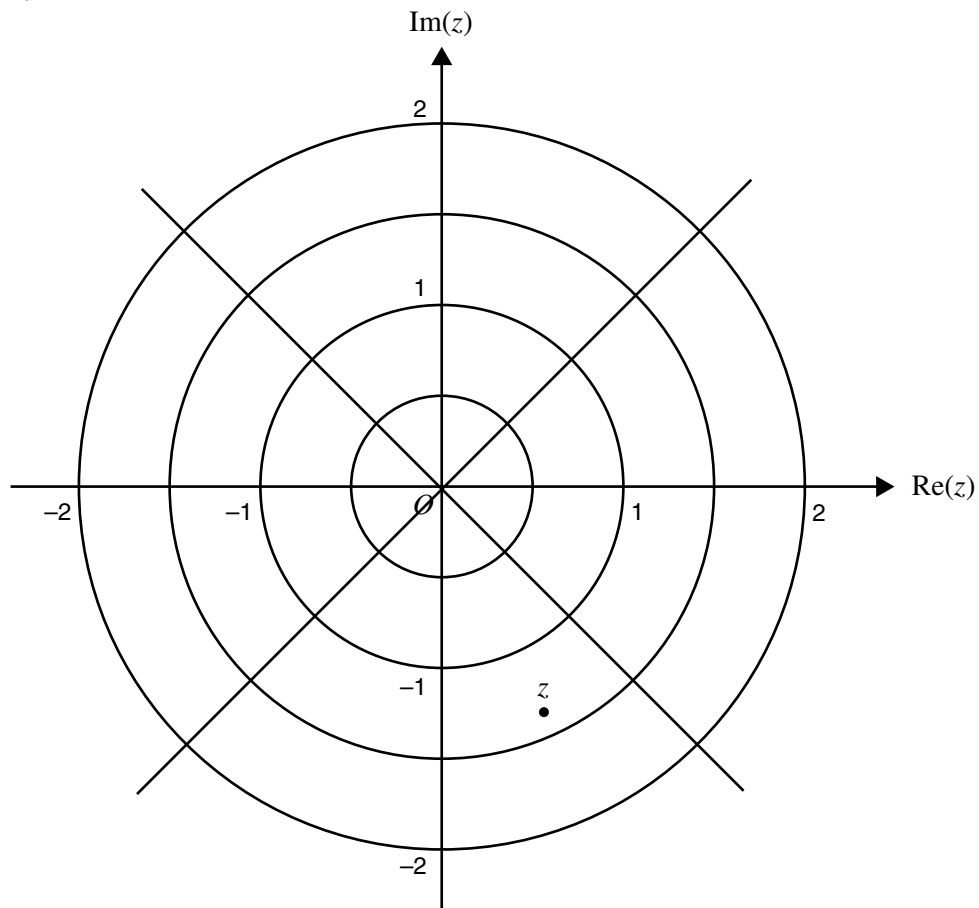
3 marks

Question 5

The complex number $z = \sqrt{2} \operatorname{cis}(\theta)$ is plotted on the Argand diagram below.

On the same diagram, plot and label clearly the following three complex numbers.

- a. $u = z^2$
- b. $v = \frac{1}{z}$
- c. $w = z^2 + \frac{1}{z}$



4 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a + b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$
hyperbola:	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a)f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + h f(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$