VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS (CAS)

PILOT STUDY

Written examination 2

(Analysis task)

Monday 10 November 2003

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	55

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and one approved CAS calculator (memory may be retained) and/or one scientific calculator. For the TI-92, Voyage 200 or approved computer based CAS, their full functionality and/or one scientific calculator may be used, but other programs or files are not permitted.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 11 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Instructions

- Answer **all** questions in the spaces provided.
- In questions where more than 1 mark is available, appropriate working must be shown.
- A decimal approximation will not be accepted if an exact answer is required to a question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A manufacturer makes metal rods whose lengths are normally distributed with mean 140.0 cm and standard deviation 1.2 cm.

a. Find the probability, correct to three decimal places, that a randomly selected metal rod is longer than 141.5 cm.

2 marks

b. A rod has a **size fault** if it is not within d cm either side of the mean. The probability of a rod having a **size fault** is 0.15. Find the value of d, correct to one decimal place.

2 marks

c. A random sample of 12 rods is taken from a crate containing a very large number of rods. Find the probability, correct to three decimal places, that the sample contains exactly 2 rods with a **size fault**.

2 marks

d. A particular box of 25 rods has 4 rods in it which have **size faults**. A sample of 12 rods is withdrawn without replacement. Find the probability, correct to three decimal places, that the sample contains at least 2 rods with a **size fault**.

2 marks

e. The sales manager is considering at what price, x dollars, to sell each rod. The materials cost \$5. The rods are sorted into three bins. 15% of all the rods manufactured have a size fault and another 17% of all the rods have other faults. The profit, Y dollars, is a random variable whose probability distribution is shown in the table below.

Bin	**Description**	**Profit(\$$y$)**	$\boldsymbol{Pr(Y = y)}$
A	Good rods – these are sold for \$$x$ each	$x - 5$	k
B	Rods with a size fault – these are not sold but are recycled	0	0.15
C	Rods with other faults – these are sold at a discount of \$3 each	$x - 8$	0.17

i. Find the value of k.

ii. Find the mean of Y in terms of x.

iii. Hence or otherwise, find, correct to the nearest cent, the selling price of good rods so that the mean profit is zero.

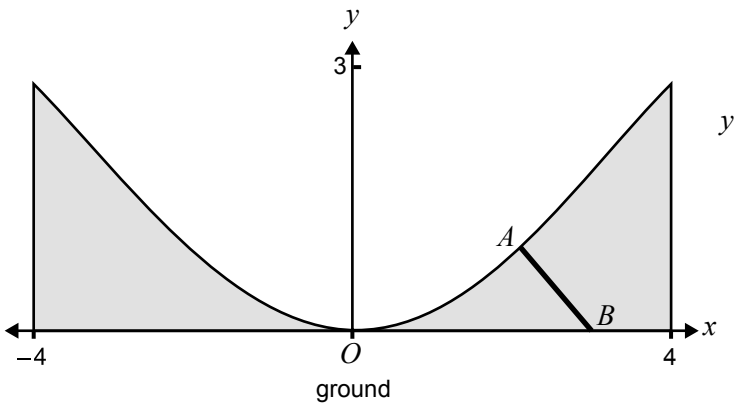
iv. The rods are stored in bins until there is a large number ready to be sold.
What proportion of the rods ready to be sold are good rods?

1 + 1 + 1 + 1 = 4 marks

Total 12 marks

Question 2

Andrew is making a skateboard ramp. He draws a cross-section diagram with coordinate axes as shown below.



$y = 2 - 2\cos\left(\frac{x}{2}\right)$

The curve has the equation $y = 2 - 2\cos\left(\frac{x}{2}\right)$, $-4 \leq x \leq 4$. All measurements are in metres; the horizontal length of the structure is 8 metres.

a. How many metres above the ground is the highest point of the ramp? Give your answer to two decimal places.

1 mark

b. Show that the gradient of the ramp is always less than or equal to 1.

2 marks

c. **i.** Write a definite integral which gives the area of the shaded region.

ii. Find the area of the shaded region, correct to two decimal places.

2 + 1 = 3 marks

There is a supporting beam AB on the structure as shown. A is a point on the curve one metre vertically above the x-axis. B is a point on the x-axis such that AB is normal to the curve at A.

d. **i.** Find the exact x-coordinate of A.

ii. Find the exact value of the gradient of the normal to the curve at A.

iii. Find the exact length of AB.

2 + 2 + 3 = 7 marks

Total 13 marks

TURN OVER

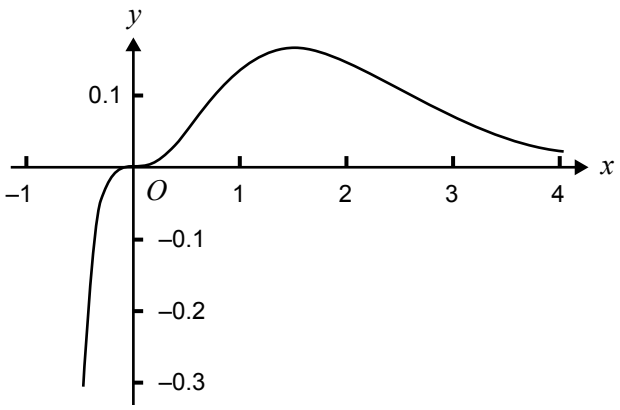
Question 3

Consider the function $f: R \to R, f(x) = x^3 e^{-2x}$

a. Find $f'(x)$.

1 mark

b. The graph of $y = f(x)$ is as shown.



Find the exact coordinates of the two stationary points and state their nature.

2 marks

c. **i.** Find an equation of the tangent to the graph of $y = f(x)$ at the point where $x = 1$.

ii. Find an equation of the tangent to the curve at the point (0, 0).

iii. Show that the tangents of parts **i.** and **ii.** are the only two tangents to the curve which pass through the origin.

3 + 1 + 3 = 7 marks

d. Consider the continuous probability density function with rule $g(x) = kx^3e^{-2x}$ for $x \geq 0$ and 0 elsewhere, where k is a positive real number.

i. Find the value of k.

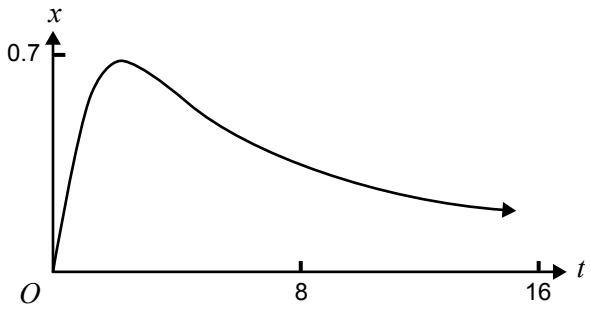
ii. Find, correct to two decimal places, the median value of the distribution of this probability density function.

2 + 2 = 4 marks

Total 14 marks

Question 4

A tranquilliser is injected into a muscle from which it enters the bloodstream. The concentration, x mg/L, of the tranquilliser in the bloodstream, may be modelled by the equation $x = \frac{3t}{5+t^2}$, $t \geq 0$, where t is the number of hours after the injection is given. The graph of this equation is shown.



a. Find the exact number of hours after the injection is given when the tranquilliser concentration is greatest. Also find the exact value of this maximum concentration.

2 marks

b. The derivative of x with respect to t gives a measure of the rate of absorption of the tranquilliser into the bloodstream.

What is the exact rate of absorption one hour after the injection is given?

1 mark

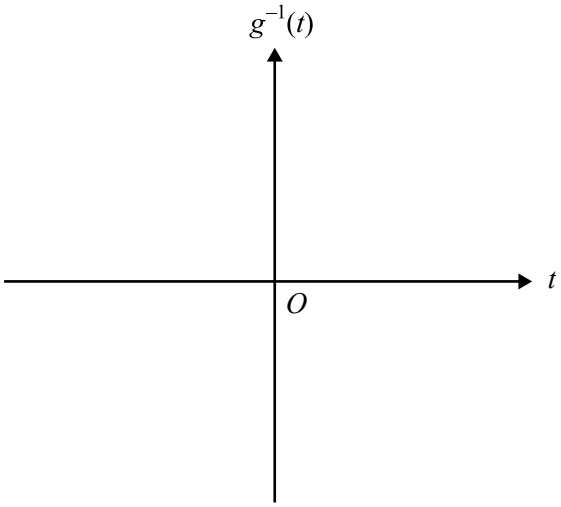
c. The tranquilliser is effective when the concentration is at least 0.4 mg/L.

Find the exact value of the length of time in hours for which the tranquilliser is effective.

3 marks

d. **i.** What is the least value of a such that the function $g\colon [a, \infty) \to R$, $g(t) = \dfrac{3t}{5+t^2}$, has an inverse function?

ii. For this value of a, sketch the graph of g^{-1} on the axes below. Label any end-point with its coordinates. Label any asymptote with its equation.



iii. Find the rule for g^{-1}.

1 + 3 + 3 = 7 marks

It is discovered that the drug will produce undesirable side-effects if its concentration exceeds 1 mg/L at any time. A modification to the drug is proposed so that the concentration in the blood, y mg/L, at time t hours after the injection is given is modelled by the equation

$$y = \frac{3t}{p + t^2}, \ 0 \leq t \leq 8, \text{ where } p \text{ is a parameter.}$$

e. Find the least value which p may take if the concentration is to be always less than 1 mg/L.

3 marks

Total 16 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

PILOT STUDY

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

approximation: $f(x+h) \approx f(x) + hf'(x)$

average value: $\frac{1}{b-a}\int_a^b f(x)dx$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x}dx = \log_e|x| + c$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Statistics

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions

	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\, {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)$

Continuous distributions

	$\Pr(a < X < b)$	mean	variance
general	$\int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty}(x-u)^2 f(x)dx$ $= \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0,1)$. $f(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$		

Table 1 Normal distribution – cdf

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET