2004 Specialist Maths: GA 3: Written examination

GENERAL COMMENTS

The number of students who sat for the 2004 examination was 6150, compared to 6231 in 2003. As in 2003, students had to answer five questions worth a total of 60 marks, with each question worth between eight and 15 marks.

The data suggests that, on average, students' performances were similar to those of students in 2003. The mean and median scores, out of a possible 60, were 26.2 and 27.2 respectively, compared with 26.3 and 27 in 2003. On the other hand, only 0.81% of students scored 90% or more of the marks, compared with 1.3% in 2003. Three students scored full marks, compared with four in 2003. Hence, it would appear that there were sufficient marks 'available' for students to show what they could do, but that it was slightly more challenging to get full marks. At the lower end of the scale, about 4.7% of students scored four or less marks.

The average score for the five questions, expressed as a percentage of the total marks available, was about 52%, 44%, 45%, 35% and 33% respectively. A smaller percentage was to be expected for Question 5, both because the final part of the question was quite challenging and because it is common for many students to have difficulty sustaining their performance until the end of the exam. However, the low average for Question 4 was disappointing and unexpected. Less than 10% of students scored full marks for Question 4e, and more than 90% did not score any marks for Question 4f. Clearly, the last parts of Question 4 were very challenging for most students.

There were two 'show that…' questions and two 'hence'-type questions on the 2004 paper. Teachers should remind students that the former question type is intended to help keep them on track and enable access to subsequent parts of the same question; while in the latter question type, previously derived results must be used.

There was scope for effective use of graphics calculators on the 2004 examination paper, and many students were able to use the technology successfully. The most obvious example of this was in Question 3aii., where the equation could not be solved analytically. Other places where the technology could be used to the student's advantage were Question 2d (solving an equation involving a circular function approximately); Question 2e (solving approximately for t); and Question 5c, where a numerical derivative (but not an analytical one) was the first stage in answering the question. Students could have used one of the 'solve' features of a graphics calculator to find the solution for m in Question 1fi.

There were still some students who disregarded instructions such as 'find the exact…' and worked numerically rather than analytically. Thus, in Question 3a some students gave a numerical approximation (6283L). This was also common in Question 3cii., where the integration was too difficult for most students, despite the lead-in parts. It needs to be emphasised that if an exact answer is required, full marks cannot be obtained if only a decimal approximation is given.

It is also necessary to remind students that appropriate working must be shown in questions worth more than one mark, and that examiners must be able to see the steps that students use to reach a solution. Marks are allocated to methods and procedures, not just to numerical or algebraic answers.

The phrase 'use calculus to…' was employed on the 2004 paper in Questions 1b and 5dii., the intention being that students would show the appropriate antiderivatives in the course of their working. On the other hand, no such instruction in Question 5c meant that a numerical derivative was acceptable (and very much preferred) in the steps towards a solution. If an analytical derivative had been required in a question like this, more marks would need to have been allocated.

As was the case in some past examinations, students often struggled with questions involving the more advanced aspects of complex numbers. In this examination, many students had difficulty with the parts of Question 4 that moved from complex numbers to vectors, and almost all students found the last two parts of this question beyond their capabilities. Perhaps more intensive revision work on complex number analysis questions would be beneficial leading up to the examination period, especially as this material is often covered early in the course.

SPECIFIC INFORMATION

Question 1ai.

Marks	0	1	Average
%	16	84	**0.8**

$6t - 3t^2$

This question was quite well done. A few students used the product rule (rather than simple expansion), often with unfortunate results. A handful integrated rather than differentiated.

Question 1aii.

Marks	0	1	2	Average
%	20	12	68	**1.6**

$a_{max} = 3$ (m/s^2)

This question was quite well done. The main errors were: solving $a = 0$; finding $t = 1$ but forgetting to state the value of a_{max}; and stating the answer without showing any working.

Question 1b

Marks	0	1	2	3	Average
%	28	13	13	46	**1.9**

$\frac{49}{4}$ (s)

This was reasonably well attempted by most students, and those who appreciated the symmetry of the situation and used calculus had little difficulty. Students who had a third integral involving T usually came to strife; and those who managed to find the correct answer without using calculus (that is, no antiderivative was shown) went unrewarded.

Question 1c

Marks	0	1	Average
%	55	45	**0.5**

as given

Less than half the students were able to write a correct equation of motion and show how it led to the **given** result. Some students simply removed the brackets from the given equation and attempted to justify the presence of the individual terms, but without success.

Question 1d

Marks	0	1	Average
%	32	68	**0.7**

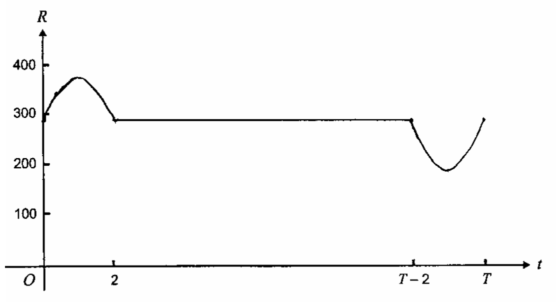
371 (N)

This question was generally well done.

Question 1e

Marks	0	1	2	3	Average
%	37	27	12	24	**1.3**

Parabolic arc $0 \le t \le 2$ with turning point at (1, 371), joining a horizontal line at $R = 284$, $2 < t < T - 2$, joining a parabolic arc $T - 2 \le t \le T$ with turning point at ($T - 1$, 197).



Few students were able to obtain the three separate sections, although some students found the first two sections correctly. Many attempts had discontinuities over the domain; others had straight line segments or cubic curves in the first and last stages of motion. This question part was a good test of students' analytical skills.

Question 1fi.

Marks	0	1	2	3	Average
%	50	7	5	38	**1.4**

54.6 (kg)

This question was not well answered. Many students showed a lot of working that led to the incorrect conclusion that the mass of the boy was equal to that of the girl. Other incorrect equations of motion for the boy sometimes led to a negative answer for the boy's mass. Some students, otherwise on the right track, over complicated their working by substituting acceleration as a formula rather than as a numerical value.

Question 1fii.

Marks	0	1	Average
%	89	11	**0.1**

no (with a correct reason)

Of the minority of students who correctly answered 'no', very few were able to explain that time $t = 1$ corresponded to the maximum reaction force for the girl but the minimum for the boy, so that at other times, $R_{boy}>371$ and $R_{girl}<371$. A few students mistakenly appealed to 'symmetry' to suggest that there would be another time, one second before the lifts stopped, when the reaction forces would be the same.

Question 2a

Marks	0	1	Average
%	41	59	**0.6**

(0, 2)

This question was reasonably well done, although many students failed to answer the question, giving their otherwise correct answer as a vector (the question specifically asked for the '**coordinates** of *P*'). A few students gave (0, 0) as their answer.

Question 2b

Marks	0	1	2	Average
%	14	14	72	**1.7**

$$\underset{\sim}{v}(t) = \frac{4}{15}\cos\left(\frac{2}{15}t\right)\underset{\sim}{i} + \left(\frac{5}{3} - \frac{5}{9}\cos\left(\frac{1}{3}t\right)\right)\underset{\sim}{j}$$

This was quite well done. Some students gave the magnitude of the velocity as their answer; and there was the usual sprinkling of sign errors.

Question 2c

Marks	0	1	2	3	Average
%	53	8	16	23	**1.1**

13.5°

This question wasn't answered as well as might have been expected. A common conceptual error was to work with the position vector, rather than the velocity vector. Many students gave the angle (76.5°) with the $\underset{\sim}{i}$ direction, rather than the (forward) $\underset{\sim}{j}$ direction.

Question 2d

Marks	0	1	2	Average
%	63	12	25	**0.7**

11.8 (s)

Not particularly well done, with many students unable to make the link that the velocity component in the $\underset{\sim}{i}$ direction must be zero. Too many students set the $\underset{\sim}{i}$ component of the position vector to zero. A few students tried to work with the acceleration vector.

Question 2e

Marks	0	1	2	3	4	Average
%	45	9	21	5	19	**1.5**

no

This question was not particularly well done, although the efficient use of a graphics calculator should have made this a straightforward question. Those students who tried to find when the ball had an x-coordinate of 1 often failed to realise that $\frac{5\pi}{4}$ corresponded to the early part of the ball's journey, whereas $\frac{25\pi}{4}$ was the relevant value of t. Those students who first found when the ball had a y-coordinate of 33 and then checked its x-coordinate often fared better.

Question 3ai.

Marks	0	1	Average
%	23	77	**0.8**

2000π (L)

This was quite well done. The obvious error was to give the numerical value 6283. Some students surprisingly gave their answer as 360 000, presumably because they had their calculators set to degree, rather than radian, mode.

Question 3aii.

Marks	0	1	Average
%	52	48	**0.5**

1.34 (m)

This question was not well done, with less than half the students able to use their calculator to solve the equation numerically.

Question 3b

Marks	0	1	2	3	Average
%	26	13	20	41	**1.8**

$$\frac{dh}{dt} = \frac{2000}{8000\text{Tan}^{-1}(h) + \frac{8000h}{1+h^2}}$$

This was reasonably well done, with most students recognising that related rates were involved. The main error was in inverting $\frac{dV}{dh}$: inverting the two terms separately was all too common.

Question 3ci.

Marks	0	1	Average
%	64	36	**0.4**

$$4\int_0^{\sqrt{3}} \left(\text{Tan}^{-1}(h) + \frac{h}{1+h^2} \right) dh$$

This was not well done by the majority of students. The most common errors included notation errors (for example, the lack of *dh*); the lack of any terminals at all; or errors in correctly using the answer from part b.

Question 3cii.

Marks	0	1	2	Average
%	87	2	11	**0.2**

as given

In this question, most students were unable to make the link with what had gone before – specifically, that the expression whose antiderivative was required was a multiple of the derivative of *V* found earlier in the question. A handful of students attempted integration by parts, usually unsuccessfully.

Question 4ai.

Marks	0	1	Average
%	12	88	**0.9**

$-1 + 2i$

This question was well done.

Question 4aii.

Marks	0	1	2	Average
%	29	23	47	**1.2**

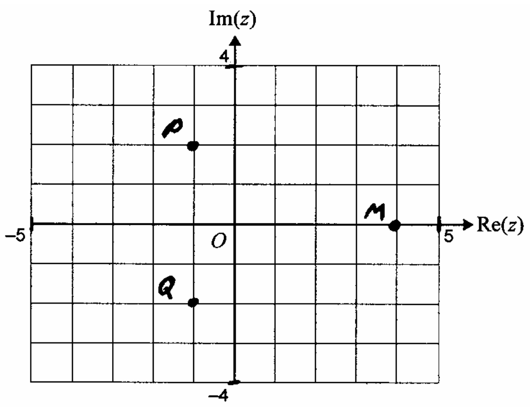
$a = -2$, $b = -3$, $c = -20$

Reasonable attempts were made in this part; however, many students got lost in the algebra needed to expand the three linear factors, and sign errors were common. Some students did not group the complex terms before finding *a*, *b* and *c*. Other students attempted to substitute each root into the cubic equation to solve for the coefficients, which proved to be a very difficult method to employ successfully due to the complexity of the substitutions.

Question 4b

Marks	0	1	Average
%	19	81	**0.8**

$P(-1, 2)$, $Q(-1, -2)$ and $M(4, 0)$.



This question was well done, but occasionally the labelling of *P* and *Q* were reversed.

Question 4c

Marks	0	1	Average
%	33	67	**0.7**

$-5\underset{\sim}{i} - 2\underset{\sim}{j}$

This was reasonably well done, with sign errors the main difficulty. Some students were unable to make the switch to vectors or use correct vector notation.

Question 4di.

Marks	0	1	Average
%	52	48	0.5

$(-d-1)\underset{\sim}{i}-2\underset{\sim}{j}$

Poor algebraic manipulation of vectors was the main difficulty here.

Question 4dii.

Marks	0	1	2	Average
%	75	9	16	0.4

–1.8

In this question, only a minority of students realised that the previous two vectors were perpendicular (an angle in a semi-circle is a right angle). Unfortunately, those students who did equate the scalar product to zero often had earlier errors, or made subsequent manipulation errors.

Question 4e

Marks	0	1	2	3	Average
%	76	7	9	8	0.5

$2 + i$ (or any non-zero real multiple)

Many students had difficulty starting this question, and fewer were able to complete it. Those that made a fair attempt at substituting for w and z often then lost their way. Of the students who found that $u = 2v$, only a minority were able to continue to a sensible result.

Question 4f

Marks	0	1	2	3	Average
%	95	2	1	1	0.1

$T = \{z: |z - 1.1| \le 2.9, z \in C\}$

or

$T = \{z: (z - 1.1)\ (\bar{z} - 1.1) \le 2.9^2, z \in C\}$ or similar

Only a small number of students were able to approach this question effectively. A few realised that a circle might be involved in some way, but were then generally unable to make any substantial progress.

Question 5a

Marks	0	1	Average
%	59	41	0.4

[0, 1) or $0 \le x < 1$

This question wasn't as well done as expected, with alternative answers such as (0, 1), (–1, 1) and even (0, 1] quite common. $R\backslash\{-1, 1\}$, or similar, was given by students who had overlooked the fact that an expression has to be positive if the fourth root is to be real.

Question 5b

Marks	0	1	Average
%	48	52	0.5

4.78

This was reasonably well done, with the main error occurring when students found the radius rather than the diameter.

Question 5c

Marks	0	1	2	3	Average
%	77	8	7	8	0.5

39° (or 141°)

This was surprisingly poorly answered, with many students not able to conceptualise the angle being sought. Some students tried to apply trigonometry using various points on the curve. Of the students who realised that they needed to find the derivative at $x = 0.5$, those who found the derivative numerically got some way towards the answer, whereas those who attempted to find an analytical derivative generally stumbled. A common error amongst the students who had made substantial progress was to give 51° as the answer.

Question 5di.

Marks	0	1	Average
%	50	50	0.5

$4\sqrt{x}$

Many students who attempted this part seemed to realise that twice the product of the two terms was required, but they were not always able to simplify this correctly. A common error was $2\sqrt{x}$.

Question 5dii.

Marks	0	1	2	3	4	5	Average
%	40	15	7	11	12	14	1.9

6.1

Many students made some progress, though some had left insufficient time and others were unable to sustain their efforts through to the end. It was pleasing to see how well the term requiring integration by substitution was handled. Most students who made a substantial attempt clearly recognised the need to integrate each term. The correct answer of 6.1 was given regularly without correct (or even any) antiderivatives, indicating that some students had ignored the direction to 'use calculus'. An upper terminal of 2.39 rather than 0.5 appeared from time to time.