**Victorian Certificate of Education
2004**

FURTHER MATHEMATICS
Written examination 1
(Facts, skills and applications)

Monday 1 November 2004

**Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)**

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	13	13			13
B	45	27	5	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 30 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Working space

SECTION A

Core

The following information relates to Questions 1 and 2.

The marks obtained by students who sat for a test are displayed as an ordered stemplot as shown.

```
0 | 9
1 |
2 | 0 1 2 5 6
3 | 0 1 1 1 3 5 5 7 8 9 9
4 | 1 2 3 4 4 6 7 7
5 | 0
```

Question 1

The number of students who sat the test is

A. 25
B. 26
C. 27
D. 32
E. 50

Question 2

The interquartile range of these test marks is closest to

A. 9
B. 13
C. 30
D. 36
E. 41

Question 3

The distribution of the weights of eggs produced by a chicken farm is approximately bell-shaped with a mean of 85 g and a standard deviation of 5 g.

Eggs weighing 95 g or more are classified as Extra Large.

The percentage of eggs that would be classified as Extra Large is closest to

A.   0.15%

B.   0.35%

C.   2.5%

D.   5%

E.   16%

The following information relates to Questions 4 and 5.

The number of DVD players in each of 20 households is recorded in the frequency table below.

Number of DVD players	Frequency
0	6
1	9
2	3
3	1
4	0
5	1
Total	**20**

Question 4

For this sample of households, the percentage of households with **at least** one DVD player is

A.   30%

B.   45%

C.   50%

D.   70%

E.   90%

Question 5

For this sample of households, the mean number of DVD players in these 20 households is

A.   0.75

B.   1.00

C.   1.15

D.   1.64

E.   2.00

The following information relates to Questions 6 and 7.

The level of Internet usage (never used, sometimes used, often used) for 217 school students sampled from Years 3 to 12 is indicated in the table below. Some of the entries in the table are missing.

Level of Internet usage	Year group 3–6	7–10	11–12	Total
never used	44	9	8	
sometimes used	16			58
often used	10		47	
Total			73	217

Question 6

For this sample of students, the total number of students who never used the Internet is

A.   44
B.   51
C.   61
D.   70
E.   217

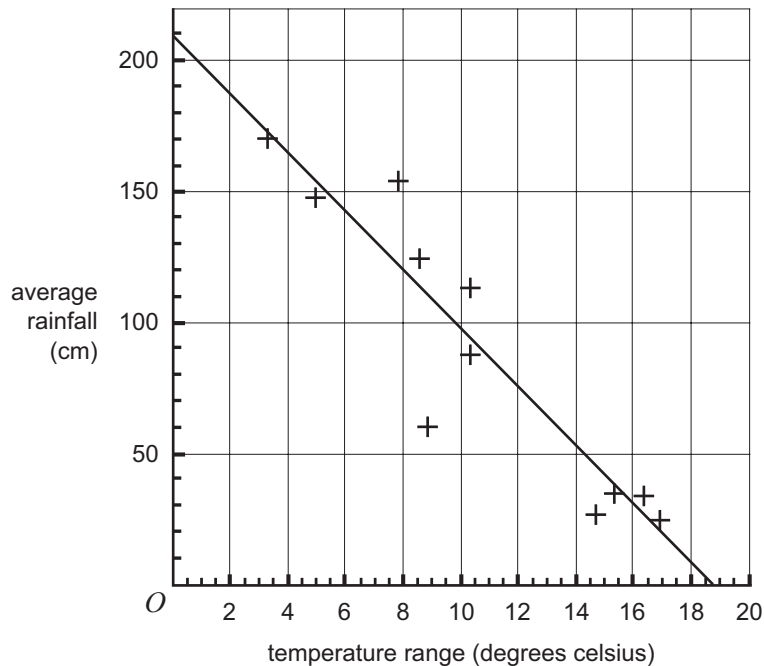## Question 7

The percentage of Year 7–10 students who sometimes used the Internet is closest to

A.   11%
B.   24%
C.   27%
D.   28%
E.   32%

The following information relates to Questions 8 and 9

The average rainfall and temperature range at several different locations in the South Pacific region are displayed in the scatterplot below.

Question 8

A least squares regression line has been fitted to the data as shown.

The equation of this line is closest to

A.   average rainfall = 210 – 11 × temperature range.

B.   average rainfall = 210 + 11 × temperature range.

C.   average rainfall = 18 – 0.08 × temperature range.

D.   average rainfall = 18 + 0.08 × temperature range.

**E.**   average rainfall = 250 – 13 × temperature range.

Question 9

The value of the product moment correlation coefficient, r, for the data, is $r = -0.9260$.

The value of the coefficient of determination is

A. -0.9260

B. -0.8575

C. 0.8575

D. 0.9260

E. 0.9623

The following information relates to Questions 10 and 11.

The time spent batting and the number of runs scored in a cricket player's last nine games are plotted on the scatterplot below.
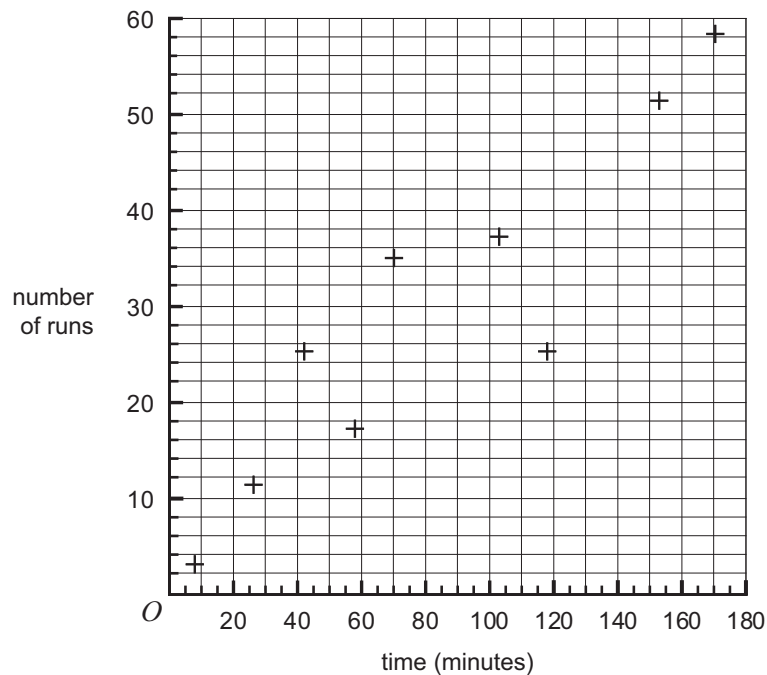
Question 10

When a three median regression line is fitted to the scatterplot, its slope is closest to

A. 0.2

B. 0.3

C. 0.4

D. 0.5

E. 0.6

Question 11

The data point (70, 55) should have been plotted instead of the point (70, 35).

If this mistake is corrected, the slope of a redrawn three median regression line will be

A. very much greater than before.

B. greater than before.

C. less than before.

D. very much less than before.

E. unchanged.

Question 12

The time series plot below shows the share price of two companies over a period of time.
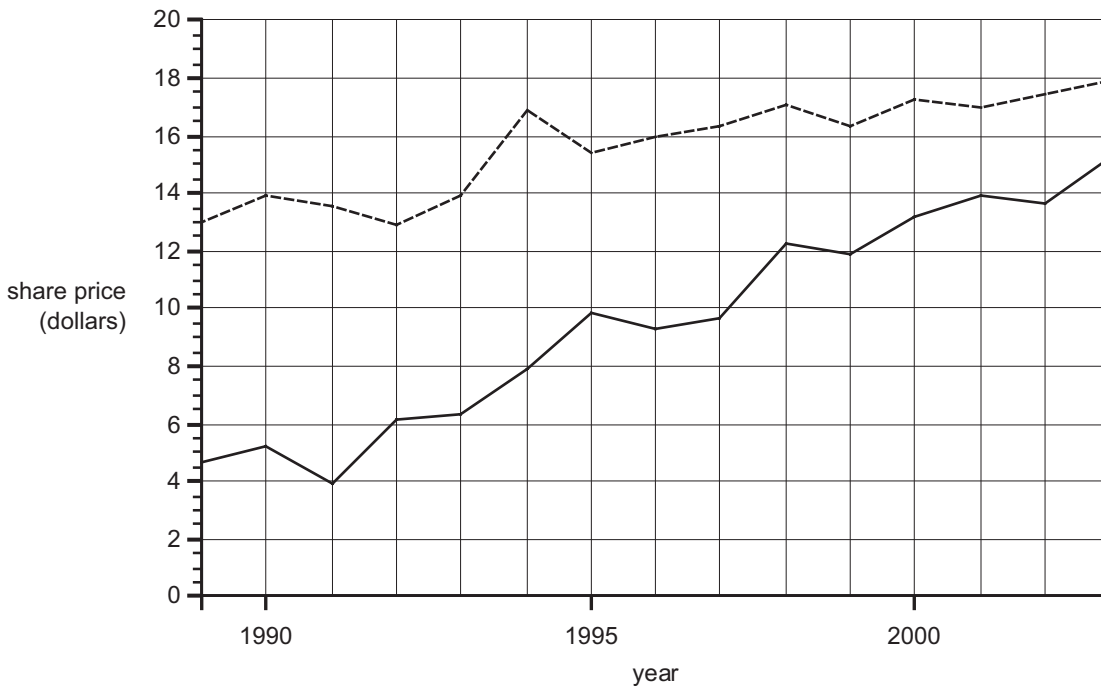
From the plot, it can be concluded that over the interval 1990–2000, the **difference** in share price between the two companies has shown

A.   a decreasing trend.

B.   an increasing trend.

C.   seasonal variation.

D.   a five-year cycle.

E.   no trend.

Question 13

The quarterly seasonal indices for mineral water sales (in litres) of a mineral water supplier are shown in the table below.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Seasonal index	1.28	1.02	0.74	0.96

When **deseasonalised** the amount of mineral water sold in Quarter 1 is 28 098 litres.

To the nearest litre, the **actual** amount of mineral water sold in Quarter 1 was

A.   7 025 litres.

B.   21 952 litres.

C.   28 098 litres.

D.   35 965 litres.

E.   112 392 litres.

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns and applications

> Before answering these questions you must **shade** the Number patterns and applications box on the answer sheet for multiple-choice questions.

Question 1

Harriet, Joshua and Ali share a bag of lollies in the ratio $5 : 3 : 2$. Ali gets six lollies.

The total number of lollies in the bag is

A.   10

B.   20

C.   30

D.   50

E.   60

Question 2

The first three terms of a geometric sequence are 10, 7 and 4.9

The fourth term in this sequence is

A.   3.43

B.   3.47

C.   3.7

D.   4.0

E.   4.2

Question 3

The sum of the infinite geometric sequence –5.4, 1.8, –0.6, 0.2, … is

A. -4.04

B. -4.05

C. -4.15

D. -8.0

E. -8.1

Question 4

The number of goats on a farm is increasing by 8% per annum. At the start of 2003 there were 600 goats on the farm.

At the start of 2005, the number of goats on the farm will be closest to

A.   616

B.   648

C.   696

D.   700

E.   756

The following information relates to Questions 5 and 6.

In the first week of training Sophia swims five laps of the pool each day. In the second week she swims seven laps each day, in the third week she swims nine laps each day, and so on.

Question 5

In the 12th week of training the number of laps that Sophia swims each day is

A.   21
B.   23
C.   25
D.   27
E.   31

Question 6

Sophia swims seven days each week.

Assuming this pattern of training continues, the total number of laps of the pool she has swum after 20 weeks is

A.   301
B.   420
C.   960
D.   2 940
E.   3 360

Question 7

A sequence follows the rule $w_{n+1} = 4w_n + 2$ where w_n is the nth term and $n = 1, 2, 3, 4 \ldots$

The value of the second term, w_2, is 10.

The value of the fourth term, w_4, is

A.   18
B.   20
C.   42
D.   170
E.   200

Question 8

A sequence is defined by the difference equation $t_{n+1} = t_n - 5$ where $t_1 = 15$.

The nth term of the sequence is given by

A. $t_n = n - 20$
B. $t_n = 10 - 5n$
C. $t_n = 15 - n$
D. $t_n = 15 - 5n$
E. $t_n = 20 - 5n$

Question 9

You need to make up an 8% detergent solution.

The volume of water you should add to 200 millilitres (mL) of a 20% detergent solution to make an 8% detergent solution is

A.   160 mL
B.   300 mL
C.   460 mL
D.   500 mL
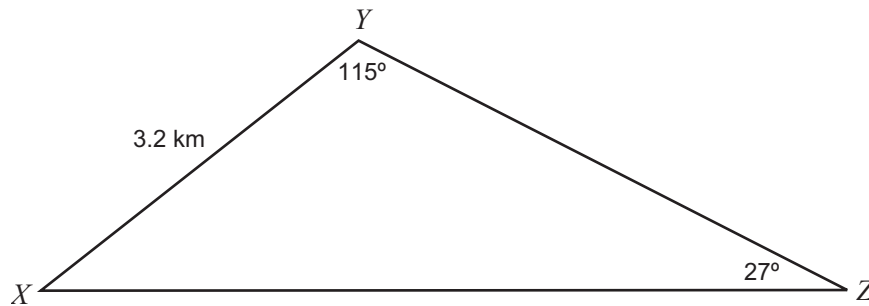E.   2 500 mL

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.
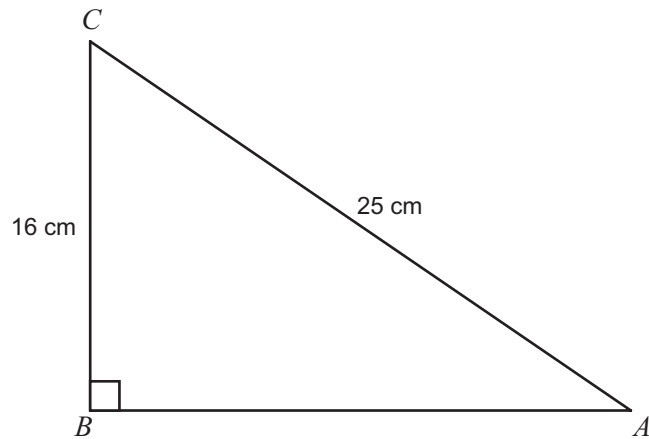
Question 1

For the right-angled triangle *ABC*, with *BC* = 16 cm and *AC* = 25 cm, the size of angle *BAC* is closest to

A.   7°

B.   25°

C.   33°

D.   38°

E.   40°

The following information relates to Questions 2 and 3.

The diagram above shows the route of a cross-country race. Point *X* lies due west of point *Z*.

Question 2

The bearing of point *Y* from point *X* is

A. 038°

B. 052°

C. 063°

D. 218°

E. 232°

Question 3

Given that the length *XY* is 3.2 km, the length *XZ* is closest to

A. 1.5 km

B. 1.6 km

C. 6.4 km

D. 7.0 km

E. 7.6 km

Question 4

A triangle has sides of length 20 cm, 48 cm and 52 cm. A second triangle which is similar to the first triangle has a longest side of 65 cm.

The perimeter of the second triangle is

A. 96 cm

B. 120 cm

C. 125 cm

D. 133 cm

E. 150 cm

Question 5

A plan for a mouse maze is drawn to a scale of $1:6$. On the plan, the mouse maze covers an area of 720 cm^2.
The area of the actual mouse maze is

A. 1 200 cm^2

B. 4 320 cm^2

C. 8 640 cm^2

D. 25 920 cm^2

E. 129 600 cm^2

SECTION B – Module 2: Geometry and trigonometry – continued

Question 6

A traverse survey of farming land has been conducted and a field sketch made as shown. The line PR runs north-south. All measurements are in metres.
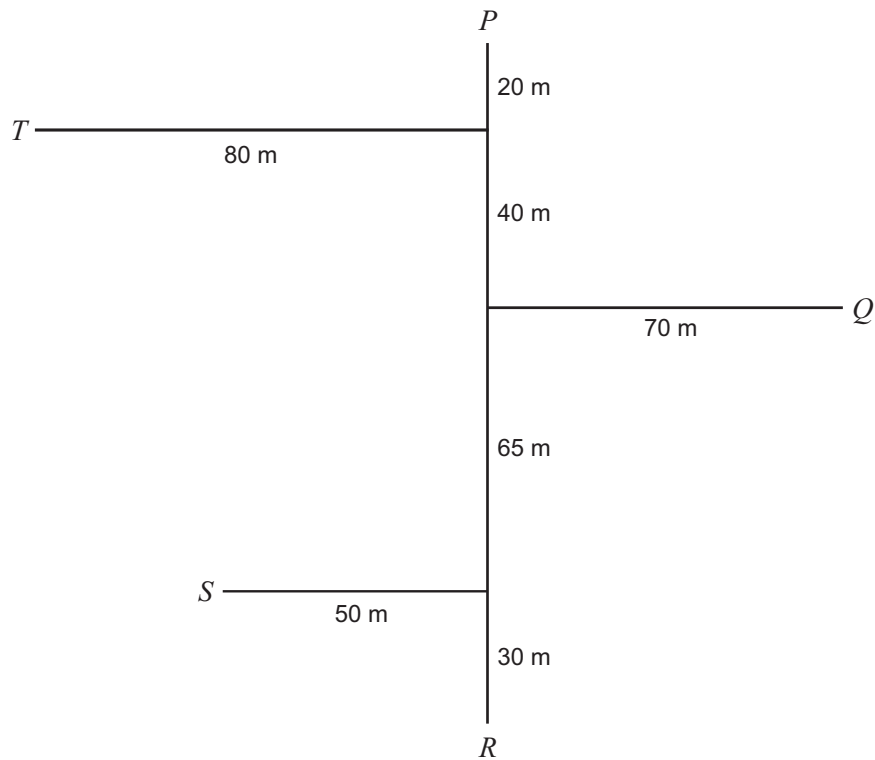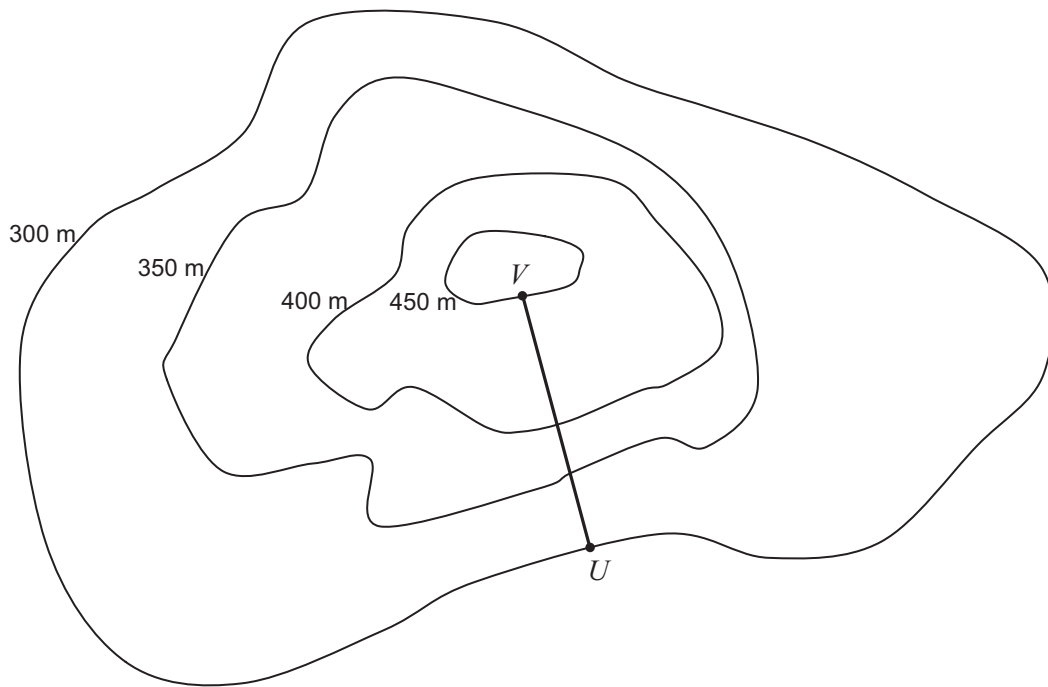

To the nearest metre, the distance ST is

A. 101 m

B. 105 m

C. 109 m
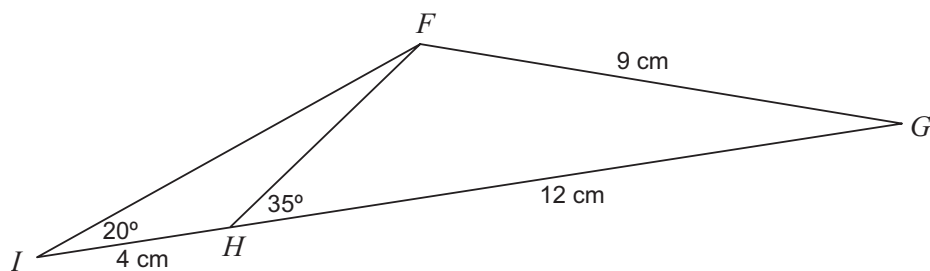
D. 132 m

E. 135 m

Question 7

The points U and V lie on the contour map as shown.

The horizontal distance between the two points U and V is 200 metres.

The average slope along the line UV is

A. 0.75

B. 1.33

C. 1.50

D. 2.25

E. 2.50

Question 8

In the diagram above, the length of FH is equal to

A. $4\tan 55°$

B. $\sqrt{\left(12^2 - 9^2\right)}$

C. $\sqrt{\left(12^2 + 9^2 - 216\cos 35°\right)}$

D. $\dfrac{4\sin 20°}{\sin 15°}$

E. $12\sin 55°$

Question 9

A right-triangular prism *ABCDEF* is as shown with lengths *AB* = 20 cm, *AC* = 100 cm and *CD* = 50 cm.
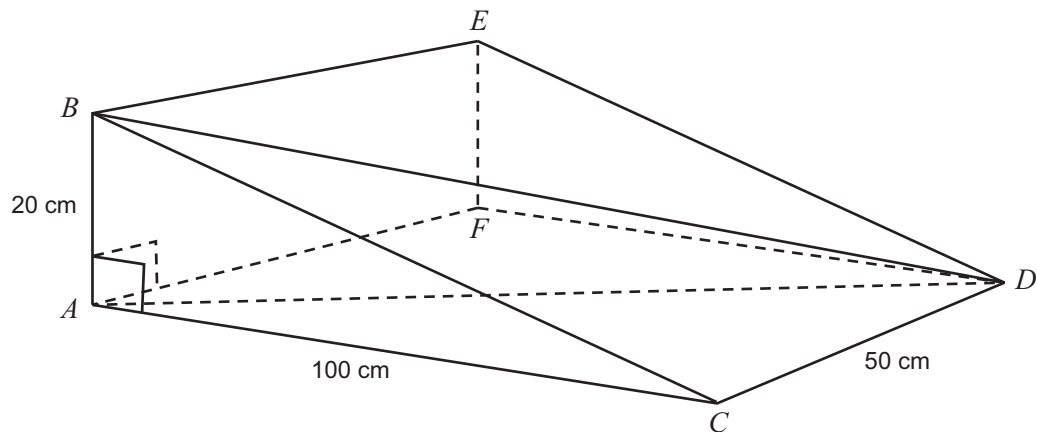
The size of angle *ADB* is

A. 10.1°

B. 11.3°

C. 30.5°

D. 59.0°

E. 59.5°

Module 3: Graphs and relations

> Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

Question 1

The graph below shows the cost (dollars) of mobile telephone calls up to 240 seconds long.
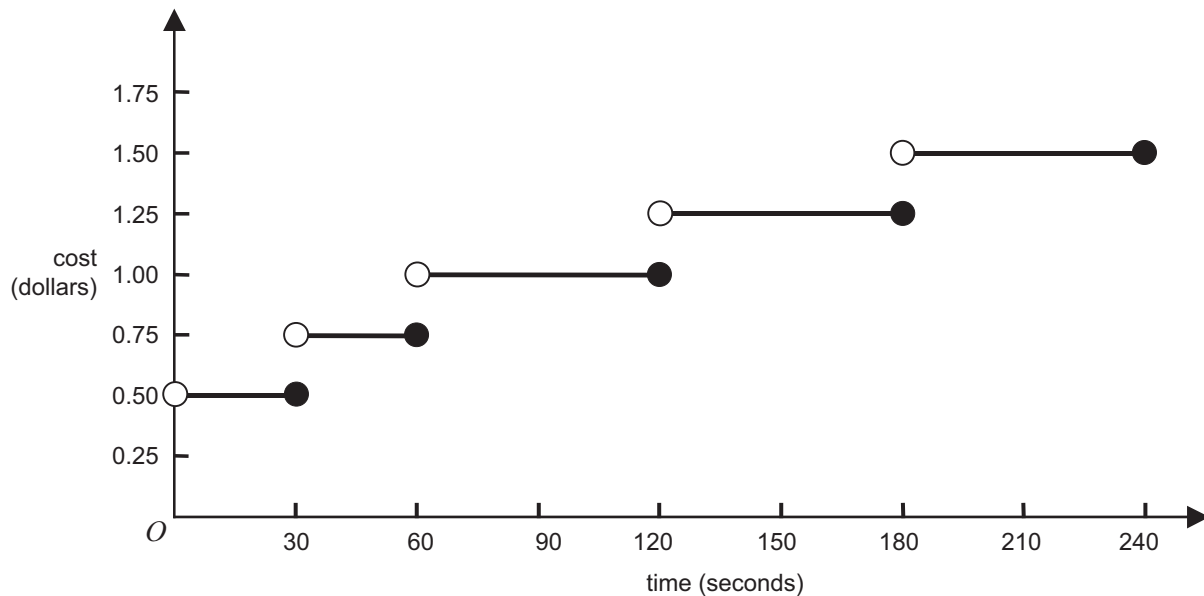
The cost of making a 90-second call followed by a 30-second call is

A. $1.00

B. $1.20

C. $1.25

D. $1.50

E. $1.75

Question 2

The point (2, 1) lies on the line $y = 3x + c$.

The value of c is

A. -7

B. -5

C. -1

D. 5

E. 7

Question 3

The lines $y + 8 = 0$ and $x - 12 = 0$ intersect at the point

A. $(-12, 8)$

B. $(-8, 12)$

C. $(0, 0)$

D. $(8, -12)$

E. $(12, -8)$

The following information relates to Questions 4 and 5.

The graph shows a distance-time graph for a car travelling from home along a long straight road over a 16-hour period.
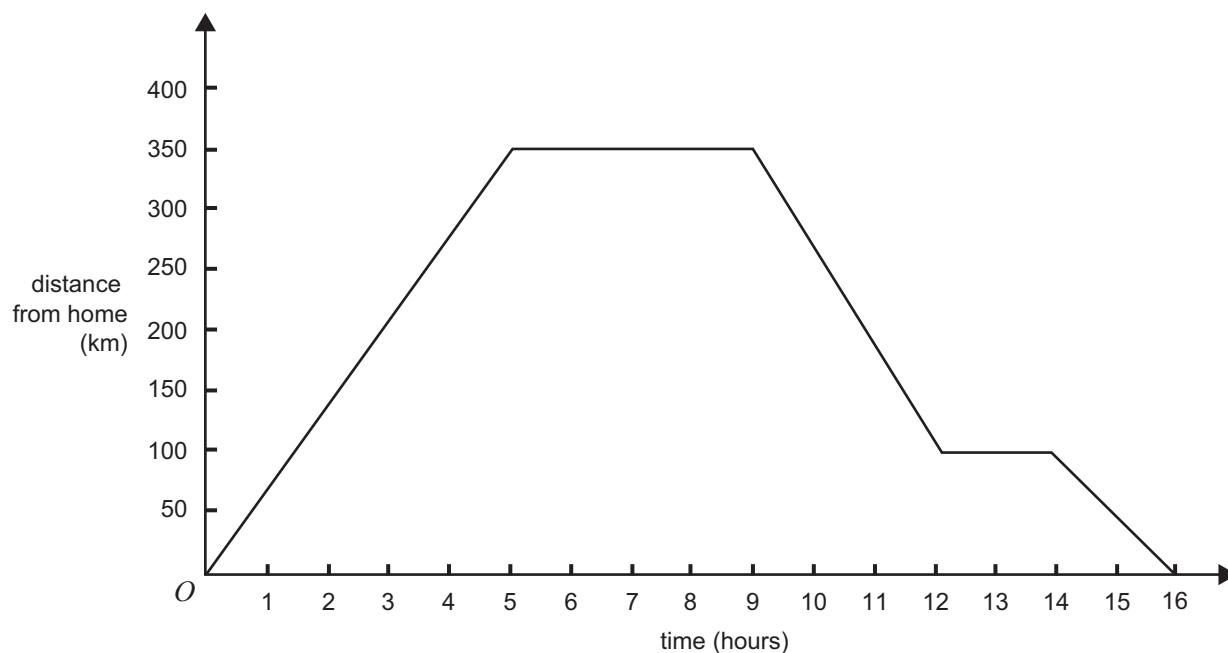
Question 4

In which one of the time intervals is the speed of the car greatest?

- **A.**   0 to 5 hours
- **B.**   5 to 9 hours
- **C.**   9 to 12 hours
- **D.**   12 to 14 hours
- **E.**   14 to 16 hours

Question 5

After twelve hours the car has travelled a total distance of

- **A.**   100 km
- **B.**   350 km
- **C.**   450 km
- **D.**   600 km
- **E.**   700 km

Question 6

The cost, C, of hiring a boat for x hours is given by the equation $C = ax + b$ where a is the hourly rate and b is a fixed booking fee.

When the boat is hired for 4 hours the cost is $320.
When the boat is hired for 6 hours the cost is $450.

When the boat is hired for one hour the cost is

- **A.** $65
- **B.** $75
- **C.** $77
- **D.** $80
- **E.** $125

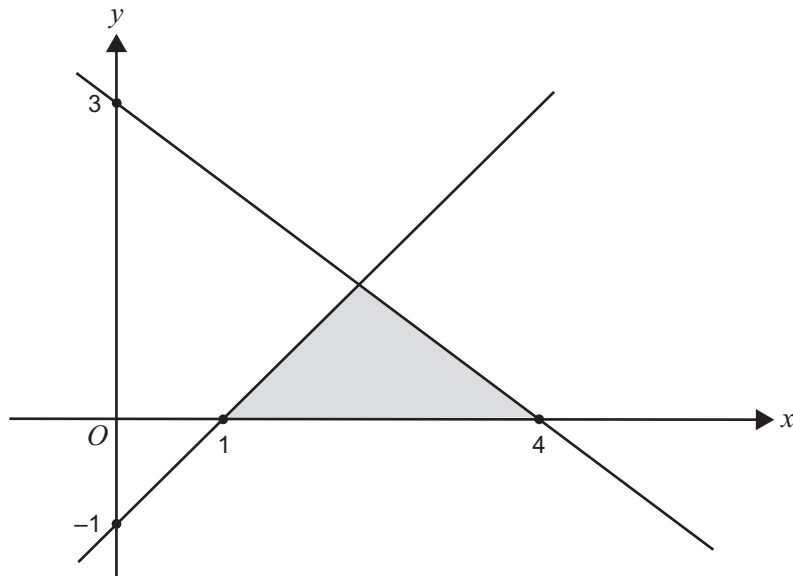**SECTION B – Module 3: Graphs and relations** – continued
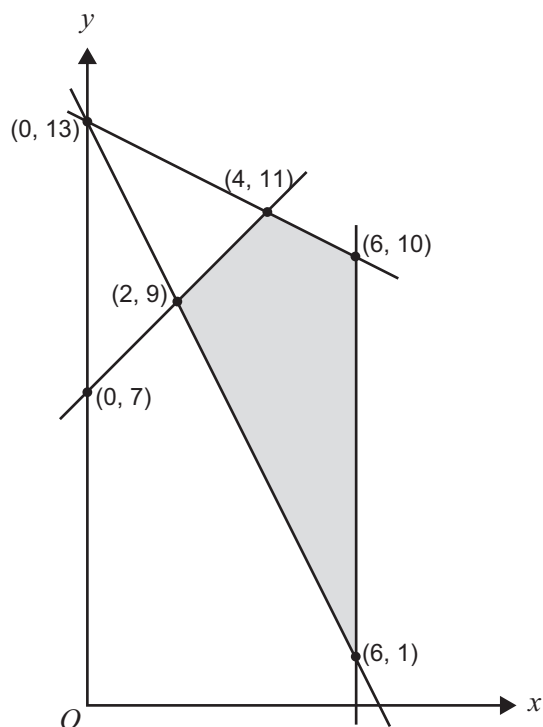
Question 7

The shaded region shown in the graph above (with boundaries included) represents the feasible region for a linear programming problem.

The maximum value of the objective function $y - 2x + 20$, for this feasible region, is
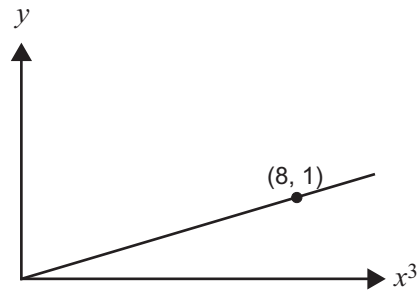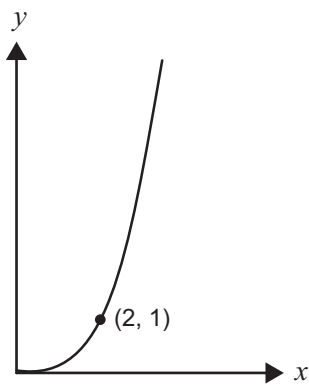
A.   18
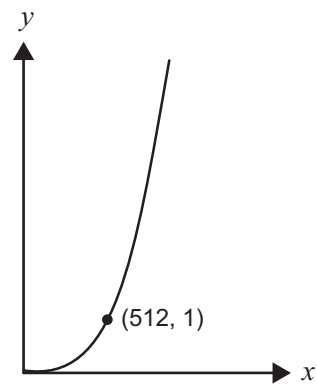B.   23
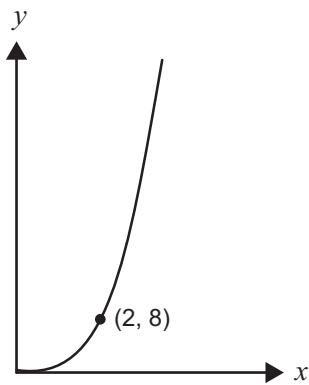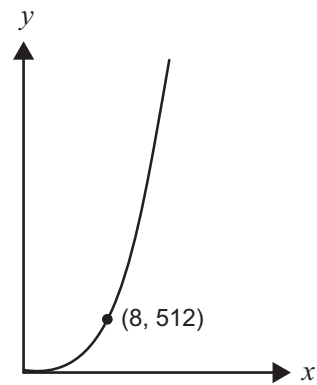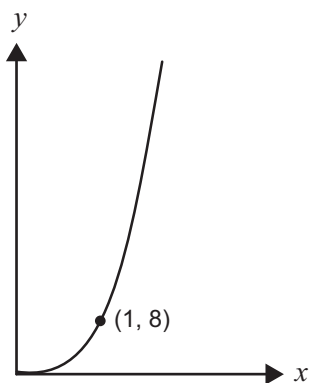C.   25
D.   27
E.   33

Question 8

The shaded region shown in the graph above (with boundaries included) is described by

A. $3x + 4y \leq 12$
    $x - y \leq 1$
    $x \geq 0$
    $y \geq 0$

B. $3x + 4y \leq 12$
    $x - y \geq 1$
    $x \geq 0$
    $y \geq 0$

C. $3x + 4y \geq 12$
    $x - y \geq 1$
    $x \geq 0$
    $y \geq 0$

D. $4x + 3y \leq 12$
    $x - y \leq 1$
    $x \geq 0$
    $y \geq 0$

E. $4x + 3y \leq 12$
    $x - y \geq 1$
    $x \geq 0$
    $y \geq 0$

Question 9

The graph above represents a relationship $y = kx^3$ for $x \geq 0$.

A graph that shows this relationship when y is plotted against x is

A.

B.

C.

D.

E.

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

Question 1

Sarah invests $37 000 at a simple interest rate of 4% per annum.

The total amount of interest earned in two years is

A. $1 480

B. $2 960

C. $5 920

D. $38 480

E. $39 960

Question 2

Ardy invests $150 000 for 6 years at an interest rate of 3.5% per annum, compounding annually.

The value of the investment at the end of the 6 years is

A. $31 500.00

B. $34 388.30

C. $178 107.00

D. $181 500.00

E. $184 388.30

Question 3

Leonard charged $36 per hour for tutoring. In September 2004, he increased his fees by 15%.

For two hours of tutoring, Leonard now charges

A. $10.80

B. $41.40

C. $47.60

D. $72.00

E. $82.80

The following information relates to Questions 4 and 5.

Chen buys a new refrigerator. The advertised price was $2 700. He chooses to pay a deposit of $500 and monthly repayments of $115 over 2 years.

Question 4

Under this arrangement, the total cost of the refrigerator is

A. $2 200

B. $2 760

C. $3 200

D. $3 260

E. $4 260

Question 5

The annual flat rate of interest paid is closest to

A. 10.4%

B. 12.2%

C. 12.7%

D. 20.7%

E. 25.4%

Question 6

A loan of $250 000 is to be paid back over a period of 20 years at an interest rate of 7.4% per annum, compounding monthly.

To the nearest dollar, the monthly repayment is closest to

A. $1 963

B. $1 999

C. $2 998

D. $4 343

E. $13 326

Question 7

Binnie invests $12 000 for 5 years at an interest rate of 3.6% per annum, compounding annually.

The amount of interest she earns during the third year of the investment is closest to

A. $463.66

B. $470.41

C. $480.36

D. $1 343.22

E. $1 823.57

SECTION B – Module 4: Business-related mathematics – continued

Question 8

A machine is purchased for $15 000. Using the reducing balance method of depreciation, its book value after six years will be $5 000.

The graph that best represents the value of the machine at the end of each year over the six-year period is

A.
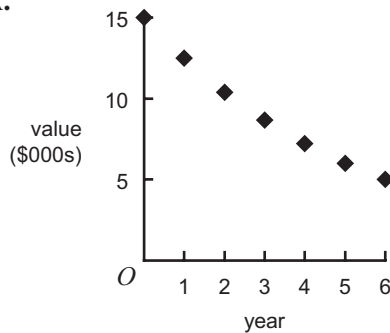
B.
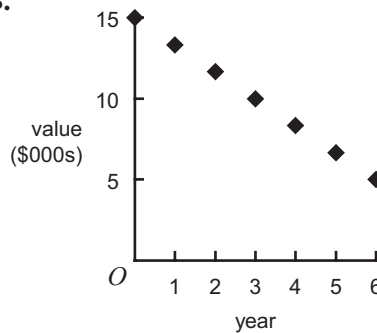
C.
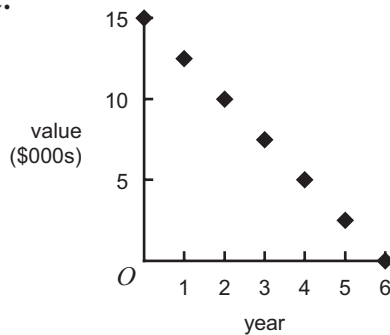
D.
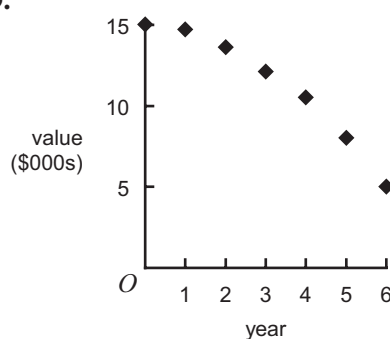
E.
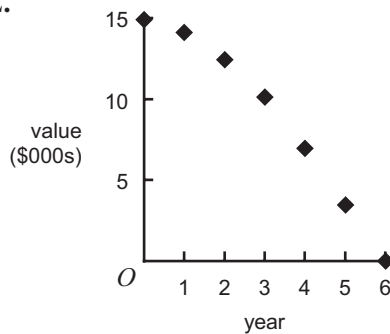
Question 9

An amount of $130 000 is borrowed at an interest rate of 7.5% per annum, compounding monthly. The loan is fully repaid over ten years with equal monthly repayments.

Which of the following statements is **not** true?

A.   The monthly interest rate is 0.625%.

B.   No money will be owed after 10 years.

C.   The total number of repayments is 120.

D.   A monthly repayment of $1 500 will reduce the length of the loan.

E.   At the end of five years, the amount of the principal still owing will exceed $65 000.

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions.
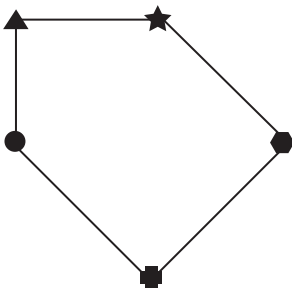
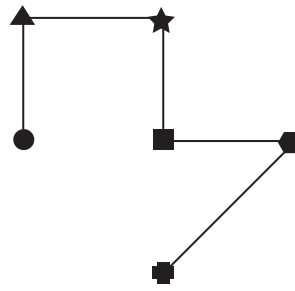### Question 1

Consider the following network graph.
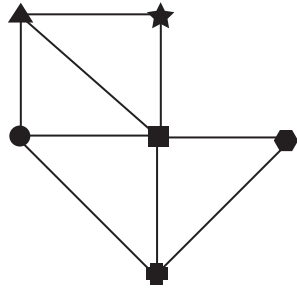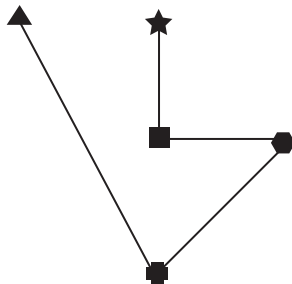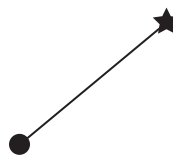

A subgraph of this graph is

A.

B.

C.

D.

E.
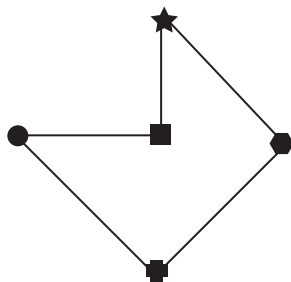
Question 2

The diagrams show four connected planar graphs.
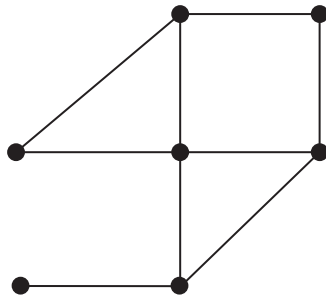
Graph *J*
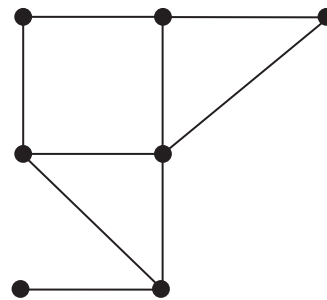
Graph *K*
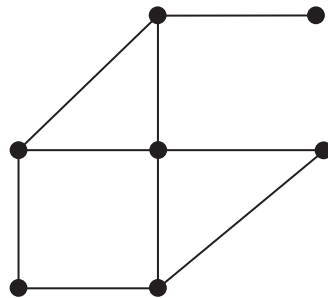
Graph *L*
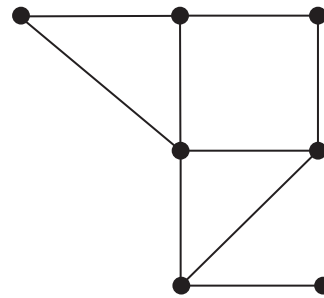
Graph *M*

Equivalent graphs are

A. *J* and *L* only.

B. *J* and *K* and *L* only.

C. *J* and *K* and *M* only.

D. *J* and *L* and *M* only.

E. *J* and *K* and *L* and *M*.

Question 3
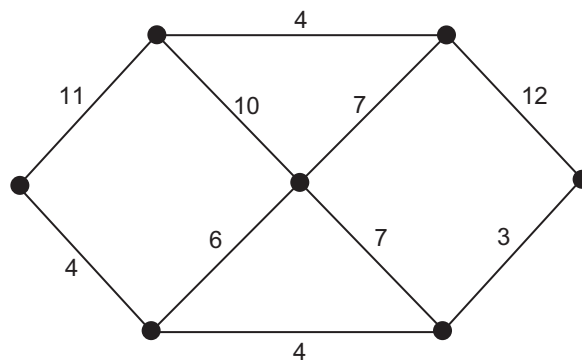
The length of the minimal spanning tree for this network is

A. 15

B. 22

C. 28

D. 34

E. 35

SECTION B – Module 5: Networks and decision mathematics – continued

TURN OVER

Question 4

The diagram below shows a map of the roads between four towns, *F*, *G*, *H* and *I*.
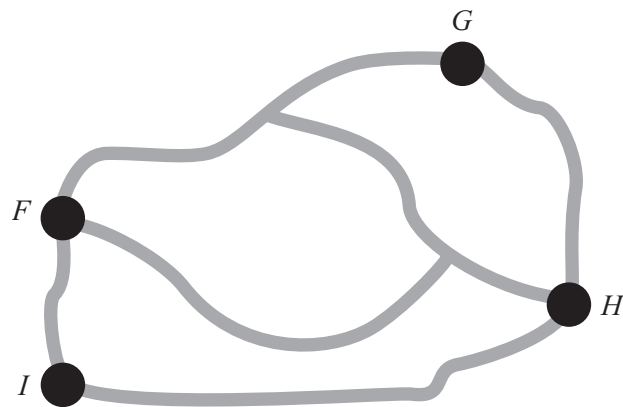
A network diagram that represents all the connections between the four towns on the map is

A.
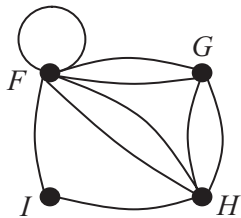
B.
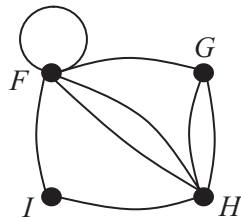
C.
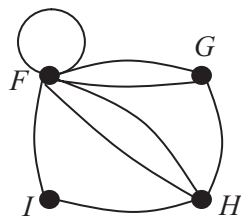
D.
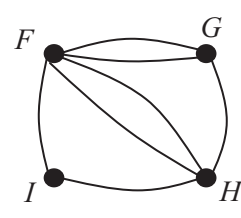
E.
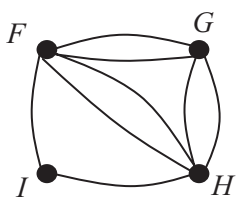
Question 5

Underground water pipes are needed to water a new golf course. Water will be pumped from the dam in the back corner of the course.

To find the smallest total length of water pipe needed, we must find

A.   a critical path.

B.   a minimal spanning tree.

C.   the shortest Euler circuit.

D.   the shortest Hamiltonian circuit.

E.   the perimeter of the golf course.

Question 6

Which one of the following is a **true** statement about a **critical path** in a project?

A.   Knowledge of the critical path can be used to decide if any tasks in a project can be delayed without extending the length of time of the project.

B.   All tasks on the critical path must be completed before any other task in the same project can be started.

C.   Decreasing the times of tasks not on the critical path will decrease the length of time of the project.

D.   The critical path must always include at least two tasks in a project.

E.   There is only one critical path in any project.

Question 7

Five people are to be each allocated one of five tasks (A, B, C, D, E). The table shows the time, in hours, that each person takes to complete the tasks.

	Task				
	A	B	C	D	E
Francis	12	15	99	10	14
David	10	9	10	7	12
Herman	99	10	11	6	12
Indira	8	8	12	9	99
Natalie	8	99	9	8	11

The tasks must be completed in the least possible total amount of time.

If no person can help another, Francis should be allocated task

A. A

B. B

C. C

D. D

E. E

SECTION B – Module 5: Networks and decision mathematics – continued

Question 8

The activities and their completion times (days) needed to complete a project are shown in the digraph below.

For the network shown, the length of the critical path is
A.   22 days.
B.   23 days.
C.   25 days.
D.   26 days.
E.   28 days.

Question 9

The table below lists the six activities in a project and the earliest start time, in hours, and the predecessor(s) of each task.

Task	Predecessor	Earliest start time
A	–	0
B	–	0
C	A	8
D	B	15
E	C	22
F	D, E	35

The time taken for activity E is two hours.

Without affecting the time taken for the entire project, the time taken for activity C could be increased by
A.   0 hours.
B.   8 hours.
C.   9 hours.
D.   11 hours.
E.   27 hours.

END OF QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Business-related mathematics

simple interest:
$$I = \frac{PrT}{100}$$

compound interest:
$$A = PR^n \quad \text{where} \quad R = 1 + \frac{r}{100}$$

hire purchase:
$$\text{effective rate of interest} \approx \frac{2n}{n+1} \times \text{flat rate}$$

annuities:
$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}, \quad where \quad R = 1 + \frac{r}{100}$$

Geometry and trigonometry

area of a triangle:
$$\frac{1}{2} bc \sin A$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3} \pi r^3$$

volume of a cone:
$$\frac{1}{3} \pi r^2 h$$

Pythagoras' theorem:
$$c^2 = a^2 + b^2$$

sine rule:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine rule:
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Graphs and relations

Straight line graphs

gradient:
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

equation:
$$y - y_1 = m(x - x_1) \qquad \text{gradient-point form}$$

$$y = mx + c \qquad \text{gradient-intercept form}$$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{two-point form}$$

Number patterns and applications

arithmetic series:

$$a + (a + d) + \ldots + (a + (n-1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

geometric series:

$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \; r \neq 1$$

infinite geometric series:

$$a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, \; |r| < 1$$

linear difference equations:

$$t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1}-1)}{a-1}, \; a \neq 1$$

$$= a^n t_0 + b\frac{(a^n - 1)}{a-1}$$

Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Statistics

seasonal index:

$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

END OF FORMULA SHEET