FURTHER MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 3 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A	13	13			13
B	45	27	5	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 30 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Working space

SECTION A

Core

The following information relates to Questions 1 and 2.

The percentage investment returns of seven superannuation funds for the year 2002 are

–4.6%, –4.7%, 2.9%, 0.3%, –5.5%, –4.4%, –1.1%

Question 1

The median investment return is

A. –4.7%

B. –4.6%

C. –4.5%

D. –4.4%

E.   0.3%

Question 2

The range of investment returns is

A. 2.6%

B. 3.5%

C. 4.0%

D. 5.5%

E. 8.4%

Question 3

The distribution of test scores obtained when 2500 students sit for an examination is bell-shaped with a mean of 64 and a standard deviation of 12.

From this information we can conclude that the number of these students who obtained marks between 52 and 76 is closest to

A.   68

B.   95

C.   850

D. 1700

E. 2375

The following information relates to Questions 4 and 5.

The mean weight of twelve people is 72 kg; the standard deviation of the weights of these twelve people is 5 kg.

Question 4

The total weight of the twelve people is

A.   77 kg

B.   360 kg

C.   864 kg

D.   924 kg

E.   4320 kg

Question 5

These twelve people are about to go on a rafting adventure. Before boarding the raft, they are all required to put on a life-saving vest that weighs 2 kg. The effective weight of each person is now their weight plus the weight of the life-saving vest.

The effective weights of the twelve people have

A.   a mean of 72 kg with a standard deviation of 5 kg.

B.   a mean of 72 kg with a standard deviation of 7 kg.

C.   a mean of 74 kg with a standard deviation of 5 kg.

D.   a mean of 74 kg with a standard deviation of 7 kg.

E.   a mean of 74 kg with a standard deviation of 10 kg.

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The following information relates to Questions 6 and 7.

The level of water usage of 250 houses was rated in a survey as low, medium or high and the size of the houses as small, standard or large. The results of the survey are displayed in the table below.

Level of water usage	Size of house		
	small	standard	large
low	15	14	9
medium	22	71	11
high	15	47	46

Question 6

The percentage of standard sized houses rated as having a high level of water usage is

A. 18.8%

B. 35.6%

C. 43.5%

D. 47.0%

E. 53.8%

Question 7

The variables, **level of water usage** and **size of house**, as recorded in this survey, are

A. both numerical variables.

B. both categorical variables.

C. neither numerical nor categorical variables.

D. numerical and categorical variables respectively.

E. categorical and numerical variables respectively.

The following information relates to Questions 8 and 9.

Eighteen students sat for a 15 question multiple-choice test. In the scatterplot below, the number of errors made by each student on the test is plotted against the time they reported studying for the test. A least squares regression line has been determined for this data and is also displayed on the scatterplot.
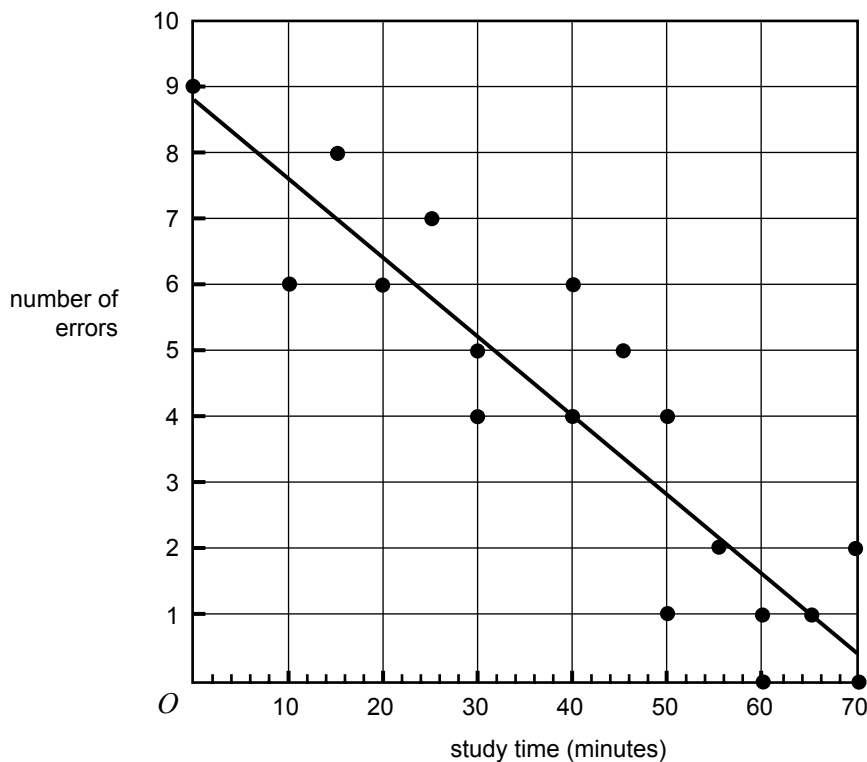
The equation for the least squares regression line is

number of errors = 8.8 – 0.120 × study time

and the coefficient of determination is 0.8198.

Question 8

Using the least squares regression line, it can be estimated that, on average, a student reporting a study time of 35 minutes would make

A.   4.3 errors.

B.   4.6 errors.

C.   4.8 errors.

D.   5.0 errors.

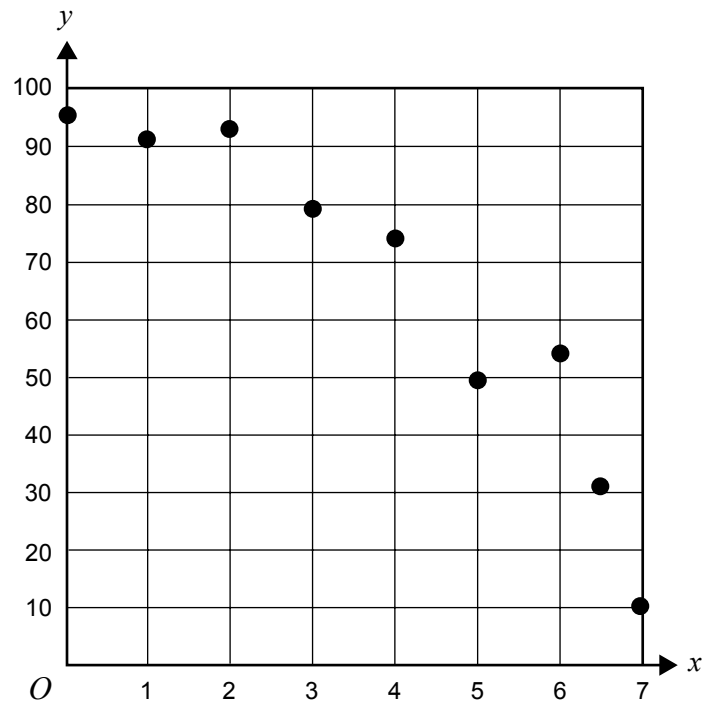E.   13.0 errors.

Question 9

The value of Pearson's product moment correlation coefficient, *r*, for this data, correct to two decimal places, is

A.   – 0.91

B.   – 0.82

C.   0.67

D.   0.82

E.   0.91

Question 10

The relationship between the two variables y and x, as shown in the scatterplot below, is nonlinear.

Which one of the following transformations, by itself, is most likely to linearise this data?

A.   a $\dfrac{1}{x}$ transformation

B.   a $\dfrac{1}{y}$ transformation

C.   an x^2 transformation

D.   a log x transformation

E.   a log y transformation

Question 11

The relationship between **resting pulse rate** (in beats per minute) and **fitness level** (below average, average, above average) is best displayed using

A.   a histogram.

B.   a scatterplot.

C.   a time series plot.

D.   parallel boxplots.

E.   back-to-back stemplots.

Question 12

The data below gives the number of accidents recorded at a city intersection each year from 1993 to 2002.

Year	Number of accidents
1993	13
1994	7
1995	3
1996	9
1997	10
1998	8
1999	7
2000	6
2001	10
2002	11

Using a **four** point moving average (mean) with centring, the smoothed value of the number of accidents in 1995 is

A. 7.25

B. 7.375

C. 7.5

D. 7.625

E. 8

Question 13

The seasonal indices for the first three quarters of a year are shown in the table below.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Seasonal index	1.05	0.84	0.92	

The seasonal index for Quarter 4 is

A. 0.88

B. 0.94

C. 1.00

D. 1.08

E. 1.19

END OF SECTION A

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns and applications

Before answering these questions you must **shade** the Number patterns and applications box on the answer sheet for multiple-choice questions.

Question 1

For the sequence

4, 10, 16, 22, …

the sum of the first ten terms is

A.   52
B.   58
C.   310
D.   340
E.   620

Question 2

In an arithmetic sequence, the second term is 36 and the fourth term is 20.

The first term is

A.   20
B.   28
C.   44
D.   52
E.   56

Question 3

A large pile of bricks is stored at a building site.

To make the pile more stable,

the bottom layer has 47 bricks

the second layer has 43 bricks

the third layer has 39 bricks

and so on.

If this pattern continues, the number of bricks in the 11th layer is

A.   3
B.   5
C.   7
D.   9
E.   11

Question 4

In an audience of 480 adults there are 180 men.

The ratio of men to women in this audience is

A.   5 : 8
B.   5 : 3
C.   3 : 8
D.   3 : 5
E.   1 : 1

SECTION B – Module 1: Number patterns and applications – continued

Question 5

Which one of the following sequences is **not** a geometric sequence?

A.   1, 0.1, 0.01, 0.001, …

B.   1, 1.1, 1.01, 1.001, …

C.   3, 3, 3, 3, …

D.   16, – 8, 4, – 2, …

E.   4, 4^2, 4^3, 4^4, …

Question 6

A tank contains 18 000 litres of water. The wall cracks and water flows out at a rate of 120 litres per minute. Three hours later, the amount of water left in this tank is

A.   0 litres.

B.   3 600 litres.

C.   7 200 litres.

D.   10 800 litres.

E.   14 400 litres.

Question 7

A shrub, 20 cm high, was planted in a pot. After it was planted in the pot, its height increased by 8 cm in the first month, by 4 cm in the second month and by 2 cm in the third month.

Assuming that this pattern of growth continues, the shrub will grow to a maximum height of

A.   35 cm

B.   36 cm

C.   37 cm

D.   38 cm

E.   40 cm

Question 8

A sequence is described by the difference equation

$$t_{n+1} = 0.4t_n \qquad \text{where} \quad t_1 = 100$$

Which one of the following best describes the sequence?

A.   a sequence which is not a geometric sequence

B.   a decreasing geometric sequence with all positive terms

C.   an increasing geometric sequence with all positive terms

D.   a geometric sequence with alternating positive and negative terms

E.   a decreasing geometric sequence with negative terms later in the sequence

Question 9

The first five terms of a sequence of numbers are
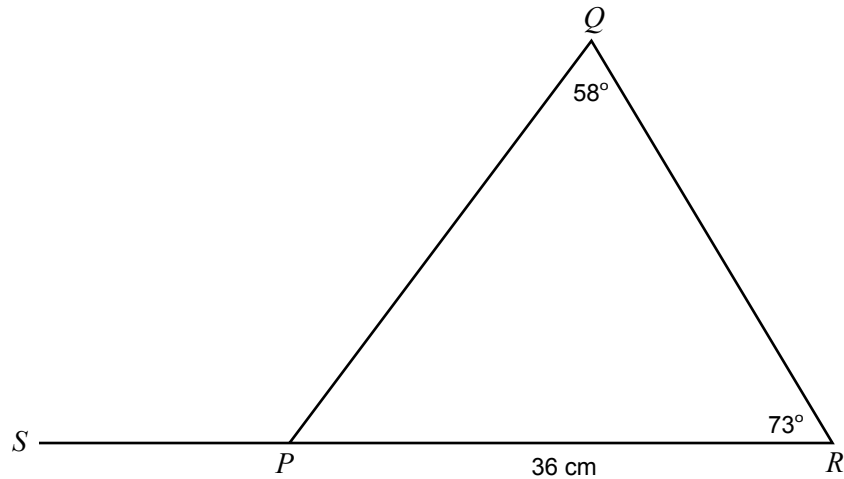
        20, 10, 20, 10, 20, ...

A difference equation that generates this sequence is

A. $t_{n+1} = 20 - t_n$

B. $t_{n+1} = t_n - 20$

C. $t_{n+1} = 0.5\, t_n$

D. $t_{n+1} = t_n - 10$

E. $t_{n+1} = 30 - t_n$

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

The following information relates to Questions 1 and 2.

Question 1

The size of ∠*SPQ* is exactly

- **A.**   41º
- **B.**   49º
- **C.**   107º
- **D.**   122º
- **E.**   131º

Question 2

Given that the length of *PR* is 36 cm, the length of *PQ* is

- **A.**   31.9 cm
- **B.**   34.4 cm
- **C.**   40.6 cm
- **D.**   42.5 cm
- **E.**   43.7 cm

The following information relates to Questions 3 and 4.

A traverse survey of a housing development site has been conducted and a field sketch made as shown. The line *PQ* runs north-south.
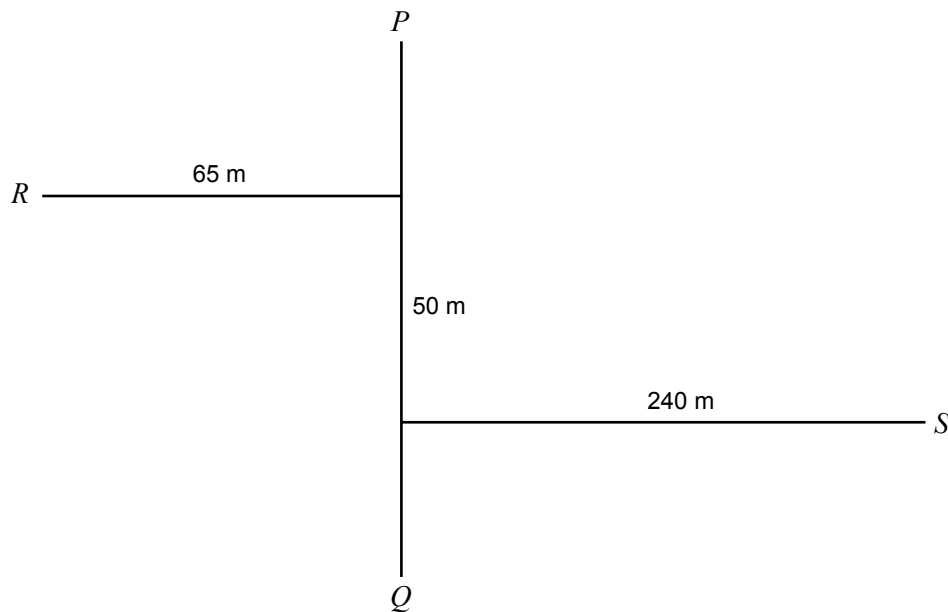
It is planned that a power cable will be run underground in a straight line from *S* to *R*.

Question 3

To the nearest metre, the length of the power cable is

A. 255 m
B. 301 m
C. 309 m
D. 311 m
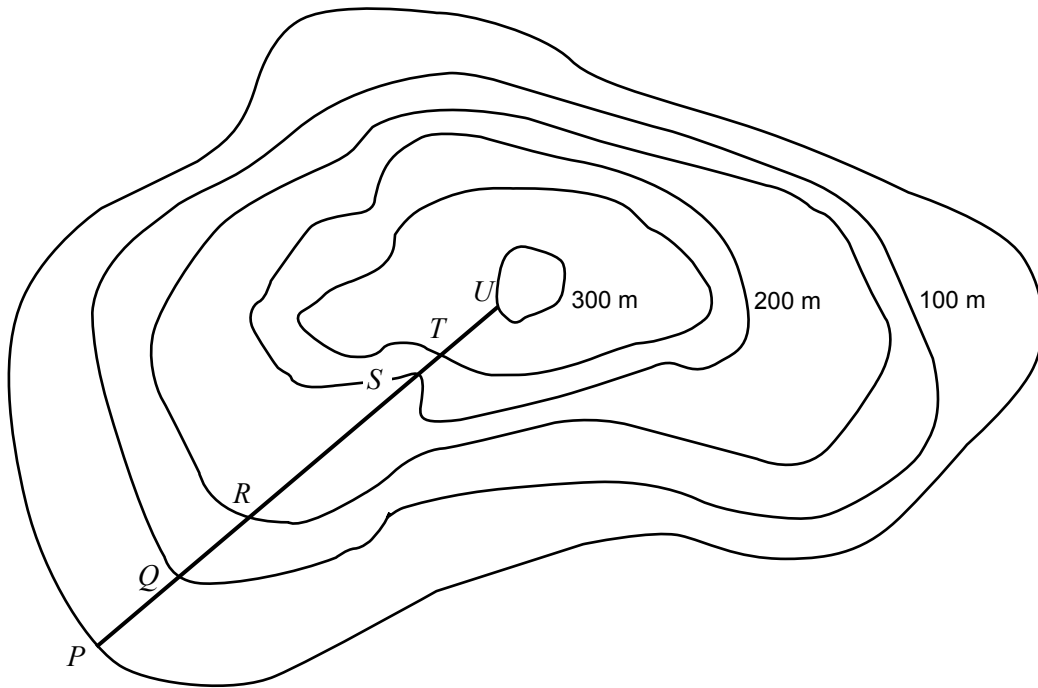E. 355 m

Question 4

The bearing of *R* from *S* is

A. 009°
B. 081°
C. 189°
D. 279°
E. 351°

Question 5

The scale on a particular map is 1:10 000.

A distance of 5 cm on this map would correspond to an actual distance of

A. 0.5 km
B. 2 km
C. 5 km
D. 20 km
E. 50 km

SECTION B – Module 2: Geometry and trigonometry – continued

Question 6

On the contour map of a hill as shown above, the steepest section of the hill along the line *PU* is

A.   *PQ*

B.   *QR*

C.   *RS*

D.   *ST*

E.   *TU*

Question 7

A juice container in the shape of a rectangular prism has a total surface area of 220 cm^2. An enlarged scale model is made so that each side is five times longer than the corresponding side of the actual container.

The total surface area of the scale model is

A.   220 cm^2

B.   1 100 cm^2

C.   5 500 cm^2

D.   22 000 cm^2

E.   27 500 cm^2

SECTION B – Module 2: Geometry and trigonometry – continued
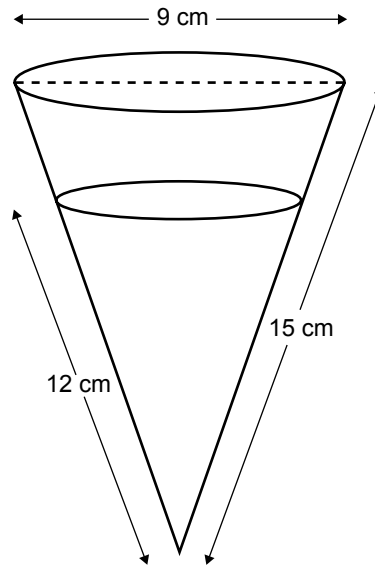
Question 8

A cross-section of a glass greenhouse is shown in the diagram below. The sides of the glass panels *TU* and *UV* are 2.1 metres and 3.5 metres long respectively. The greenhouse is 4.2 metres wide. The walls *ST* and *WV* are vertical and equal in height.
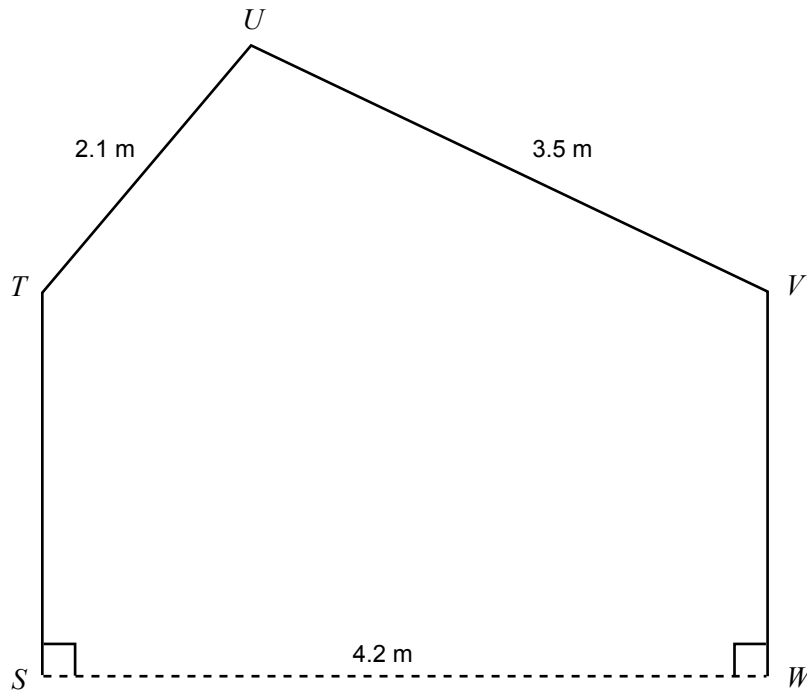
The size of ∠*TUV* is

A.   44.4°

B.   45.6°

C.   86.2°

D.   93.8°

E.   109.6°

Question 9

Two right cones, as shown above, have the same angle at the base. The larger cone has a slant height of 15 cm and the smaller cone has a slant height of 12 cm. The diameter of the larger cone is 9 cm.

The diameter of the smaller cone is

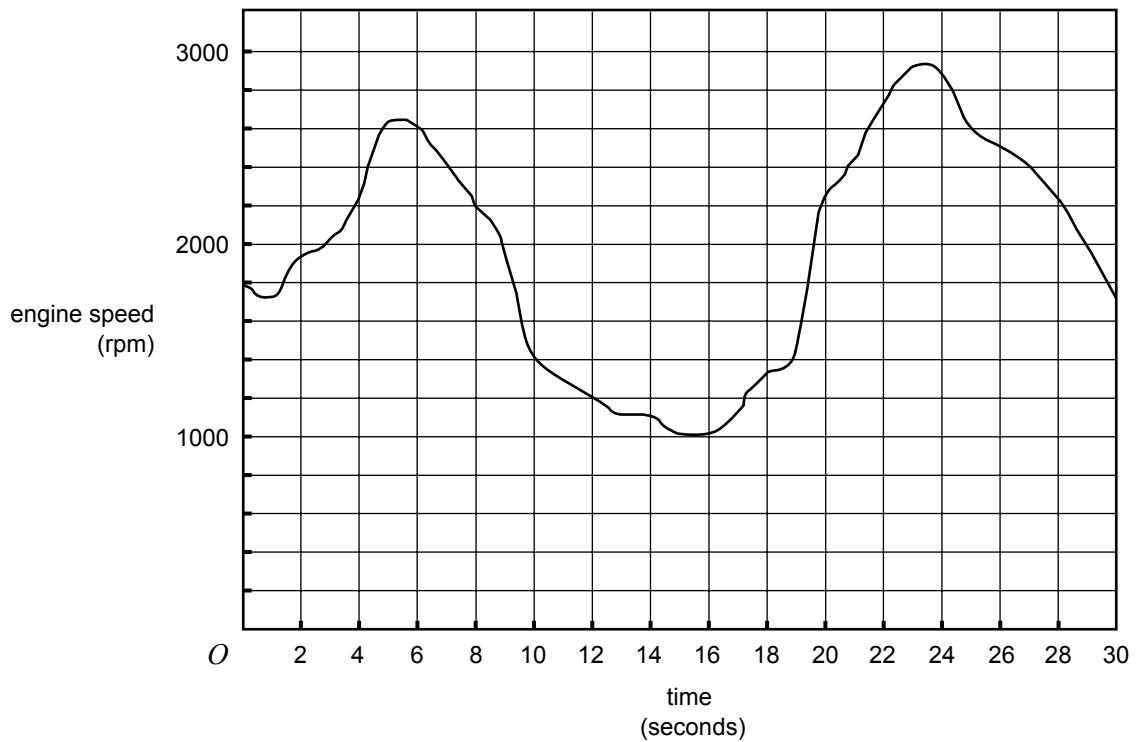A. 2.0 cm
B. 3.6 cm
C. 4.5 cm
D. 6.0 cm
E. 7.2 cm

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Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

Question 1

The graph below shows the engine speed of a car measured in revolutions per minute (rpm) over a period of 30 seconds.
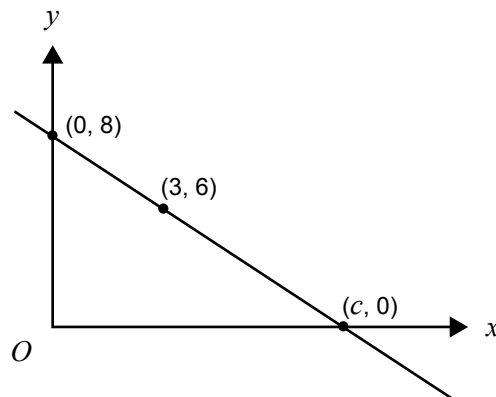

The total time that the engine speed was above 2200 rpm is

A.   4 seconds

B.   6 seconds

C.   12 seconds

D.   20 seconds

E.   24 seconds

Question 2

If the point (3, –2) lies on the curve with equation $y = \dfrac{k}{x^2}$, then the value of k is

A. -18

B. -12

C. -6

D. 12

E. 18

Question 3

For the straight line graph above, the value of c is

A. 8

B. 11

C. 12

D. 14

E. 16

SECTION B – Module 3: Graphs and relations – continued

The following information relates to Questions 4 and 5.

A publisher produces a restaurant guide each year.

To produce x copies, the cost is C dollars, where

$$C = 15\,000 + 15x$$

If all of the x copies produced are sold, then the revenue gained is R dollars, where

$$R = 25x$$

Question 4

Which one of the following statements is **not** true?

- **A.** The cost and revenue equations are linear.
- **B.** The selling price for each copy of the guide is $25.
- **C.** It will cost $30 000 to produce 1000 copies of the guide.
- **D.** The revenue from selling 1000 copies of the guide is $15 000.
- **E.** The revenue is more than the cost if 1600 copies of the guide are sold.

Question 5

If x copies of the guide are produced and sold, then the profit made is P dollars, where P is given by

- **A.** $P = 15\,000 - 10x$
- **B.** $P = 10x - 15\,000$
- **C.** $P = 15x - 15\,000$
- **D.** $P = 40x - 15\,000$
- **E.** $P = 15\,000 - 40x$

Question 6

For the pair of simultaneous equations

$$2x - 3y = 7 \quad \text{and} \quad 3x = 5 - y$$

the solution is

- **A.** $x = -2, \quad y = -1$
- **B.** $x = -1, \quad y = -3$
- **C.** $x = -1, \quad y = 2$
- **D.** $x = 2, \quad y = -3$
- **E.** $x = 2, \quad y = -1$

Question 7

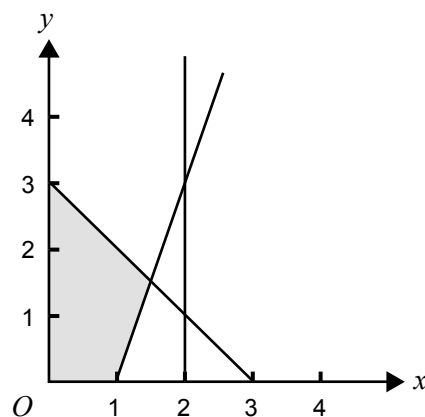The following inequalities define a region in the *x-y* plane.
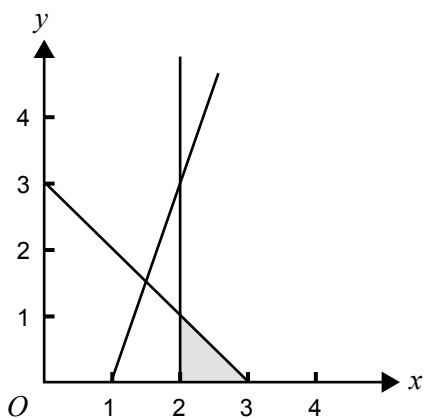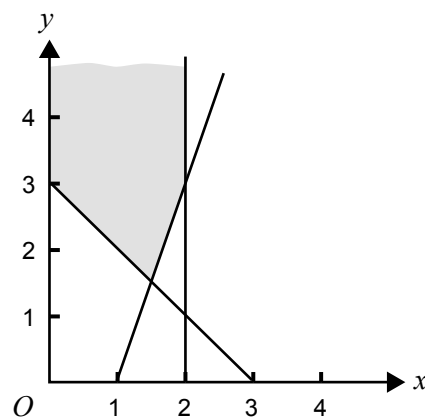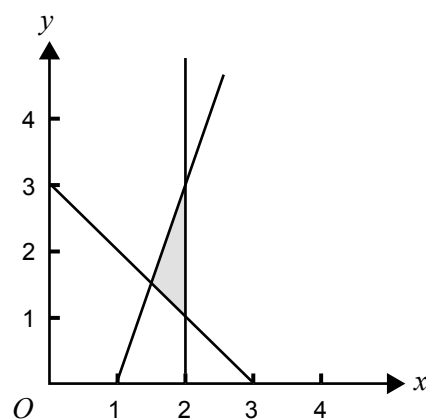
$$x \geq 0$$
$$x \leq 2$$
$$y \geq 0$$
$$3x - y \geq 3$$
$$x + y \leq 3$$

Which one of the following diagrams represents this region?

A.

B.

C.

D.

E.

SECTION B – Module 3: Graphs and relations – continued
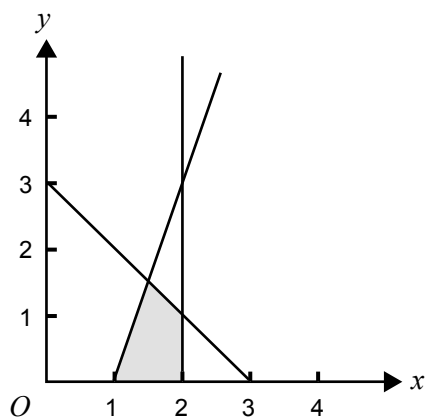
Question 8

In the diagram below, the shaded region (with boundaries included) represents the feasible region for a linear programming problem with the objective function $Z = 5x + 3y$.
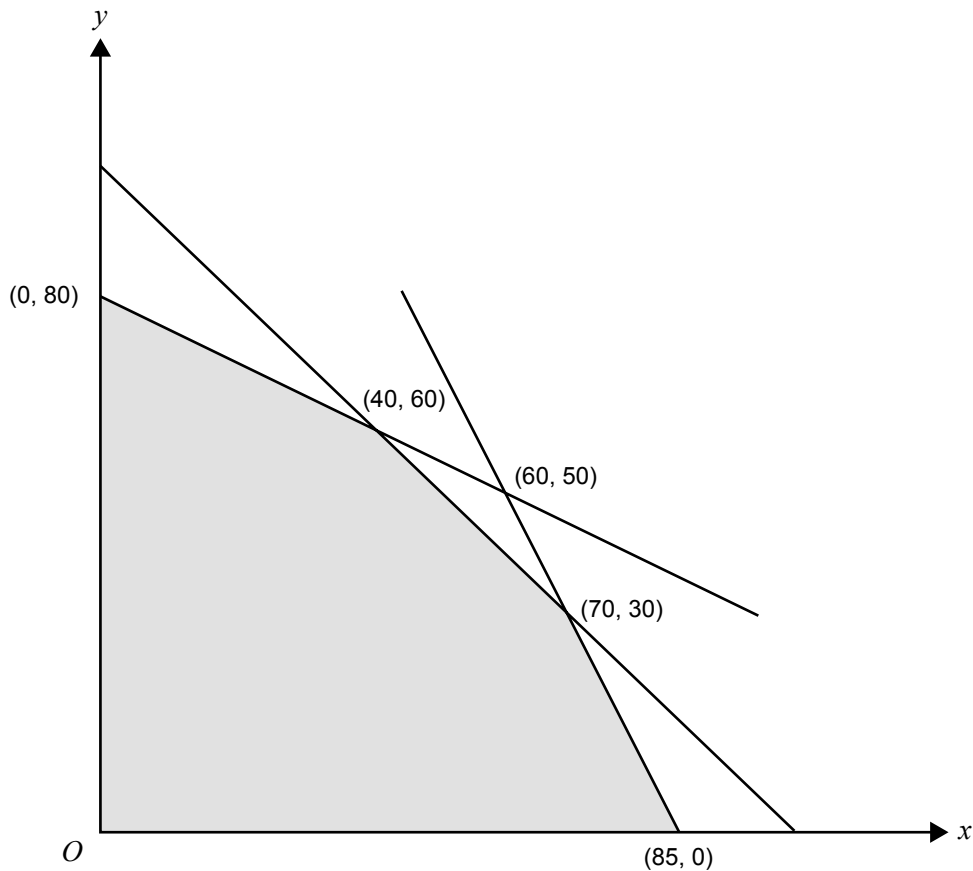
The **maximum** value of Z for this feasible region occurs at the point with coordinates

A. (0, 80)

B. (40, 60)

C. (60, 50)

D. (70, 30)

E. (85, 0)

Question 9

Jensen has two jobs, one at a nursery and the other in a restaurant. Each week he works for at least 18 hours; he works at least 4 hours at the nursery and at most 16 hours in the restaurant.

Also, each week, Jensen works at least twice as many hours in the restaurant than he does at the nursery.

Let

    x be the number of hours per week that Jensen works at the nursery

and

    y be the number of hours per week that Jensen works in the restaurant.

The set of constraints that apply to Jensen's working hours is

A. $x \leq 4,$ $y \geq 16,$ $x + y \geq 18,$ $y \geq 2x$

B. $x \leq 4,$ $y \geq 16,$ $x + y \geq 18,$ $2y \geq x$

C. $x \geq 4,$ $y \leq 16,$ $x + y \leq 18,$ $2y \geq x$

D. $x \geq 4,$ $y \leq 16,$ $x + y \geq 18,$ $y \geq 2x$

E. $x \geq 4,$ $y \leq 16,$ $x + y \geq 18,$ $2y \geq x$

Module 4: Business-related mathematics

> Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

Question 1

Under a hire purchase agreement, Sheng will pay a total of $960 for a television set. He is required to pay a deposit of $120 and to pay the balance in regular equal monthly payments over 6 months.

The monthly repayments are

A. $70

B. $80

C. $140

D. $160

E. $180

Question 2

Derek invested $26 000 for eighteen months and earned $975 in simple interest.

The annual interest rate for the investment is

A.   0.025%

B.   0.0563%

C.   2.5%

D.   3.75%

E.   5.63%

Question 3

Heather invests $45 000 at 4% per annum for 5 years compounding annually.

The total amount of interest earned is

A. $1 800

B. $2 100

C. $9 000

D. $9 750

E. $54 750

Question 4

Swee borrowed $150 000 at 6.2% per annum compounding monthly. The repayments are $1100 per month.

The balance of the loan at the end of five years is closest to

A. $0

B. $84 000

C. $127 000

D. $137 000

E. $148 000

Question 5

Zoltan is running a convenience store. He purchases equipment for $6500. It is anticipated that the equipment will last 5 years and have a depreciated value of $2000.

Assuming the straight line method of depreciation, the equipment is depreciated annually by

A. $400

B. $900

C. $1027

D. $1300

E. $4500

Question 6

Interest is paid monthly into an account at a rate of 3% per annum. Each month, immediately after the interest is paid, the account is debited $5 in fees. No other transactions take place. The initial amount of money in the account is $12 200.

After all interest has been paid and fees debited, the balance in the account at the end of **two** months is

A. $12 251.06

B. $12 261.08

C. $12 271.09

D. $12 932.83

E. $12 953.13

Question 7

Lim invested $8000 in an investment account, earning r % interest per annum, compounding quarterly.

The balance in dollars, after 5 years, is given by

A. $8000\left(1+\dfrac{r}{100}\right)^5$

B. $8000\left(1+\dfrac{r}{100}\right)^{20}$

C. $8000\left(1+\dfrac{r}{400}\right)^5$

D. $8000\left(1+\dfrac{r}{400}\right)^{20}$

E. $8000\left(1+\dfrac{r}{1200}\right)^{60}$

SECTION B – Module 4: Business-related mathematics – continued

Question 8

The following is an extract from a bank account showing all transactions for the period 1 January to 30 June, 2003.

Date	Particulars	Credit	Debit	Balance
01 Jan 2003	Brought Forward			4320.00
15 Mar 2003	Deposit	2100.00		6420.00
31 Mar 2003	Interest	32.40		6452.40
22 May 2003	Withdrawal		460.00	5992.40
30 June 2003	Interest			

Interest on this account is calculated at a rate of 0.25% per month on the minimum monthly balance and paid into the account quarterly. Interest for the June period (April to June) is paid on 30 June.

The balance in the account after interest is paid on 30 June 2003 is

A. $6039.64

B. $6038.49

C. $6024.76

D. $6023.51

E. $6022.36

Question 9

Peter borrows $80 000 for 10 years at 5.6% per annum, compounding monthly, with monthly repayments of $555.

Which one of the following statements is **true**?

A. The loan will be fully paid out in ten years.

B. At the end of five years, the balance of the loan will be $40 000.

C. The amount of interest paid each month during the loan increases.

D. Weekly repayments of $132 compounding weekly would reduce the period of the loan.

E. If one extra payment of $2000 is to be made, it would be better to make it at the end of year eight than at the end of year two.

Module 5: Networks and decision mathematics

> Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions.

Question 1

The bipartite graph below shows the tasks that each of five people are able to undertake.
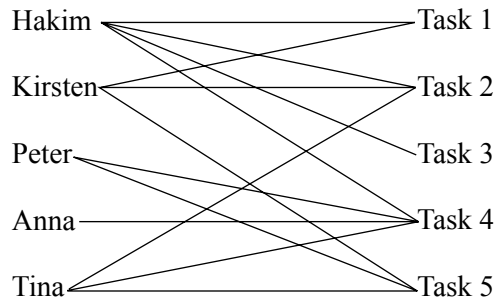
If each person is to be allocated one task only, then a feasible task allocation is

A.

Hakim	3
Kirsten	1
Peter	5
Anna	4
Tina	2

B.

Hakim	3
Kirsten	2
Peter	5
Anna	4
Tina	1

C.

Hakim	3
Kirsten	1
Peter	2
Anna	4
Tina	5

D.

Hakim	3
Kirsten	5
Peter	1
Anna	4
Tina	2

E.

Hakim	3
Kirsten	5
Peter	1
Anna	2
Tina	4

Question 2
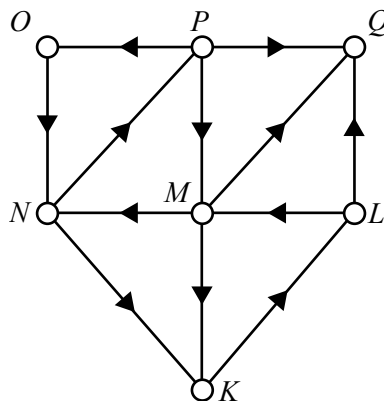
For the directed graph shown above, vertex O can **not** be reached from vertex
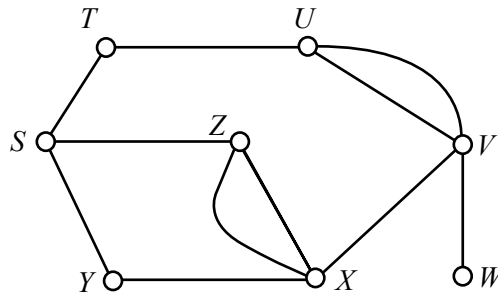
A. L

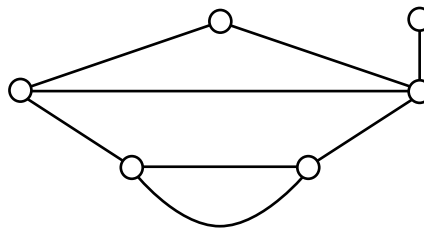**B.** M

C. N

D. P

E. Q

Question 3

In the network below, an Euler path can be created by adding **one** new edge.

Adding which one of the following edges creates an Euler path?

A. *ST*

B. *SU*

C. *SX*

D. *XW*

E. *ZY*

Question 4

The sum of the degrees of all the vertices in the network above is
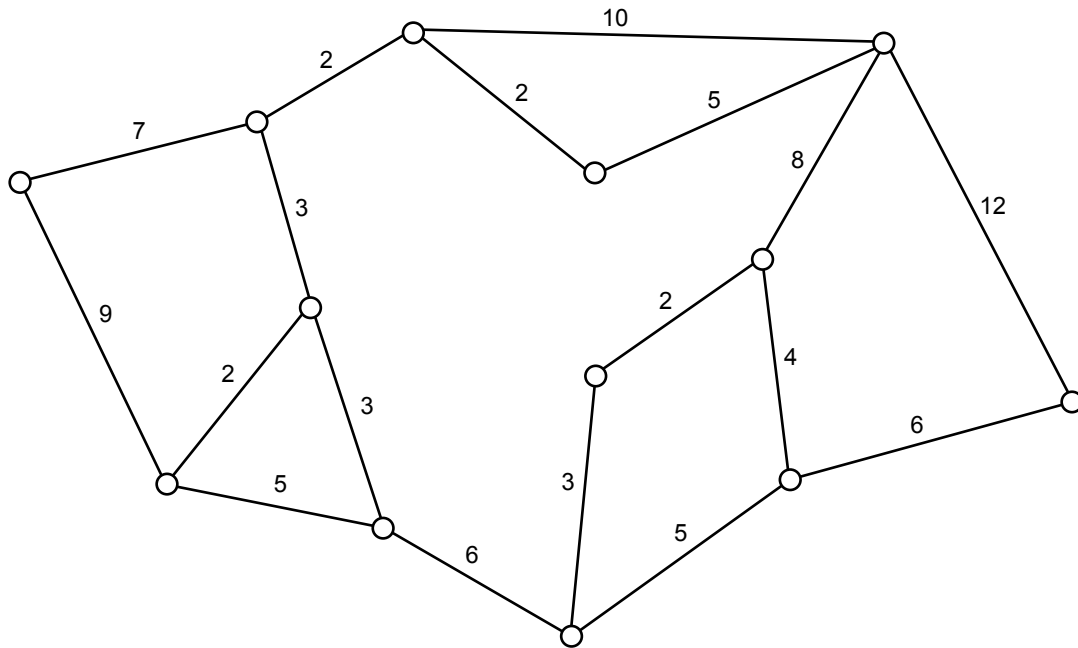
A.   6

B.   7

C.   8

D.   15

E.   16

Question 5

A connected planar graph has an **equal** number of vertices and faces.

If there are 20 edges in this graph, the number of vertices must be

A.   9

B.   10

C.   11

D.   20

E.   22

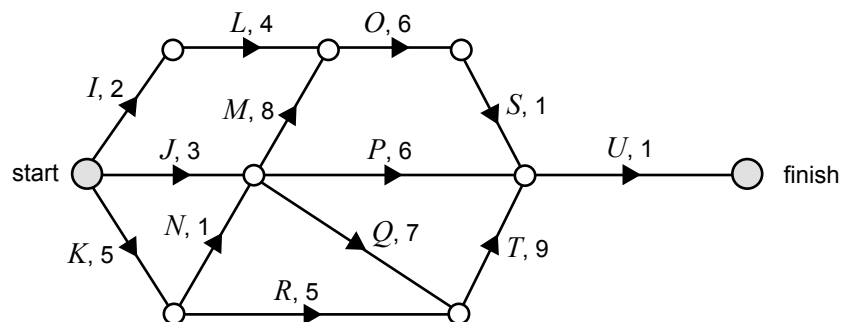**SECTION B – Module 5: Networks and decision mathematics** – continued

Question 6

The length of the minimal spanning tree for this network is

A.   37

B.   38

C.   45

D.   47

E.   51

Question 7

The following network gives the times in hours to complete the 12 tasks required to finish a project.
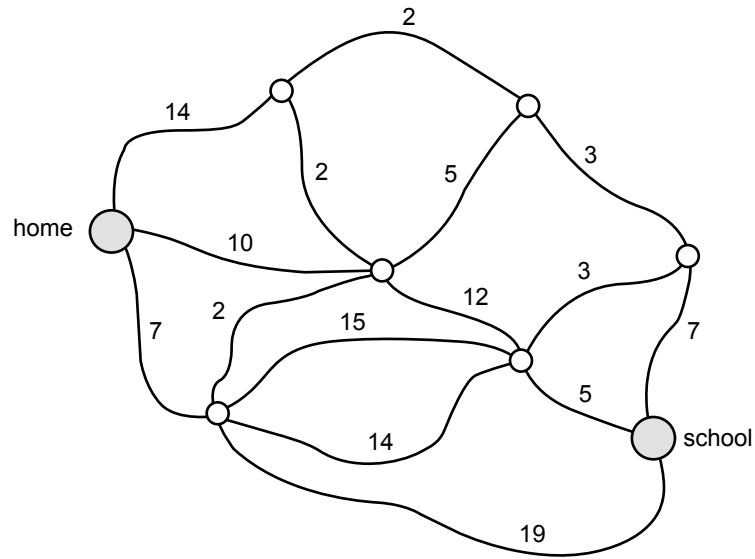

The critical path for this project is

A.   *J-P-U*

B.   *K-R-T-U*

C.   *J-M-O-S-U*

D.   *K-N-Q-T-U*

E.   *K-N-M-O-S-U*

Question 8

The network below shows the travel times, in minutes, along a series of roads that connect a student's home to school.

The shortest time, in minutes, for this student to travel from home to school is

A.   22
B.   23
C.   24
D.   25
E.   26

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SECTION B – Module 5: Networks and decision mathematics – continued

Question 9

Five graphs are each represented by an adjacency matrix as shown below.

Graph	Adjacency matrix
M	$\begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
N	$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$
O	$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$
P	$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
Q	$\begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Which adjacency matrix represents a **connected** graph?

A. *M*

B. *N*

C. *O*

D. *P*

E. *Q*

END OF QUESTION BOOK