



Victorian Certificate of Education 2002

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 4 November 2002

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 18 pages, with a detachable sheet of miscellaneous formulas in the centrefold and two blank pages for rough working.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part I

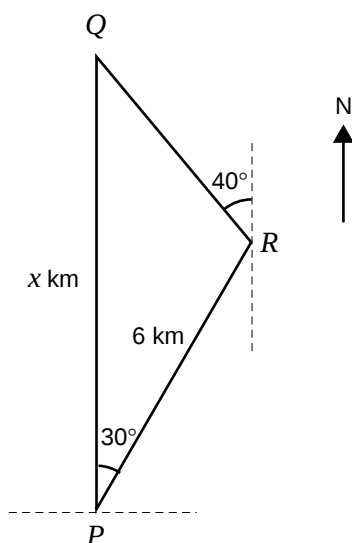
Answer **all** questions in pencil, on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

Question 1

A ship leaves a port, P , and sails 6 km on a heading of $N30^\circ E$ to position R . It then heads $N40^\circ W$ until it reaches a port, Q , which is directly north of P .



The distance x km from P to Q is given by

- A. $\frac{x}{\sin(40^\circ)} = \frac{6}{\sin(110^\circ)}$
- B. $\frac{x}{\sin(30^\circ)} = \frac{6}{\sin(110^\circ)}$
- C. $\frac{x}{\sin(30^\circ)} = \frac{6}{\sin(40^\circ)}$
- D. $\frac{x}{\sin(40^\circ)} = \frac{6}{\sin(30^\circ)}$
- E. $\frac{x}{\sin(110^\circ)} = \frac{6}{\sin(40^\circ)}$

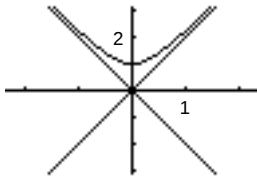
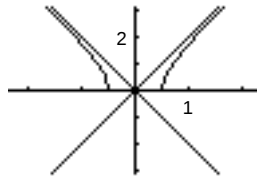
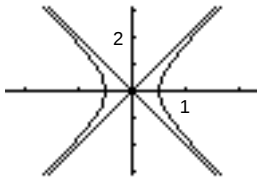
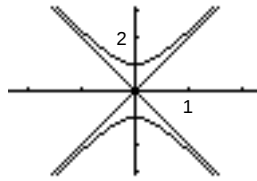
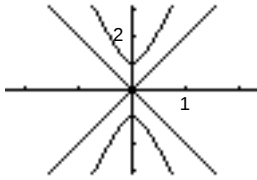
Question 2

The graph of $y = \frac{x^2 + 4}{x^2}$ has

- A. a single asymptote $x = 0$, and no turning points.
- B. two asymptotes $x = 0$ and $y = 0$, and no turning points.
- C. two asymptotes $x = 0$ and $y = 1$, and no turning points.
- D. a single asymptote $x = 0$, and turning points at $x = \pm\sqrt{2}$.
- E. a single asymptote $x = 0$, and intercepts at $x = \pm 2$.

Question 3

Which one of the following could be the graph of the curve with equation $4x^2 - y^2 + 1 = 0$?

A.**B.****C.****D.****E.****Question 4**

Let $f: (0, \frac{\pi}{2}) \rightarrow \mathbb{R}$ where $f(x) = \sec^2(x) + \operatorname{cosec}^2(x)$.

Which one of the following statements is true?

- A.** f is identical to the function $h: (0, \frac{\pi}{2}) \rightarrow \mathbb{R}$ where $h(x) = \sec^2(x)\operatorname{cosec}^2(x)$
- B.** f is identical to the function $g: (0, \frac{\pi}{2}) \rightarrow \mathbb{R}$ where $g(x) = \tan^2(x) + \cot^2(x)$
- C.** f is equal to 1 for all values of x in $(0, \frac{\pi}{2})$
- D.** f has range $[1, \infty)$
- E.** f has range $[2, \infty)$

Question 5

The exact value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\cos^{-1}\left(\frac{1}{2}\right)$ is

- A. 0.822
- B. 2700
- C. $\frac{7\pi}{24}$
- D. $\frac{\pi^2}{24}$
- E. $\frac{\pi^2}{12}$

Question 6

If $z = x + yi$, where x and y are non-zero real numbers, which one of the following **must** be a pure imaginary number (that is, a complex number with real part zero)?

- A. $z - \bar{z}$
- B. $z + \bar{z}$
- C. iz
- D. $i\bar{z}$
- E. $\text{Im}(\bar{z})$

Question 7

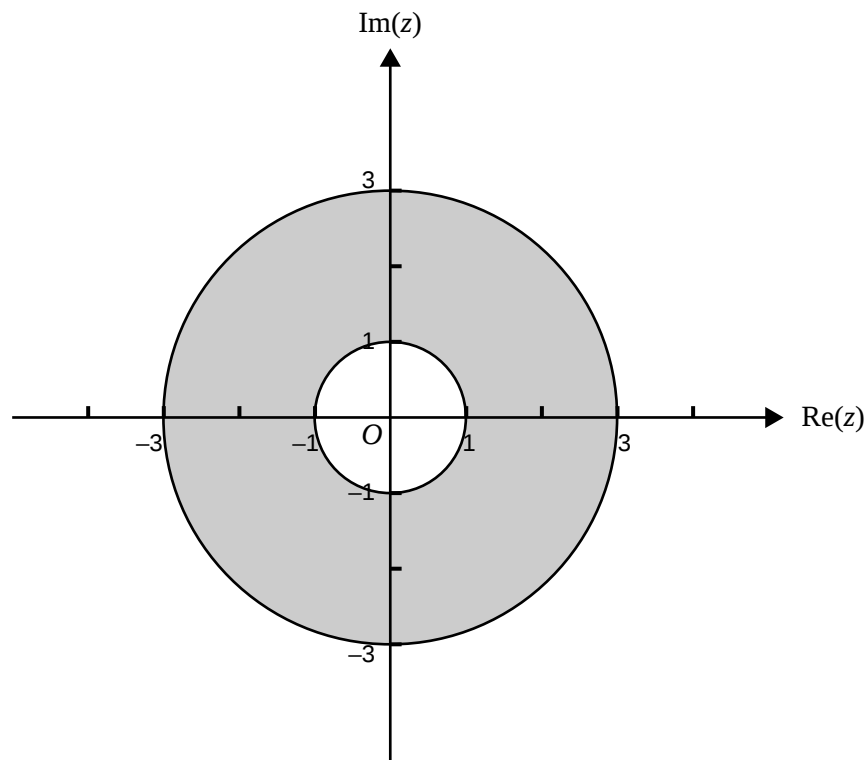
If $z = -2i$, then $|z^2|$ and $\text{Arg}(z^2)$ are respectively

- A. 4 and 0
- B. 4 and π
- C. 4 and $-\pi$
- D. -4 and π
- E. -4 and $-\pi$

Question 8

Given that $(z - 2 + i)$ is a factor of $P(z) = 2z^3 - 7z^2 + 6z + 5$, which one of the following statements must be true?

- A. $P(-2 + i) = 0$
- B. $P(-2 - i) = 0$
- C. $P(z) = 0$ has no real roots
- D. $P(z) = 0$ has one real root and two complex roots
- E. $P(z) = 0$ has two real roots and one complex root

Question 9

Given that $z \in \mathbb{C}$, the shaded region (with boundaries included) is best described by

- A. $\{z: 1 \leq z\bar{z} \leq 3\}$
- B. $\{z: 1 \leq z\bar{z} \leq 9\}$
- C. $\{z: 1 \leq |z| \leq 9\}$
- D. $\{z: 1 \leq |z|^2 \leq 3\}$
- E. $\{z: (|z| \leq 3) - (|z| \leq 1)\}$

Question 10

$\frac{3x+4}{(x-4)^2}$ expressed in partial fractions has the form

- A. $\frac{A}{(x+4)} + \frac{B}{(x-4)}$
- B. $\frac{A}{(x-4)} + \frac{B}{(x-4)}$
- C. $\frac{A}{(x-4)} + \frac{B}{(x-4)^2}$
- D. $\frac{A}{(x-4)} + \frac{Bx+C}{(x-4)^2}, B \neq 0$
- E. $\frac{A}{(x-4)^2} + \frac{B}{(x-4)^2}$

Question 11

The derivative of $x \cos^{-1}(x)$ with respect to x is $\cos^{-1}(x) - \frac{x}{\sqrt{1-x^2}}$.

It follows that an antiderivative of $\cos^{-1}(x)$ is

- A. $x \cos^{-1}(x) + \int \frac{x}{\sqrt{1-x^2}} dx$
- B. $\int x \cos^{-1}(x) dx + \int \frac{x}{\sqrt{1-x^2}} dx$
- C. $x \cos^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}} dx$
- D. $\int x \cos^{-1}(x) dx + \frac{x}{\sqrt{1-x^2}}$
- E. $x \cos^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$

Question 12

An antiderivative of $\frac{e^{2x}}{2e^{2x}-1}$ (for $e^{2x} > \frac{1}{2}$) is

- A. $\frac{1}{2}(x - e^{2x})$
- B. $4\log_e(2e^{2x} - 1)$
- C. $2\log_e(2e^{2x} - 1)$
- D. $\frac{1}{2}\log_e(2e^{2x} - 1)$
- E. $\frac{1}{4}\log_e(2e^{2x} - 1)$

Question 13

Using an appropriate substitution, $\int_0^4 (2x + 3)\sqrt{2x + 1} \, dx$ is equal to

A. $\frac{1}{2} \int_0^4 (u + 2)\sqrt{u} \, du$

B. $\frac{1}{2} \int_1^3 (u + 2)\sqrt{u} \, du$

C. $\frac{1}{2} \int_1^9 (u + 2)\sqrt{u} \, du$

D. $\int_1^9 (u + 2)\sqrt{u} \, du$

E. $2 \int_1^9 (u + 2)\sqrt{u} \, du$

Question 14

The value of $\int_0^1 \frac{x^2 - 2x}{2 \cos(x)} \, dx$, correct to four decimal places, is

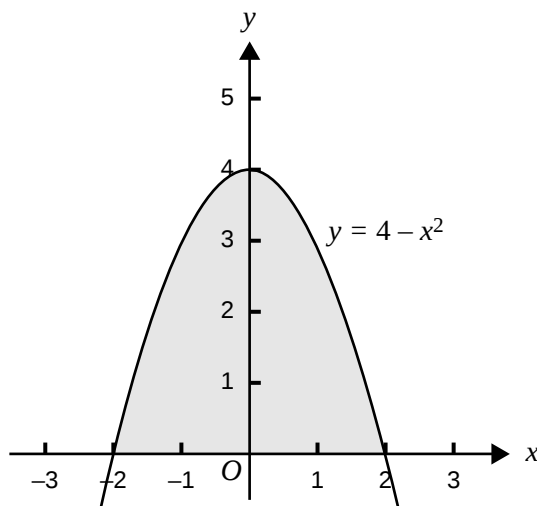
A. -0.4369

B. -0.3334

C. -0.2622

D. -0.0484

E. 0.4369

Question 15

The shaded region is bounded by the x -axis and the curve with equation $y = 4 - x^2$.

This region is rotated about the y -axis to form a solid of revolution.

The volume of the solid, in cubic units, is

- A. 6π
- B. 8π
- C. $\frac{32\pi}{3}$
- D. 16π
- E. $\frac{64\pi}{3}$

Question 16

If $y = e^{kx}$ satisfies the differential equation $\frac{d^2y}{dx^2} = 5\frac{dy}{dx} - 6y$, the possible values for k are

- A. -6 and 1
- B. -1 and 6
- C. -5 and 6
- D. -3 and -2
- E. 2 and 3

Question 17

The radius of a sphere is increasing at a rate of 3 cm/min.

When the radius is 8 cm, the rate of increase, in cm^3/min , of the volume of the sphere is

- A. $85\frac{1}{3}\pi$
- B. 256π
- C. $682\frac{2}{3}\pi$
- D. 768π
- E. 2048π

Question 18

To reach a lookout from a campsite, a bushwalker walks 2 km due south up a slope which is inclined at 30° to the horizontal.

Let $\hat{i}$ be a unit vector due east, $\hat{j}$ a unit vector due north, and $\hat{k}$ a unit vector vertically up.

The position vector of the lookout relative to the campsite is

- A. $-\hat{j} + \sqrt{3}\hat{k}$
- B. $-\sqrt{3}\hat{j} + \hat{k}$
- C. $-2\hat{j} + \hat{k}$
- D. $\sqrt{2}(-\hat{j} + \hat{k})$
- E. $-2\hat{j} + \sqrt{3}\hat{k}$

Question 19

A vector perpendicular to $2\hat{i} - 2\hat{j} - 3\hat{k}$, and with magnitude 4, is

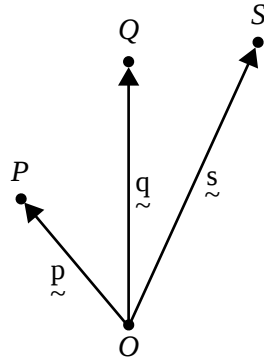
- A. $\frac{4}{3}(\hat{i} - 2\hat{j} - 2\hat{k})$
- B. $\frac{4}{9}(\hat{i} - 2\hat{j} + 2\hat{k})$
- C. $4(\hat{i} - 2\hat{j} + 2\hat{k})$
- D. $\frac{4}{3}(-2\hat{i} + \hat{j} - 2\hat{k})$
- E. $\frac{4}{9}(-2\hat{i} + \hat{j} - 2\hat{k})$

Question 20

The vector resolute of $-\hat{i} - \hat{j} + 3\hat{k}$ in the direction of $\hat{i} + 2\hat{j} - 2\hat{k}$ is $-\hat{i} - 2\hat{j} + 2\hat{k}$.

The vector resolute of $-\hat{i} - \hat{j} + 3\hat{k}$ perpendicular to $\hat{i} + 2\hat{j} - 2\hat{k}$ is

- A. $\hat{j} + \hat{k}$
- B. $-\hat{j} - \hat{k}$
- C. $-2\hat{i} - 3\hat{j} + 5\hat{k}$
- D. $2\hat{i} + 3\hat{j} - 5\hat{k}$
- E. $\hat{i} + 2\hat{j} - 2\hat{k}$

Question 21

Let the three distinct points P , Q and S have non-zero position vectors $\tilde{p}$, $\tilde{q}$ and $\tilde{s}$ respectively.

To prove that P , Q and S lie in a straight line, it is sufficient to show that

- A. $\tilde{s} = k(\tilde{q} - \tilde{p}), k \in R$
- B. $(\tilde{q} - \tilde{p}) = k(\tilde{s} - \tilde{q}), k \in R$
- C. $(\tilde{q} - \tilde{p}) \cdot (\tilde{s} - \tilde{q}) = 0$
- D. $\tilde{p} + \tilde{q} + \tilde{s} = \tilde{0}$
- E. $\tilde{p} + \tilde{q} + \tilde{s} \neq \tilde{0}$

Question 22

The velocity of a particle at time t , $t \geq 0$, is given by $\tilde{v}(t) = \sin(t-1)\tilde{i} + 5\tilde{j} - 3e^{(1-t)}\tilde{k}$.

The acceleration of the particle when $t = 1$ is

- A. $\tilde{i} + 3\tilde{k}$
- B. $-\tilde{i} + 3\tilde{k}$
- C. $\tilde{i} - 3\tilde{k}$
- D. $\tilde{i} + 5\tilde{j} + 3\tilde{k}$
- E. $-\tilde{i} + 5\tilde{j} + 3\tilde{k}$

Question 23

At time $t \geq 0$, a particle has displacement

$$\vec{r}(t) = (3t - 6)\vec{i} - (t^2 - 6t - 16)\vec{j}$$

where $\vec{i}$ is a horizontal unit vector and $\vec{j}$ is a unit vector in the vertically up direction.

The particle reaches its maximum height when t is

- A. 1
- B. 2
- C. 3
- D. 8
- E. 10

Question 24

The acceleration of a particle at time t , $t \geq 0$, is given by $\vec{a}(t) = 2\sin(t)\vec{i}$.

The velocity of the particle when $t = \pi$ is $2\vec{i} + 2\vec{j}$.

The **initial velocity** of the particle is

- A. $-2\vec{i}$
- B. $2\vec{j}$
- C. $-2\vec{i} + 2\vec{j}$
- D. $2\vec{i} + 2\vec{j}$
- E. $6\vec{i} + 2\vec{j}$

Question 25

A particle is acted on by two forces, one of magnitude 5 newtons acting due north, and the other of magnitude x newtons acting due east. The magnitude of the resultant force is 8 newtons.

The number x is

- A. 3
- B. $\sqrt{39}$
- C. 7
- D. $\sqrt{89}$
- E. 13

Question 26

A particle of mass 2 kg has a horizontal velocity component of magnitude 12 m/s, and a vertical velocity component of magnitude 5 m/s.

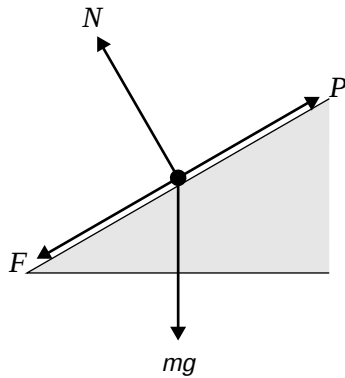
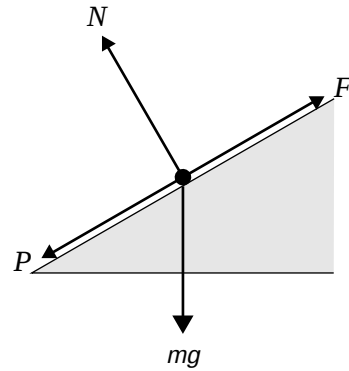
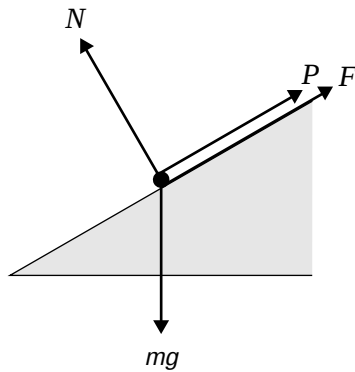
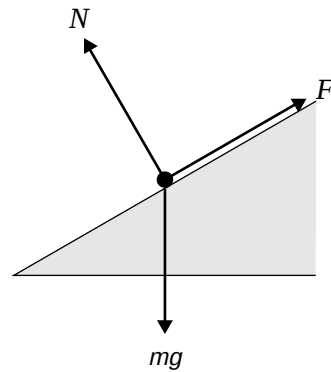
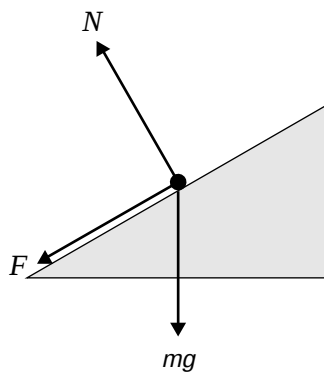
The magnitude, in kg m/s, of the momentum of the particle is

- A. 10
- B. 14
- C. 24
- D. 26
- E. 34

Question 27

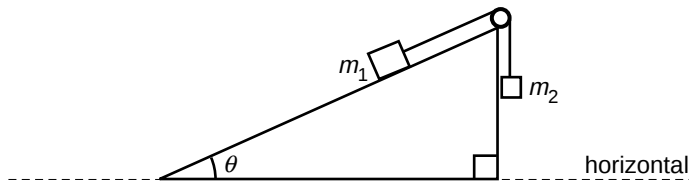
A particle of mass m kg rests on a rough inclined plane. The particle is stationary. There is a normal reaction of magnitude N newtons, and F newtons is the magnitude of the force due to friction. Where present, P newtons is the magnitude of a force applied to the particle parallel to the plane.

Which one of the following diagrams **cannot** be a correct representation of the forces acting on the particle?

A.**B.****C.****D.****E.**

Question 28

Two particles of masses m_1 and m_2 are connected by a light string that passes over a smooth pulley as shown in the diagram. The particle of mass m_1 rests on a smooth, inclined plane.

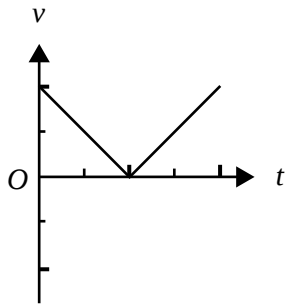
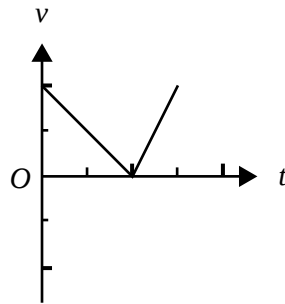
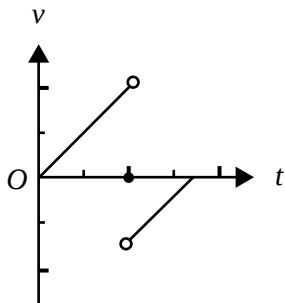
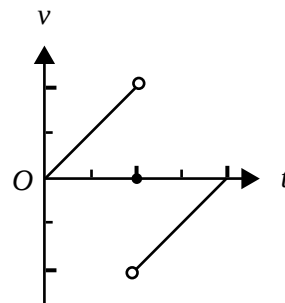
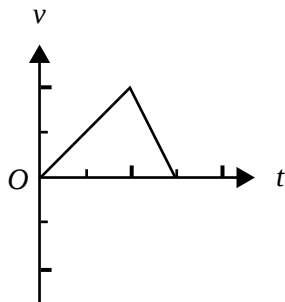


If the system is in equilibrium, $\frac{m_1}{m_2}$ is equal to

- A. $\sin(\theta)$
- B. $\cos(\theta)$
- C. $\frac{1}{\sin(\theta)}$
- D. $\frac{1}{\cos(\theta)}$
- E. 1

Question 29

A ball is dropped from rest on to a concrete floor and bounces vertically to half its drop height. Which one of the following velocity-time graphs could represent the motion of the ball?

A.**B.****C.****D.****E.**

Question 30

A 150 litre cylinder of air contains 20% oxygen. The amount of oxygen in the cylinder is to be increased by pumping in pure oxygen at a constant rate of 10 litres/minute, while removing the uniformly mixed air at the same rate.

If P litres is the volume of oxygen in the cylinder at time t minutes after the pumping begins, a differential equation for P in terms of t is

- A. $\frac{dP}{dt} = 8P; t = 0, P = 30$
- B. $\frac{dP}{dt} = 8P; t = 0, P = 150$
- C. $\frac{dP}{dt} = 30 + 10t; t = 0, P = 30$
- D. $\frac{dP}{dt} = 10 - \frac{P}{15}; t = 0, P = 30$
- E. $\frac{dP}{dt} = 10 - \frac{P}{15}; t = 0, P = 150$

Working space

TURN OVER

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK



Victorian Certificate of Education 2002

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

SPECIALIST MATHEMATICS

Written examination 1 (Facts, skills and applications)

Monday 4 November 2002

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II QUESTION AND ANSWER BOOK

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<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
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Materials supplied

- Question and answer book of 8 pages.

Instructions

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- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part II

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an **exact** answer is required for a question, appropriate working must be shown.

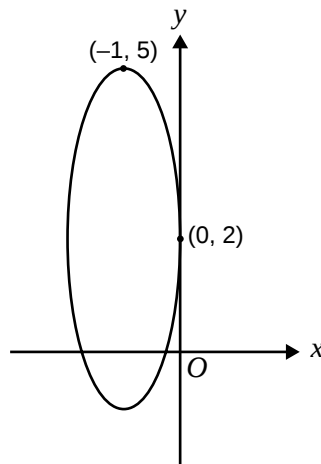
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The diagram shows an ellipse with axes of symmetry parallel to the coordinate axes.



- a. Write down the equation of the ellipse.

2 marks

- b. Describe a sequence of transformations which, when applied to the unit circle with equation $x^2 + y^2 = 1$, produces this ellipse.

2 marks

PART II – continued

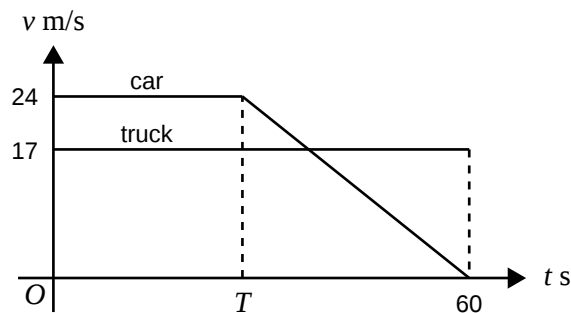
www.theallpaper.com

Question 2

A car travelling at 24 m/s overtakes a truck travelling at a constant speed of 17 m/s along a straight road. T seconds later, the car decelerates uniformly to rest.

The truck continues at constant speed and it passes the car at the instant the car comes to a stop. This is exactly 60 seconds after the car had passed the truck.

The velocity-time graph representing this situation is shown below.



Find T .

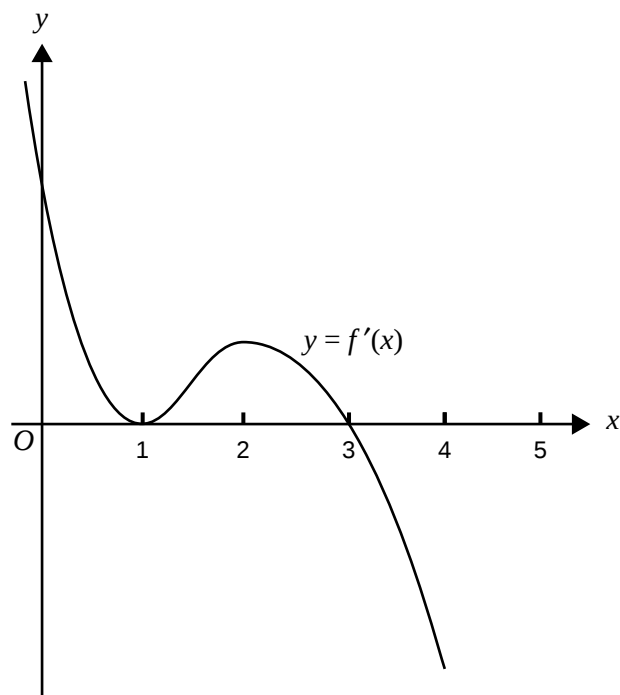
3 marks

Question 3

Find an antiderivative of $\frac{\sin(\frac{x}{2})}{\cos^2(\frac{x}{2})}$.

2 marks

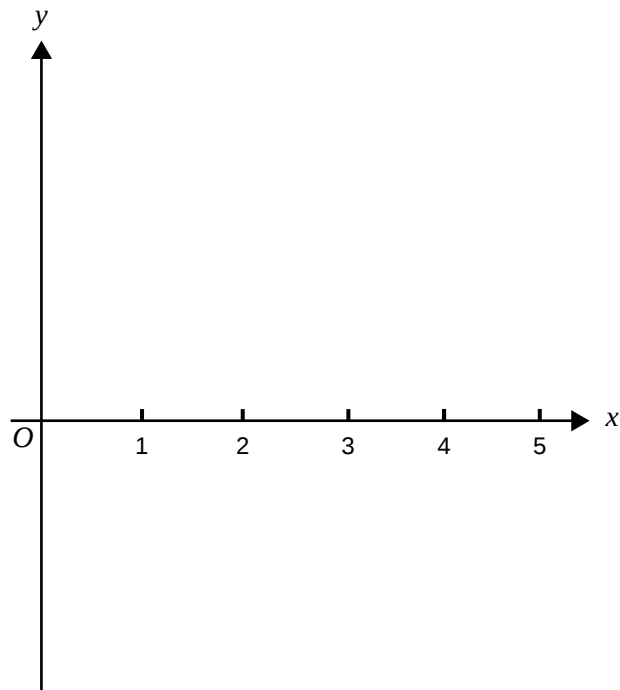
Working space

Question 4

The graph of the derivative of a function f is shown above.

On the axes below, draw **two** graphs each of which could be the graph of the function f .

In each case, show clearly any stationary points.



4 marks

PART II – continued

Question 5

Let $\vec{u} = \vec{i} - \vec{j} - \vec{k}$ and $\vec{v} = \vec{i} + a\vec{j} + \vec{k}$, where $a \in \mathbb{R}$. The angle between $\vec{u}$ and $\vec{v}$ is 60° .

Find the exact value of a .

4 marks

Working space

Question 6

A particle travels in a straight line with velocity v m/s at time t s. The acceleration of the particle, a m/s², is given by $a = -2 + \sqrt{v^2 + 5}$.

Find, correct to two significant figures, the time taken in seconds for the speed of the particle to increase from 3 m/s to 10 m/s.

3 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a + b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$
hyperbola:	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

mid-point rule:

$$\int_a^b f(x) dx \approx (b-a) f\left(\frac{a+b}{2}\right)$$

trapezoidal rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + h f(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$