Specialist Mathematics

2013 Chief Assessor's Report

Government
of South Australia

SACE
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SPECIALIST MATHEMATICS

2013 CHIEF ASSESSOR'S REPORT

OVERVIEW

Chief Assessors' reports give an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, the quality of student performance, and any relevant statistical information.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

Most samples of work displayed a range of routine and complex tasks, which gave students the opportunity to achieve at all levels of the performance standards. Although early in the year tests may not be able to assess the most complex level, overall teachers had the necessary balance of complex and routine questions within the set tasks. In most samples the tasks were packaged in the order in which they were completed. This assisted moderators to see the progress of a student throughout the year. An indication of the degree of difficulty of a question was also useful for moderators.

For this assessment type, it is highly recommended that teachers structure questions to allow capable students to achieve above a C grade level.

It is helpful to moderators if teachers clearly relate their assessment decisions to the performance standards. For example, marks and percentages should be matched to the performance standards so that moderators have the opportunity to confirm teachers' decisions.

Assessment Type 2: Folio

Folios should be submitted in a report format, as per the subject outline guidelines, with an introduction, analysis, and conclusion. A report format is more easily assessed against the performance standards.

Folios in the assignment style often do not allow students to achieve above a C grade level in the Mathematical Modelling and Problem-solving (MMP) assessment design criterion. Pages 45 and 46 of the 2014 subject outline detail the requirements of this task and the expectation of a response in a report format. Folios should allow students the opportunity to develop and display the higher-order problem-solving skills listed in the MMP criterion and to communicate their work according to the specific features of the Communication of Mathematical Information (CMI) criterion.

When teachers supply information about which specific features are to be assessed in a task, it assists the moderators to confirm the grade given by the teacher. For instance, if conjecture and proof is not being assessed then teachers should omit MMP5 from their task outline and/or delete it from the performance standards for the assessment.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination

SECTION A

Of the nine questions in Section A, seven had average marks between 66% and 73% of the marks allocated for the question, while the remaining two had average marks of 49% and 42% of the marks allocated for the question. The latter two questions dealt with inductive argument and the triangle inequality. This will be referred to below in the discussion of individual questions.

Question 1

This question had an average mark of 72% of the marks allocated for the question, indicating a sound understanding of parametric equations, the use of technology, and basic parametric calculus. This conclusion is supported by the fact that 30% of the cohort earned full marks for this question.

Students lost marks in part (b) for not understanding, or ignoring, the implication of the word ‘exact’, which was used twice in part (b), and in part (c) because they could not clearly represent the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ on the graph by drawing the tangent to the curve at point P.

Another, less pleasing aspect of the responses is that 5% of students did not gain any marks for this straightforward application of the key concepts, processes, and tools of the Specialist Mathematics course.

Question 2

The average mark for Question 2 was also 72% of the marks allocated for the question, and its modal mark was the maximum for the question. In fact, for Question 2, more students earned full marks and fewer students earned no marks than for Question 1. Marks were lost for poor use of the relevant trigonometric relationships. Either the sine addition formulae or the unit-circle symmetry relationships $[\sin(\frac{\pi}{2} \pm h) = \cos h]$ could be used to show the required relationship in part (c). Other errors were lines misplaced on the graph, a lack of labels, and mishandling ‘2’ when finding $f(\frac{\pi}{4} \pm h)$.

Question 3

While only 28% of those who attempted this question received full marks, the modal mark was a healthy 6 out of 8 and the average mark was 69% of the marks allocated for the question. It is worrying that 11% of the cohort earned no marks at all.

For those who did make an attempt to answer the question, marks were deducted because students did not give reasons, failed to name relevant theorems to substantiate mathematical logic, and/or omitted the required working. That notwithstanding, the average mark is significantly higher than the mean mark for similar questions over the past few years.

Question 4

Twenty-seven per cent of students earned full marks for Question 4, so the maximum mark was also the modal mark. The average mark for the question was 66% of the marks allocated for the question, indicating a notional 3 marks lost on average. Many students lost 2 marks for their inability to differentiate using the product rule, and many also lost a mark for not presenting a fully factorised polynomial as directed. While 4% of students earned no marks for this question, the majority demonstrated a sound grasp of polynomials and their applications.

Question 5

Students found this question the most difficult in Sections A and B. Approximately 15% of students gained no marks for this question, which dealt with the triangle inequality applied to complex numbers. Of these, a third did not attempt the question and the remaining two-thirds were unable to earn a mark. In stark contrast to the majority of questions in Section A (for 7 out of 9 questions the maximum mark was the modal mark), the maximum mark was the *least* frequent result.

While the majority of students realised that $|z^2|+|z|>|z^2-z|$ was true by the triangle inequality, there was a significant minority who did not. Additionally, many attempts to answer part (b)(ii) did not include the rearrangement $|z^2-z|+|z|>|z^2|$ (true by the triangle inequality), which gave the required result.

Part (c) was well done by the few students who knew $z=2\operatorname{cis}\theta \Rightarrow |z|=2$ and there was some good work done establishing the numerical result for part (d) with some students making proper use of their graphics calculator. The average mark for this question was 44% of the marks allocated for the question.

Question 6

This question caused the least difficulty for students. All but 1% of students earned marks by attempting to answer the question and the average mark was just above 73% of the marks allocated for the question. The vast majority of students did the basics well, although some did not distinguish between the verification of a specific case in part (a)(iii) and the proof of the general case required in part (b).

Students lost marks in part (b) for poor use of vector notation and gaps in logic and/or working. For example, many immediately wrote $|\boldsymbol{c}|^2$ on expansion, rather than $\boldsymbol{c} \bullet \boldsymbol{c}$. The better students expanded the equation fully, noted the commutativity of $\boldsymbol{c} \bullet \boldsymbol{d}$, used $\boldsymbol{c} \bullet \boldsymbol{c} = |\boldsymbol{c}|^2$ to complete the proof, and showed all the required steps. The term $\boldsymbol{c}^2$ was frequently misused.

The modal mark was the maximum mark, which was obtained by 25% of the cohort, indicating that there were many correct responses to the numerical applications of the result proved in part (b). However, a surprisingly large proportion of students who managed to complete the first application were unable to interpret the implications of the negative sign in part (c)(ii).

Question 7

In Question 7, the maximum mark was again the modal mark. The average mark was 69% of the marks allocated for the question and 5% of students were unable to earn any marks for this question.

There were many pleasing aspects to the responses to this question. The proof of the relationship between the varying quantities was very well done and those students who then chose to differentiate implicitly, reached the result for $\frac{\mathrm{d}x}{\mathrm{d}t}$ more efficiently than those who chose to make x the subject before differentiating.

Finding the required rate was sometimes made more difficult by a student's superficial interpretation of the information given in part (c).

The better responses gave exact answers to part (d), although this was not a requirement.

Question 8

Again, the modal mark was the maximum mark for the question. The question was very well done by the majority of students; only 2% of the cohort was unable to earn any marks.

Students lost marks when they confused x and t; these students did not really understand the role of t, as the parameter, in calculating x and y. Another weakness had to do with significant figures. Many students gave answers to three significant figures, as directed by the front page and part (c), but for too many students those figures were not correct. Students need to be aware of the impact of rounding intermediate values in a calculation, and to work to the accuracy of their calculators, rounding only the final answer. The average mark for Question 8 was 72% of the marks allocated for the question.

Question 9

Students found this one of the most difficult questions in Sections A and B, with 13% of students unable to gain any marks and an average mark for the cohort of 49% of the marks allocated for the question. The modal mark was 2 out of a possible 9 marks. Too many students simply listed successive statements without attempting to show the truth of one statement following from the truth of its predecessor. There is strong evidence that many students were unable to construct an inductive argument.

SECTION B

The average performance for the four questions in Section B ranged from 52% to 62% of the marks allocated for the question and averaged 57% of the marks allocated for the question, which could be classified as a borderline C/C+.

Question 10

The average mark for this question was 62% of the marks allocated for the question. The modal mark was 13, two short of the maximum, perhaps indicating that the last 2 marks in this question really had to be earned. Fewer than 2% of students achieved full marks and 4.5% earned no marks at all. Most students used scalar multiples to prove the ratio of division. Some students tried to use lengths only to prove the ratio of division and omitted to establish collinearity. Most of the question was well done except for parts (d) and (e); many students were unable to find the distance between two planes and missed the logic of finding a point on P_3 to substitute into the equation and thus find the value for λ.

Question 11

One of the markers commented that 'many [students] struggled with this question. Either they knew it or they didn't. Generally it was either 5 marks or 0 marks'. While this may be an overstatement, it goes some way towards explaining the average mark of 52% of the marks allocated for the question. Some candidates ignored the direction to 'solve the differential equation' and attempted to verify the given solution. Others were unable to separate the variables correctly, with many students not identifying the multiplicative connection between the differentials ($\mathrm{d}y$ and $\mathrm{d}t$) and the other quantities in the differential equation.

Notwithstanding, there were some good responses and 10% of students achieved full marks. The slope field was done well, although some students started the solution curve at (2, 0) rather than (0, 2).

Obtaining the required results for the second and first derivatives was reasonably well done. Some of the better solutions substituted the given solution into the differential equation and simplified the equation to get $\frac{\mathrm{d}y}{\mathrm{d}t} = (k - 2\alpha)e^{-\alpha t}$.

Differentiating this leads to the second derivative result. Substituting $k = 2\alpha$ gives the value of the first derivative.

The proportion of the cohort who received no marks for this question was 11%.

Question 12

Students could well be forgiven a sense of déjà vu as they began Question 12, but any such feeling would have disappeared as they wrote the complex number $u = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ in the form $r\operatorname{cis}\theta$. This was reasonably well done but students would be well advised to draw the complex number on an Argand diagram as an aid to obtaining the correct argument.

The modal mark for this question was 12 (out of 15). This was earned by 15% of the cohort. The average mark was 61% of the marks allocated for the question. The majority of students found it too difficult to relate the algebraic work on the complex numbers to the geometrical properties of complex numbers and could not earn marks for parts (d) and (e). Full marks were earned by 5% of students, while 3% earned no marks at all.

Question 13

This question had an average of 56% of the marks allocated for the question. Marks were lost early in the question as many students could not complete the table using Euler's method. Of those students who did attempt the table, many confused $W(1)$ with W_1 and gave an answer for which rounding to four significant figures made no sense. Students who cleared these two hurdles still had problems with significant figures, some giving imprecise answers by working incorrectly with rounded intermediate values.

The majority of students made a good job of solving the logistic differential equation in part (d), but some had trouble graphing it and interpreting the graph and the equation. Students should be aware that logistic graphs do not go through (0, 0), that the point of inflection occurs halfway to the maximum function value, and the reason for this.

The modal mark was 11 out of 16 earned by 8% of students, while 6% earned no marks and 4% earned full marks.

SECTION C

Of those students who attempted one of the two questions in Section C, 22% chose Question 14 and 78% chose Question 15. This is the most lopsided split since the inception of the Specialist Mathematics curriculum. While three-dimensional curves have appeared in the examination before, the students were probably more familiar with the differential system in Question 15 than the helix in Question 14.

Question 14

One student achieved full marks, and 28 students earned no marks despite making an attempt to answer the question. The modal mark was 2 and the average mark was 4.5 out of a total of 15 marks allocated for the question. Nevertheless good responses were presented by some students. Some students could perform the manipulations requested, but were circumspect about interpreting their results.

Question 15

Of the 957 students who attempted this question, 107 students achieved full marks. The average mark was 56% of the marks allocated for the question and 13% of students earned no marks. The modal mark was 14. Students lost marks by:

- not giving exact answers where requested
- confusing the values of t and x in parametric equations
- neglecting to draw the graph of $y=\frac{5}{2}x$ on the slope field
- failing to ‘show’ when requested.

When asked to show, explain, or prove, students are best advised to do so using mathematical language and notation, giving all logical steps and stating relevant reasons.

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