Specialist Mathematics

2010 ASSESSMENT REPORT

Mathematics Learning Area

Government of South Australia

SACE Board of SA

SPECIALIST MATHEMATICS

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GENERAL COMMENTS

The 2011 Specialist Mathematics external examination contained questions which covered a range of difficulty levels and covered a balanced mixture of graphics calculator work and algebraic solutions. The curriculum threads of the Triangle Inequality and Inductive Argument maintained their presence as did the iterative methods sub- topics, Euler's method, slope fields, and quadratic iteration. Graphs were well drawn, good use was made of the handheld technology and solution curves through slope fields were generally drawn well.

Students therefore demonstrated good knowledge and understanding of some of the newer aspects of this subject but many were less successful in some of the more traditional aspects of this subject. Particularly, many in this year's cohort did not handle the requirement for 'exact' solutions when this was specified, and the ability to effect the instruction to 'show' was also not a strength for a significant proportion of the cohort.

ASSESSMENT COMPONENT 1: EXAMINATION

Section A

This year's cohort was unable to take full advantage of the variety of single topic, shorter style routine questions.

Question 1

Of the 1678 students who attempted this question 922 achieved full marks and another 455 lost just 1 mark. An average of 84% was achieved by the cohort for this initial question. A substantial majority used the given information to correctly find the angle between the line and the plane's normal but significant numbers did not go on to deduce the angle between the line and the plane.

Question 2

With an average mark of 89% question 2 was the 'best' done question in the paper. Maximum marks were achieved by 1036 students, with a further 448 losing only one mark. Many of the remaining students did not do enough to 'show clearly' as required by the question's wording.

Question 3

After questions 1 and 2, the geometry content of question 3 saw a slide in the average mark down to 68%. Contributing to this diminution were the 73 students who did not attempt question 3 and the 90 who achieved no marks despite an attempt. It is concerning that 10% of students could not access any marks in this question on geometry. On the other hand nearly 28% of students achieved the maximum 6 marks for this question.

Question 4

The logistic differential equation also proved a harder topic, with an average of 60%. The standard for this question was well below that of previous cohorts.

Question 5

Many students could do some early parts of this question. The attrition rate was quite high as individuals progressed through the question. A mere 68 students achieved maximum marks and many students failed to make the connection in parts (a) and (b) that if $(x^2+1)=(x-i)(x+i)$ in part (a) then it was still true, but remained unused in part (b). The average mark for this question was 39%, the worst done question in the paper, with a non-achieving count of 301.

Question 6

This year, induction was not the most poorly done question in the paper (by a bare 0.88%). The introductory work was fairly well done here but an unfortunately common mistake was $\sin 4\theta = \sin(2\theta + 2\theta) = \sin 2\theta + \sin 2\theta$. Errors of this nature caused considerable problems throughout the question. The induction was poorly done, with many students listing the result for consecutive cases but neglecting to make the inductive link between them by proving that truth for one case implied the truth of the next case.

Question 7

Many students demonstrated a sound facility with the procedures for approximate solutions to differential equations. The slope field solution curves were well drawn and many students managed to implement the Euler's method iterations effectively on their calculators. A small minority forgot the requirement for radians when doing calculus with trig functions. Although operating well with a numerical/algebraic approach, many lacked the insight of a geometrical nature when it came to understanding whether their approximate result was an over- or under-estimate. At 55% the average returned to acceptable levels, on a par with the overall average for the paper. Despite this, 135 students did not gain any marks for this question.

Question 8

Vector proof was well done by many, but significant numbers had trouble with circular arguments. Some good work was done with $|\mathbf{a}| = |\mathbf{b}|$ and with a 60ºangle, $\mathbf{a} \bullet \mathbf{b} = \frac{1}{2}|\mathbf{a}||\mathbf{b}|$. The average for this question was 51%, 241 students achieve maximum marks and 179 students received no marks.

Question 9

Like question 6, question 9 dealt with a curriculum thread, the triangle inequality. Unlike question 6, which was the least attempted in the paper, question 9 was attempted by the majority, with many students making good use of the introductory marks for the question. The average mark was 48% reflecting the difficult choices made later in the question when higher order thinking was needed to draw the logical conclusion to the question.

Question 10

With an average of 63% there were many good responses and although only 33 achieved maximum marks, the bulk of the responses were in the 65-80% range, with most showing a nice appreciation of the geometry associated with iteration in the complex plane. Marks were lost, however, on the circle work, with many claiming a centre of -1 rather than $(-1, 0)$ and $r = \frac{1}{4}$ was missed by many who incorrectly wanted $r = \sqrt{\frac{1}{4}} = \frac{1}{2}$. Many students correctly carried out a few early iterations to reveal exact 2-cycle behaviour, but many did not appreciate the role of technology when later determining the long term behaviour for $z_0 = 0$.

Section B

Question 11

With 61% of students scoring from 50-88% this question was well done as students exhibited sound knowledge of vectors and the triple scalar product. Not all showed working, however, and this presented problems when answers were incorrect. Most students successfully negotiated the early parts of the question but problems came in part (c) with significant numbers not able to find point H. Successful attempts were made by those who made use of bisecting diagonals, while others successfully used the longer procedure of finding and solving equations of the relevant straight lines. Part (d) was well done with the main problem being use of the correct form of the line, $10x - 6y + z - 16 = 0$. Part (e) was attempted successfully by only those students who were most successful overall. A variety of solution methods was used but some did not heed the direction to give final answers correct to 3 significant figures. The most frequent method of solution was to use the normal to the plane through P and find its intersection with the plane. The average for the question was 58%, there were 32 candidates who scored maximum marks and 58 students received no marks.

Question12

The best done question in Section B, question 12 required sound techniques and careful interpretation. The average mark was 61% , maximum marks were achieved by 15 students, 61% achieved from 57-93%. Marks were lost for technical errors: e.g. using $\ln V$ rather than the correct $\ln|V|$, not showing $c = V_0$, and not using appropriate algebra when showing or proving a given result. Marks were also lost for misinterpretation: in part (a) $\frac{dh}{dt} = 0.2$ was well recognised but many had problems with $\frac{dr}{dt}$, wanting to halve the annual growth ring of 0.032, not recognising that it occurred all around the circumference of the tree, in part (e) many wrote at length but included no mathematics nor referred to their previous results to describe the situation. Many did not realise that a growth rate=0.358-0.901=0.268 explained that an infected tree grew less each year than a healthy tree.

Question 13

Question 13 proved the one topic where the 2010 cohort performed more successfully than cohorts from previous years. The marks distribution was mostly uniform, but that uniformity in the upper marks range made this cohort more consistent than those from previous years. Many students handled the mixture of surds and complex numbers competently and moved smoothly between Cartesian, polar and geometric representations of complex numbers. Part (e) proved difficult for those who had trouble with the diagram. Those with good diagrams who recognised the relevance of Pythagoras' theorem went on to find correct values for $|z_1 - z_2|$ and $|z_1 - z_3|$.

Question 14

Like question 13, the marks earned for question 14 were distributed fairly uniformly and the maximum mark, achieved by 169 students, was very nearly the modal mark. Two features combined in this question were use of the graphics calculator and the requirement for results to be stated exactly, rather than approximately (as given by the calculator). The uniform marks' distribution shows that this was achieved at a variety of levels as students grappled with signed areas, technology and exact manipulations for exact answers. The average for this question was 59%.

Section C

Section C was not attempted by 105 students. Of those who did attempt it, 36% chose question 15 and 64% chose question 16. The combined average mark for section C was 48% and 15 students scored maximum marks. There were 129 students who accessed no marks from this section.

Question 15

The slope field was well done and those students who could use the product rule went on to complete part (b) satisfactorily. Part (c) proved problematic for those who were unable to link the periodic functions in their solutions to 'periods of growth' in the latter parts of the question. Partial credit was earned by those candidates who attempted some form of interpretation of their results. The average mark was 45% and 4 students achieved maximum marks.

Question 16

It was pleasing to see a majority of students choose this question, tackle the new concept of curvature and implement its use on the graphics calculator. The average mark was 50% (making it better performed than three section A questions). Maximum marks were obtained by 11 students, 9 students did not achieve any marks despite an attempt; and 63% of students achieved between 33-67%.

Chief Assessor
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