Mathematical Studies

2011 Assessment Report

Government of South Australia

SACE Board of SA

MATHEMATICAL STUDIES

2011 ASSESSMENT REPORT

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

In skills and applications tasks, students need to have the opportunity to provide evidence of their learning at all levels of the performance standards, in each of the topics. Most sets of tasks included a balance of questions from routine to those requiring a high level of mathematical knowledge, skills, and understanding in a range of theoretical and applied contexts. Students were given opportunities to demonstrate their mathematical modelling and problem-solving skills by the inclusion of conjecture and proof questions. The communication of mathematical information was shown through well-developed solutions and the use of appropriate notation, representations, and terminology.

In some classes students had insufficient opportunities to perform at the higher achievement levels. Reasons commonly related to tasks that were routine and to the absence of questions assessing one or more specific features from the mathematical modelling and problem-solving criterion. Appropriate examples of questions that enable students to provide evidence of their learning are in past external examinations and support materials on the website (www.sace.sa.edu.au). It was also of concern that some classes were set tasks containing predominantly complex questions, which limited the opportunities for all but the best students to provide evidence of their learning.

It is important that teachers develop a process to evaluate the complexity of their tasks and to ensure that each of the three assessment design criteria is assessed across the set of tasks. Many teachers used percentages to evaluate student performance in individual tasks but this can be deceptive when a final grade is assigned. When the final grade is awarded, the decision about the quality of the student's work should be made by referring to the performance standards. The grade should reflect the student's performance across the topics, with some allowance for a single poor result during the year. An A+ student should not be expected to achieve perfection in all tasks but an overall judgement needs to be made about the quality of the student's responses, with the necessary consideration of the relative complexity of the tasks; no weakness should be demonstrated in any of the topics.

A small number of schools did not show the marking of the assessment tasks; this made it difficult to ascertain the correctness of the mathematics. It is helpful if teachers include information demonstrating the process that has been used to determine the grade awarded for the assessment type so that the moderators can confirm the assessment decision. It is also helpful when teachers include a simple breakdown of the tasks, showing the complexity of each question, particularly when mathematical modelling and problem-solving are assessed.

Assessment Type 2: Folio

Most schools included two investigations in their folio; frequently one of these focused on conjecture with proof and the other focused on mathematical modelling. These tasks were assessed against the performance standards and the final grade reflected the quality of the student's evidence of learning in the two tasks.

Many tasks were too directed, limiting the opportunities for the students to demonstrate their performance at the highest level. Many successful tasks included an 'in-class' component, which required students to explore an application of mathematics, and an investigation component, which enabled students to extend this application and gave them opportunities to achieve at the highest levels. One investigation should include collaborative work but each student is required to prepare and submit an individual final report. It is worth noting that tasks with only one solution give students limited opportunities to demonstrate their ability to work independently, and that many algebraic proofs are readily available on the Internet.

Students should present their folio tasks in a report format, with an introduction, an analysis section, and a conclusion. The introduction should explain the purpose and context of the investigation, outlining the problem to be explored and the method to be used in the investigation. The analysis section should consist of any data collected, a description of the methods used in collecting the data, and an analysis of the data with a clear explanation of the mathematical processes used, including discussion of any limitations to the investigation. The conclusion should be in the context of the original problem and provide an evaluation of the result of the investigation. Appendices and a bibliography should be included, as appropriate. Teachers and students should refer to pages 46 and 47 of the subject outline and to the description of the specific features at each grade in the performance standards.

Investigations in the form of an extended assignment limited the opportunity for students to demonstrate their mathematical modelling and problem-solving skills and their ability to demonstrate their communication skills through the report format. Students could only rarely be awarded more than a C grade for their performance in these tasks.

This year was considered to be a transition year and hence some allowances were made when identifying evidence against the criteria for the two folio tasks. For example, when the first folio task did not require students to present their solution in report format, greater emphasis was given to the evidence provided in the second folio task. In 2012 moderators will base their decision on the evidence provided for each of the tasks in the assessment type.

OPERATIONAL ADVICE

It was a positive aspect of the quality assurance of school assessment that most schools followed the procedures for packaging and presenting materials. It is important that moderators have access to a set of solutions for the skills and applications tasks so that the complexity of the tasks can be evaluated effectively. When student materials were placed in folders or plastic sleeves, the work of moderators was made substantially more demanding and this practice is discouraged.

Many learning and assessment plans were modified during the year as the course progressed. It is expected that in the second year these learning and assessment plans will be further refined and that an accurate addendum will be included with the submission of samples. It is important that supporting evidence accompanies any variations in materials for the sample to explain how the grade was determined for the assessment type.

When classes are combined to form one assessment group it is strongly advised that at least some common assessment tasks are used, and that the teachers collaborate to ensure that students' work is assessed with reference to the performance standards. Moderation of the assessment group is based on a sample drawn from all the classes that form the assessment group. If an adjustment is made to particular grades for an assessment type it applies to all students within those grades.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination

This year's examination drew on a range of questioning strategies to elicit evidence of student learning. The learning being assessed incorporated specific features from each of the three assessment design criteria: mathematical knowledge and skills and their application; mathematical modelling and problem-solving; and communication of mathematical information.

The following features of the 2011 examination, and of students' responses, are worth noting:

- An assessment of the link between linear equations and matrices: the student body indicated a degree of unfamiliarity with this link.
- The assessment of the first-principles process showed a dichotomy between students who were confident with this process and those who were unfamiliar with it.
- A question in which the curve-sketching capability of electronic technology nears its limit and further mathematical strategies need to be applied: very few students had both the skills and the confidence to use their own knowledge to augment the information provided by technology.
- A hypothesis test given in a less structured format: students handled this well.
- An assessment of the correspondence between definite integrals and areas: students continue to have difficulties with the subtleties of this relationship.
- Aspects of the examination that were presented in less structured ways offered students opportunities to solve problems using the mathematics with which they were familiar. Students handled these opportunities with a good degree of success.

Attention to the precise wording of a number of questions in the examination was of great benefit to some students. A lack of such attention limited the achievement of others; this included choosing an unhelpful method of answering a question when the content presented previously in the question, as well as the marks allocation and the space provided, suggested a better method (see, for example, the notes on Question 2). Similarly, the use of the marks allocation to indicate the need for additional information (see Question 3) was sometimes overlooked. Many students handled 'show' questions poorly, suggesting that more practice during the year would have been beneficial (see, for example, Question 12). Marks were lost for not observing words such as ‘Hence’ (Question 10) or ‘accurately marking’ (Question 9). Attention to these issues in the lead-up to examinations may help future students to provide evidence of their learning.

Question 1

This question gave students a chance to demonstrate a range of integration and differentiation skills, which many did with much success, half of all students earning 7 or 8 marks out of 8. A common minor error was the omission of the modulus signs in part (a); less common was the omission of the constant of integration in parts (a) and (c). The integration called for in part (c) was done well by the many students who used the ‘u-substitution’ algorithm. Those who did not use this algorithm were, once again this year, very unsuccessful in handling this aspect of the examination.

Question 2

This question involved some simple operations illustrating the association between systems of linear equations and matrices. The necessary skills were handled well in parts (a) and (b), but a few students struggled to work successfully with notation, representations, and terminology. In part (a) this included writing the determinant of the 3×3 matrix as the linear sum of three 2×2 matrices, and then performing matrix addition. In part (b) this included using an augmented form rather than a matrix equation. These weaknesses were relatively rare, and nearly three-quarters of students earned 4 or more marks out of 6. Of these, less than 40% earned 5 or 6 marks out of 6. Students commonly did not see the link between the determinant from part (a) and the unique solution of the system of equations in part (b). This important connection between linear equations and matrices clearly needs greater emphasis in some cases. Many students used row operations in part (c), despite the question presentation and the marks allocation, both of which suggested that a simpler method would be more appropriate.

Question 3

This question gave students a chance to demonstrate their knowledge of calculus techniques that can be applied to determine average and instantaneous velocity, and to distinguish between the two. The calculus techniques were generally performed well, except by students who used the derivative function to find the gradient of the chord in part (a). The performance of these techniques helped two-thirds of all students to achieve at least 5 marks out of 7. The discriminating aspect of this question was the need for students to interpret mathematical results in the context of the question. Given that the required interpretations related to the slope of a chord in part (b) and a derivative value in part (d), it was expected that students would draw the clear distinction between the average velocity for the first 2 seconds of motion and the instantaneous velocity at the moment when time equals 2. The need for additional information, rather than just ‘velocity’, was reinforced by the allocation of 2 marks to part (b).

Question 4

In this question students were asked to perform calculations and interpretations relating to a confidence interval. Part (b) was done very well; the few errors were made by students who chose a ‘by-hand’ method to calculate a confidence interval. The interpretation assessed in parts (b) and (c) was handled well by a number of students. Marks were lost by those who thought that inclusion of the threshold value of 0.75 within the confidence interval indicated that the threshold would be met.
In part (e) students needed to make the decision to round up a decimal that would normally be rounded down (i.e. 596.3 to 597 or 909.3 to 910). Given the importance

of this decision in the context of this technique, it is recommended that students show this rounding process explicitly. In spite of these issues, 85% of students earned 3 or more marks out of 8 for this question, nearly 60% earned 5 or more, and more than 20% earned full marks.

Question 5

It was expected that this relatively straightforward example of differentiation by first principles would provide students with a chance to demonstrate their grasp of this important technique. This was generally the case, with more than half the students earning full marks. However, the 24% of students who earned no marks in this question clearly reflected a major omission in the skill set being developed. Students who earned some marks erred in the substitution of ($x+h$) and associated algebra. In general, notation was used well.

Question 6

This question offered students an opportunity to demonstrate their skills in, and understanding of, matrix multiplication. More than two-thirds of students earned 7 or 8 marks out of 8. In part (b) some students erroneously solved the matrix equation by post-multiplying by the inverse matrix. In part (a), where four equations are given in terms of two unknowns, it was desirable for students to ensure that their solution satisfied all four equations. If it did not (if, for example, an error in part (i) was carried into part (ii)), students were expected to indicate and comment on this.

Question 7

This question was handled confidently by students, with more than two-thirds earning 6 or 7 marks out of 7. The work of some students was weakened because they erroneously started the implicit differentiation process in part (a) with '$dy/dx=$' or left the equation of the tangent in part (b) as $(y-3)/(x-3) = -1$.

Question 8

This question gave students a chance to demonstrate a range of skills associated with the normal distribution. This proved to be a routine task for more than 30% of students who earned 8 or 9 marks out of 9. Unfortunately a significant number were quite uncomfortable with this aspect of the curriculum, with more than half the students earning 3 or fewer marks out of 9. Some of these students might have improved their performance if they had drawn the statistical information provided in part (b). Some students did parts (i) and (ii) together, successfully earning the marks via the use of simultaneous equations. Of those who were unable to complete part (b), few saw that part (c) could be successfully completed without part (b), by using the information provided elsewhere in the question. Some assigned two sensible values to mu and sigma, and successfully used them to show the knowledge being assessed in part (c), despite not having completed part (b).

Question 9

This question presented some straightforward curve-sketching tasks, as well as some more challenging elements involving calculus and the mathematical use of technology. In part (a) many students were unable to get beyond the fact that a graph generated using electronic technology did not provide the solution required. Some of those who used other methods of electronic solution in part (a) were uncomfortable with the complex numbers provided, and erroneously recorded their real part as a solution, rather than discarding them (or recording them) entirely. For most of those who successfully answered part (a), the link with part (b) was not apparent: students did not indicate on their graph in part (b) the *x*-intercept that they had found in part (a). Imprecise graphing skills weakened the responses of some students. Despite being asked to accurately mark intercepts and stationary points, many students provided only a rough sketch. To maximise their marks, students needed to use their electronic graph analysis tools, as well as the scale provided, to give the required information. If this information is not clear on their graph, students are encouraged to mark significant points with their coordinates. Further marks were available for working well with technology in part (c). Some students attempted this part algebraically, with limited success. Of those who used technology, some lost marks because they did not correctly determine the *y*-intercept. In part (d) many students differentiated successfully and considered that derivative equal to (or not equal to) zero. Making sense of the equation that resulted was a challenge for all but the strongest students. As a result of this combination of routine and complex skills, nearly half of all students earned half marks or better but less than 3% earned 11 or 12 marks out of 12.

Question 10

This question allowed students to work with matrices both with and without technology and to form conjectures based on their results. Part (a) was done well, with only a few students calculating matrix products by hand, and even fewer erroneously squaring matrix elements. The work of some students elsewhere in this question was limited by a disregard for the requirements given. For example, parts (b) and (d) required students to answer 'On the basis of your results …'. For this reason a general statement such as *A*^*n*=*A*^(*n*–1)**A* earned no credit, as it did not draw on the information gained previously. Similarly in part (c) (ii), students were directed to 'Hence derive …' and so needed to use the given result in order to earn full credit. Overall, however, students handled this question well; 80% earned better than half marks and almost 40% earned 8 or 9 marks out of 9.

Question 11

This question presented a hypothesis test in a less structured fashion, something that, in general, students coped with well. In part (b) very few linked the presence of bias to a difference on average or in general. In part (c) the null hypothesis was erroneously accepted by fewer students than observed in previous years. Some students did not draw 'a conclusion about … bias', as asked for in part (c), and so could not earn full credit. It was disappointing that almost 40% of students were unfamiliar with the hypothesis-testing aspect of the course, earning 0 or 1 mark out of 6. The fact that less than 2% of students earned full marks for this question, largely due to a lack of insight in part (b), also reflected the limitations of students' statistical knowledge.

Question 12

This question involved working with definite integrals and areas in a fashion that was clearly unfamiliar to many students; more than a quarter of students earned no marks. Of those who engaged with the question, some made errors in part (b) by equating definite integral to area, regardless of the area's orientation in relation to the *x*-axis. Others handled the 'Show that' structure poorly. In such questions it is much more fruitful to start with a true statement drawn from the information provided, and work to obtain the necessary statement, than to start with the statement that needs to be shown. Some students seemed to have had little experience of this style of assessment question. In answers to part (c) it was common to see a parabolic-shaped graph, rather than the quartic-shaped one that was required. In spite of these errors, more than a third of students earned 6 or more marks out of 8.

Question 13

This question required students to engage with an involved statistical context and use a range of skills. Many were able to handle elements of the question, with 44% earning better than half marks, but few were fully successful; only 3% earned 10 or 11 marks out of 11. In part (b) (i) some students seemed unfamiliar with the request to 'State the distribution' and did not identify the distribution as normal. In part (b) (ii) many students seemed unsure of how to show their calculation, given that a value was provided, and surprisingly few gave an answer that contained more information than was given in the question (i.e. the value to more than two decimal places). In part (c) a number of students confused an average weight of 50 grams with an individual weight of 50 grams and suggested that a combined weight of less than 300 grams meant that all individual sausages weighed less than 50 grams. In part (e) many students failed to provide any evidence of how they arrived at an answer of fifty additional sausages, and many did not add that to 5400 to get the total number of sausages needed. These errors lowered their overall level of achievement.

Question 14

This question presented an opportunity for students to demonstrate a high level of knowledge of and skill in integration and its relationship with area, as well as to work algebraically with exponential form. Given the challenging nature of what was asked, the most able students distinguished themselves, with only 28% earning half marks or better and only 12% earning 6 or 7 marks out of 7. In part (a) students made errors when working with areas and integrals; those who worked algebraically and split the integral into two proved more successful than those who reasoned in words or used other approaches. In part (b) many students integrated successfully but then encountered difficulties in solving the resultant equations, often using the laws of logarithms incorrectly.

Question 15

This question gave students a chance to demonstrate a range of skills involved in working with linear equations. Many handled the question successfully, with 54% earning better than half marks. A considerable degree of success was shown by the 20% of students who earned 12 or 13 marks out of 13. Common errors included failing to use an 'average' equation as the basis for their solution in parts (a) and (d), procedural errors in completing row operations in part (e), and using y=55 rather than x=55 in part (f). This last error suggested that students would benefit from seeing a wider range of notation use before the examination.

Question 16

Given that aspects of this question required students to work accurately with a function featuring parameters, it was handled well; most students completed the derivative in part (a) successfully. In part (b) it was pleasing that relatively few students chose to substitute the given value of x as a means of checking, as opposed to showing the result as asked. In part (c) a common error was to set x=15 or x=1200. In part (e) many students were able to solve the equation $C(x)$=0.4 but then did not choose the largest solution as required. Students who avoided these errors scored well, with a third of them earning 9 or 10 marks out of 10.

Question 17

This question provided some straightforward work with functions and graphs as well as a challenging opportunity to demonstrate skills relating to conjectures and proofs.

Given the relative simplicity of parts (a) and (b), it was disappointing that 20% of students earned no marks in this question. As there was no evidence that the examination was too long, the most likely explanation is that students perceived this question to be harder than it was. For those who completed them, parts (a) to (d) proved to be a source of good marks. Student work in part (e) was at times very good, showing some confidence in applying a familiar algorithm in terms of the parameter k. Those who erred often did so by failing to substitute $x=1$ appropriately or by assigning numerical values for k — a fruitless strategy. There was evidence that some students overlooked part (e) of this question. This is a reminder that careful reading is required at all times and that students should use all their examination time to check their paper for missed work. Given the simplicity of the work before part (e), and the frequency of demanding 'conjecture and proof' questions in the later stages of past examinations, it is surprising that even a cursory check of the paper did not uncover an oversight.

Mathematical Studies
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