Mathematical Studies

2010 ASSESSMENT REPORT

Mathematics Learning Area

Government of South Australia

SACE Board of SA

MATHEMATICAL STUDIES

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ASSESSMENT COMPONENT 3: EXAMINATION

GENERAL COMMENTS

The Mathematical Studies external examination gave students an opportunity to demonstrate their knowledge, skills, and understanding in relation to all aspects of the Mathematical Studies course.

There were numerous opportunities to demonstrate routine, as well as complex, mathematical skills. These were met with varying degrees of success, including some excellent responses to the more challenging elements that allowed differentiation between students right across the grade bands. Overall, based on student responses, this examination was slightly easier than some set in previous years.

In order to fully assess the content of Mathematical Studies as described in the curriculum statement, the examination contained questions that focused on the students' understanding of concepts. These proved to be the elements of the examination that students found most difficult. While such elements can be naturally quite challenging, student responses overall suggested that many had limited experience with this style of assessment, especially when working with statistics.

Students who wrote clear, logical, and sequential answers were the most successful. This could be seen when students presented a structured response that contained an error, because they were rewarded with marks in ways that students relying on correct answers alone could not be. The advantage of good communication could also be seen in students' worded responses. Students who could make a clear, concise response with the necessary detail were rewarded accordingly; the onus is on the student to communicate their knowledge clearly.

It was positive this year to see evidence of students working with more confidence with algebraic structures containing parameters. This has traditionally been an area of weakness across the cohort, but this was less evident in 2010.

It was disappointing to see students lose marks because they disregarded or overlooked instructions indicating the types of responses that a question required. In particular, this included instances where it was made clear that previous results must be used to obtain a desired result. In such situations, no marks can be awarded for alternative approaches.

Question 1
This question provided an opportunity for students to demonstrate the ability to use calculus algorithms. In part (a), the product rule was used well, although some students omitted the negative sign when differentiating $-2\ln x$. In part (b), a fairly common error was the omission of the modulus when integrating $\frac{1}{x}$. Otherwise, most students were highly successful in parts (a) and (b). Part (c) provided a point of differentiation between students who were comfortable with this aspect of the course

and those who were not. The half that was comfortable with this integral generally used a substitution algorithm. The other half struggled to make any progress in the absence of such an approach.

Question 2
Students clearly found this to be the easiest question of the examination, with over 60% earning full marks. Most students handled the finding of the determinant in part (a) well, computational errors notwithstanding. In part (b), many students misread the question and focused on the values of k *k* for which the inverse does *not* exist. Those who communicated this reading error were not penalised, but those who solved the determinant equal to zero (rather than not equal to zero) without any comment were not awarded the final mark.

Question 3
This question was generally approached well, with 60% achieving 7 or more out of 9, and 40% achieving full marks. Students who were comfortable with later parts made errors in part (a) with their rectangles, including inaccurate drawing, determination of width, and use of function values. In part (b)(i), marks were lost by errors in integration, and in part (b)(ii), by the use of electronic technology, because the use of a previous answer was mandated.

Question 4
Questions such as this one, that assess a student's understanding of concepts and the way they interact, rather than their computational prowess, continue to make up the most challenging components of the examination. A significant number of students are uncomfortable with this form of question, with 40% achieving 2 marks or less out of 8, and only 37% achieving more than half marks. Common errors affecting otherwise correct responses in parts (a) and (b) included the handling of asymptotes and difficulties describing a region precisely using notation. In part (c), a number of otherwise correct sketches had the curve touching the *x*-axis at the point where $x = 2$.

Question 5
The degree of success that students had with this question varied greatly. For many it was a very accessible question that rewarded the ability to work accurately with the binomial distribution; 43% of students earned 8 or 9 out of 9 marks. For others, worded questions examining statistical content continue to present major challenges, regardless of content; 33% earned 2 marks or less. The most common single error was the misinterpretation that 'at most three' corresponds to $x \leq 2$.

Question 6
For most students the computations required in this question posed few problems, including the 'by hand' matrix multiplication. This helped 75% of students achieve 4 or more out of 6 marks. Marks were mainly lost because students struggled to describe 'the specific features of the matrices' that prevented parts (b) and (c) from being evaluated. In part (b), many identified that the matrices to be added were 'different shapes', but could not be more specific in part (c) by identifying the specific 'order mismatch' that was preventing multiplication.

Question 7
This question was quite accessible, with more than 80% of students achieving over half marks, and 46% of students earning 9 or more out of 11. Despite this, parts (b) and (e) provided opportunities for students to demonstrate their accurate computation and reasoning. In part (b), some students ignored the numerous 'clues'

and adopted an algebraic rather than a technology approach. The majority, however, used technology to draw a graph of the derivative and then located a turning point as an efficient way of finding the *x*-coordinate of the inflection point. Unfortunately, most of these students gave the *y*-coordinate from the turning point of the graph of the derivative as the *y*-coordinate of the inflection point. This was the main reason why only 10% of students earned full marks for this question.

Question 8
Like Question 5, student responses to this question varied greatly; 21% earned full marks, while 13% earned no marks, despite the simplicity of some of the computations. Student responses included a range of computational errors in parts (a) and (b). In part (c), the common error was to focus on the fact that the confidence interval did not contain the 140 mmHg value, rather than the fact that the confidence interval for μ was below 140 mmHg.

Question 9
As with Question 4, the relationship between concepts, in this case concepts of area and definite integral, caused many students significant difficulty. Nearly 30% of students earned 0 or 1 mark, and only 25% were able to earn 5 or 6 out of 6. In general, students seemed uncomfortable with using areas to determine definite integrals and, to a lesser extent, with using supplied definite integrals to determine areas. Negative signs were either used indiscriminately or used in a way that showed the confusion many students have with this aspect of the course.

Question 10
This question, like Question 9, dealt with definite integrals and areas. It took a more computational approach to these concepts and so, despite the proliferation of pronumerals, it was handled well by students. Over 60% of students earned 7 or more marks out of 9. A common error was to consider the white area above the parabola to be the shaded area. When followed through, this error was not penalised as students were able to complete the question and obtain equivalent results.

Question 11
In general, students were quite successful in working with the central limit theorem, with over half of them earning 7 or more marks out of 10. Common errors included: in part (b), not using the given information to sketch a sufficiently narrow distribution of sample means, and in part (c), confusing the spend of an individual at lunchtime with the average spend for a lunchtime.

Question 12
Students responded well to this question, with 70% earning 5 or more out of 8, and more than 40% achieving full marks. It was pleasing to see such successful work with pronumerals. Students who could not do so in part (b) often ignored the right-hand side of the equations or assigned values to *a*, *b,* and *c*. This earned little or no credit.

Question 13
Many students were able to provide evidence of their ability to work with this mathematical model, with 43% earning 7 or more marks out of 9. Some were unable to access the question successfully, perhaps due to reading issues, with nearly 30% earning 3 marks or less. From their work, it could be seen that some students are less confident with a rate function, with some focusing on the difference in function values in part (a) rather than using an integral. In part (c), many students showed good examination technique, getting back into the question after having had limited

success with parts (a) and (b). Given the 'show' structure of part (c)(i), it was important for students to present the working that leads to the given result; for example, showing as part of their differentiation '$\ldots -1776 \times -0.0625 \ldots$' or equivalent. Responses to part (c)(ii) suggested that many students are unfamiliar with the defining properties of the logistic function.

Question 14
Students were less comfortable with a Z-test that focused on 'preference' rather than 'probability'. Despite this, many were successful in completing the more procedural elements of the question, but 53% earned only 3, 4, or 5 marks out of 8.

The idea that a P-value represents 'the probability of achieving what was observed or a more extreme result, calculated assuming the null hypothesis to be true' is presented in the Mathematical Studies Curriculum Statement and is central to an understanding of hypothesis tests. Despite this, part (c)(i) was the most challenging part of this examination, with few students (less than one in 200) providing anything close to this response. It is recommended that teachers provide their students with opportunities to explain their understanding of statistical concepts as a complement to presenting their computations.

Question 15
Most students were able to work successfully with a model based on transition matrices, with 58% achieving 10 or more marks out of 16. Marks were lost by some students for providing non-integer numbers of vans, providing numbers of vans that did not sum to 60, or providing an answer rather than a calculation in part (a). Aspects of the question that asked students to interpret mathematical results gave an opportunity for students to be rewarded for their higher levels of understanding, with only 9% of students earning 15 or 16 marks. Some students did not include 'Day 1' in part (b)(ii) and, more commonly, students were not able to confidently discuss the concept of a steady state in part (c) and, even more so, in part (e)(iii).

Question 16
As in previous years, the last question of the examination contained a number of opportunities for all students to achieve a degree of success, but also opportunities for students to demonstrate higher-order problem-solving skills. As a result, 48% of students earned more than half marks, but only 5% earned more than 12 out of 15. In part (a)(i), some students erred with implicit differentiation, in particular in dealing with the '1'. Some students chose to isolate y before differentiating, but most did not consider the '$\pm$' that was needed. In part (a)(ii), it was common to see students overlook the need for a negative *y*-value. Given the familiarity of the function, it was surprising to see the number of poor graphs that were drawn in part (b). Clearly part (d) was a significant challenge for many students. It was positive to see students equate tangent slopes and/or *y*-intercepts, but relatively few were able to work successfully with these two equations. Many struggled to work purposefully with the algebra involved, and often did not see the advantage of using technology in this context.

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