Government of South Australia

SACE Board of SA

External Examination 2013

2013 MATHEMATICAL APPLICATIONS, Semester 1

FOR OFFICE USE ONLY

SUPERVISOR CHECK

RE-MARKED

SACE REGISTRATION NUMBER

SEQ | FIGURES | CHECK LETTER | BIN

MATHEMATICAL APPLICATIONS, Semester 1

Graphics calculator ☐

Brand _____

Model _____

Computer software ☐

Thursday 6 June: 9 a.m.

Time: 1½ hours in total (to complete two question booklets, one on each topic studied in Semester 1)

Pages: 10
Questions: 4

TOPIC 4

Topic 4: Matrices

Examination material: two question booklets
one SACE registration number label

Approved dictionaries, notes, calculators, and computer software may be used.

Instructions to Students

1. You will have 10 minutes to read the question booklets. You must not write in your question booklets or use a calculator during this reading time but you may make notes on the scribbling paper provided.
2. Each of the following five topics is printed in a separate question booklet. ***Tick the boxes by the two topics you have studied in Semester 1***:
   - Topic 2: Investment and Loans ☐
   - Topic 4: Matrices ☐
   - Topic 5: Optimisation ☐
   - Topic 6: Share Investments ☐
   - Topic 7: Statistics and Working with Data. ☐
3. The total mark for each topic is 35.
4. Answer ***all*** parts of Questions 1 to 4 in the spaces provided in this question booklet. There is no need to fill all the space provided.
5. Show all working in this booklet. (You are strongly advised ***not*** to use scribbling paper. Work that you consider incorrect should be crossed out with a single line.)
6. Write on page 10 if you need more space. Make sure to label each answer carefully.
7. Use only black or blue pens for all work other than graphs and diagrams, for which you may use a sharp dark pencil.
8. Appropriate steps of logic and correct answers are required.
9. Marks may be deducted if you do not clearly show all steps in the solution of problems, if your answers have an inappropriate number of decimal places, or if you use incorrect units.
10. Diagrams, where given, are not necessarily drawn to scale.
11. Complete the box on the top right-hand side of this page with information about the electronic technology you are using in this examination.
12. Attach your SACE registration number label to the box at the top of this page on one of your question booklets. Copy the information from your SACE registration number label into the box on the front cover of your other question booklet.
13. At the end of the examination, place one question booklet inside the back cover of the other question booklet.

www.theallpapers.com

QUESTION 1

Nicola is a local butcher who is interested in finding out the trends in the weekly buying habits of her customers. She sells three main types of meat: beef (B), lamb (L), and chicken (C). Nicola compiled the following information, displayed in transition matrix T:

$$T = \begin{array}{c} \\ \mathrm{B} \\ \mathrm{L} \\ \mathrm{C} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{B} & \mathrm{L} & \mathrm{C} \end{array} \\ \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.25 & 0.50 & 0.25 \\ 0.23 & 0.12 & 0.65 \end{bmatrix} \end{array}$$

(a) Interpret the meaning of 0.05 in transition matrix T above.

(1 mark)

(b) By calculating T^2, find and interpret the meaning of $T^2_{3,1}$. (There is no need to write down the full matrix T^2.)

(2 marks)

(c) When Nicola opened her shop 60% of customers initially bought beef and 15% bought lamb.

Write down the initial state matrix, S.

(1 mark)

www.theallpapers.com

(d) If Nicola served 200 customers each week, determine how many people would be buying beef 4 weeks after she opened the shop.

(3 marks)

(e) Find the steady state and interpret it in the context of this question.

(2 marks)

(f) State *one* underlying assumption that you have made in using the transition matrix model to find your answer to part (e) above.

(1 mark)

(g) After an advertising campaign the percentage of customers who bought lamb again the next week increased to 70%. The resulting decrease in sales of other types of meat was shared equally by beef and chicken.

Complete the new transition matrix, T_{new}, to show this information.

$$T_{new} = \begin{array}{c} \\ B \\ L \\ C \end{array} \begin{array}{c} \begin{array}{ccc} B & L & C \end{array} \\ \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ ____ & ____ & ____ \\ 0.23 & 0.12 & 0.65 \end{bmatrix} \end{array}$$

(1 mark)

www.theallpapers.com

QUESTION 2

Frances and Christine are shopping for a number of items of clothing and plan to check prices at two different stores. Frances needs four shirts, three pairs of jeans, and a hoody. Christine needs six shirts, a pair of jeans, and three hoodies.

(a) Write the clothing requirements of the two girls as a 2×3 matrix, R.

(2 marks)

(b) At Shop X, shirts cost \$35, jeans cost \$62, and hoodies cost \$51.

At Shop Y, shirts cost \$28, jeans cost \$75, and hoodies cost \$49.

Write the costs of the items of clothing as a 3×2 matrix, C.

(1 mark)

(c) Calculate matrix RC.

(2 marks)

www.theallpapers.com

(d) Interpret the values in the first column of matrix RC.

(2 marks)

(e) Using a matrix calculation, or otherwise, explain which store has cheaper prices overall for the clothing that Frances and Christine need.

(2 marks)

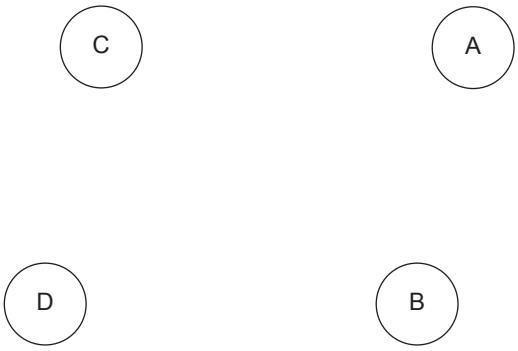
www.theallpapers.com

QUESTION 3

Four rural towns (A, B, C, and D) each have their own water storage for use in emergency bushfire situations. Water can also be pumped between the towns if necessary, as shown in the connectivity matrix, M, below:

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a) Using the nodes below, show these connections as a network diagram.



(2 marks)

(b) (i) (1) Find $P = M + M^2$.

(2 marks)

(2) Why are the main diagonal elements ($P_{1,1}$, $P_{2,2}$, $P_{3,3}$, and $P_{4,4}$) not useful in this context?

(1 mark)

www.theallpapers.com

(ii) Using your result from part (b)(i)(1), or otherwise, explain which town or towns would be most at risk in a bushfire situation.

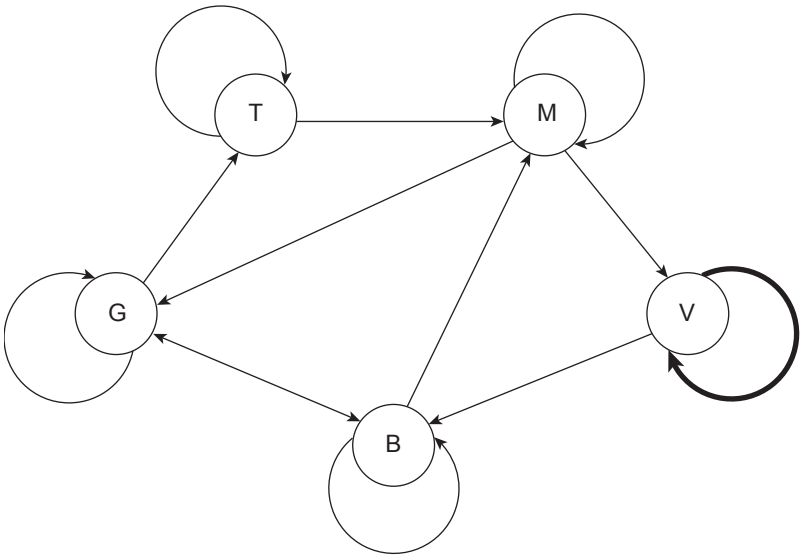
(2 marks)

www.theallpapers.com

QUESTION 4

Gianni is about to set off on a holiday to northern Italy. His travel agent suggests that Gianni should consider visiting five major cities in this region: Turin, Milan, Verona, Bologna, and Genoa. The regional tourist association offers a package that allows tourists to travel through the region without having to decide their itinerary in advance.

Tourists buy a bus pass that is valid for up to 2 weeks on the buses that travel around and between the five cities every day. The network diagram below shows the routes these buses take, with each link on the network representing 1 day's travel. Each day tourists can get on a bus and continue exploring a city or travel to another city.



(a) State the meaning of the link shown in bold on the network diagram above.

(1 mark)

(b) Complete the following connectivity matrix, C, for this network.

$$C = \begin{array}{c} \\ T \\ M \\ V \\ B \\ G \end{array} \begin{array}{c} \begin{array}{ccccc} T & M & V & B & G \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & __ & 1 & __ & __ \\ 0 & __ & __ & 1 & __ \\ 1 & __ & __ & __ & 1 \end{bmatrix} \end{array}$$

(2 marks)

www.theallpapers.com

(c) Matrix C^3 has been calculated and the result is shown below:

$$C^3 = \begin{array}{c} \\ T \\ M \\ V \\ B \\ G \end{array} \begin{array}{c} \begin{array}{ccccc} T & M & V & B & G \end{array} \\ \begin{bmatrix} 2 & 3 & 3 & 2 & 3 \\ 3 & 4 & 3 & 6 & 5 \\ 1 & 3 & 2 & 4 & 4 \\ 4 & 5 & 3 & 6 & 7 \\ 4 & 6 & 2 & 4 & 6 \end{bmatrix} \end{array}$$

Explain the meaning of the value 2 in the fifth row of matrix C^3. Support your answer with one path from the network diagram.

(2 marks)

(d) Gianni decides to fly to Bologna, spend 7 days using a bus pass to travel around the region, and then fly out from Genoa.

(i) Using a matrix calculation, find out how many ways Gianni has of travelling during the 7 days. (There is no need to write down the full matrix answer but you must indicate the calculation used.)

(2 marks)

(ii) State *one* limitation of using matrices to solve problems such as the one in part (d)(i).

(1 mark)

www.theallpapers.com

You may write on this page if you need more space to finish your answers to Topic 4. Make sure to label each answer carefully (e.g. 'Question 1(d) continued').

© SACE Board of South Australia 2013

www.theallpapers.com

www.theallpapers.com

www.theallpapers.com

SACE BOARD OF SA