Government of South Australia

SACE Board of SA

External Examination 2012

2012 MATHEMATICAL APPLICATIONS, Semester 1

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ATTACH SACE REGISTRATION NUMBER LABEL TO THIS BOX

Graphics calculator ☐

Brand _____

Model _____

Computer software ☐

Thursday 7 June: 9 a.m.

Time: 1½ hours

Pages: 11
Questions: 3

TOPIC 4

Topic 4: Matrices

Examination material: two question booklets
two SACE registration number labels

Approved dictionaries, notes, calculators, and computer software may be used.

Instructions to Students

1. You will have 10 minutes to read the question booklets. You must not write in your question booklets or use a calculator during this reading time but you may make notes on the scribbling paper provided.

2. Each of the following five topics is printed in a separate question booklet. ***Tick the boxes by the two topics you have studied in Semester 1***:

   Topic 2: Investment and Loans ☐
   Topic 4: Matrices ☐
   Topic 5: Optimisation ☐
   Topic 6: Share Investments ☐
   Topic 7: Statistics and Working with Data. ☐

3. The total mark for each topic is 35.

4. Answer ***all*** parts of Questions 1 to 3 in the spaces provided in this question booklet. There is no need to fill all the space provided.

5. Show all working in this booklet. (You are strongly advised ***not*** to use scribbling paper. Work that you consider incorrect should be crossed out with a single line.)

6. Write on page 7 if you need more space. Make sure to label each answer carefully.

7. Use only black or blue pens for all work other than graphs and diagrams, for which you may use a sharp dark pencil.

8. Appropriate steps of logic and correct answers are required.

9. Marks may be deducted if you do not clearly show all steps in the solution of problems, if your answers have an inappropriate number of decimal places, or if you use incorrect units.

10. Diagrams, where given, are not necessarily drawn to scale.

11. Complete the box on the top right-hand side of this page with information about the electronic technology you are using in this examination.

12. Attach one of your SACE registration number labels to the box at the top of this page.

13. At the end of the examination, place one question booklet inside the back cover of the other question booklet.

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QUESTION 1

A mechanic (M), a salesperson (S), and an accounts clerk (C) work in a local car yard. The hours that they work each day in 1 week are shown in matrix H below:

$$H = \begin{array}{c} \\ \text{M} \\ \text{T} \\ \text{W} \\ \text{T} \\ \text{F} \end{array} \begin{array}{c} \begin{array}{ccc} \text{M} & \text{S} & \text{C} \end{array} \\ \begin{bmatrix} 7.0 & 9.0 & 8.0 \\ 7.5 & 8.0 & 9.0 \\ 8.0 & 8.5 & 8.0 \\ 8.0 & 8.0 & 7.5 \\ 5.0 & 0.0 & 8.5 \end{bmatrix} \end{array}$$

(a) The gross (before tax) hourly wage is \$18.50 for the mechanic, \$32 for the salesperson, and \$26.50 for the accounts clerk.

(i) Construct a column matrix, G, for these gross hourly wages.

(1 mark)

(ii) Using matrix methods, calculate the total amount of gross wages paid each day.

(2 marks)

(iii) Show how matrix methods are used to calculate the total amount of gross wages paid for the week and find this value.

(2 marks)

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(b) Let A = [1 1 1 1 1].

Explain what the calculation $A \times H$ shows in the context of this question.

(1 mark)

(c) The amount of tax deducted from the gross hourly wage of each of the workers is shown in matrix T:

$$T = \begin{matrix} \text{M} \\ \text{S} \\ \text{C} \end{matrix} \begin{bmatrix} 3.43 \\ 7.36 \\ 5.84 \end{bmatrix}$$

Find the matrix N that shows the net (after tax) hourly wage for each worker.

(1 mark)

(d) Circle the matrix expression below that would calculate the total amount of tax deducted for the week.

- $T \times A \times H$
- $A \times H \times T$
- $G \times H - N \times H$

(1 mark)

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QUESTION 2

In a brood of fifty hens, each hen usually lays one egg per day. On some days the hens lay very large eggs but on the next day they often do not lay any eggs at all. There are four categories for the eggs laid: extra large (XL), large (L), standard (S), or none (N).

The results obtained from several months of recording the weights of eggs are shown in matrix T below:

$$T = \begin{array}{c} \\ \text{Size today} \end{array} \begin{array}{c} \begin{array}{cccc} \text{XL} & \text{L} & \text{S} & \text{N} \end{array} \text{ Size tomorrow} \\ \begin{bmatrix} 0.01 & 0.07 & 0.12 & 0.80 \\ 0.01 & 0.55 & 0.39 & 0.05 \\ 0.02 & 0.47 & 0.45 & 0.06 \\ 0.01 & 0.33 & 0.65 & 0.01 \end{bmatrix} \end{array} \begin{array}{l} \text{XL} \\ \text{L} \\ \text{S} \\ \text{N} \end{array}$$

(a) Explain the meaning of $T_{1,4}$.

(1 mark)

(b) (i) Part of matrix T^2 is given below.

Complete this matrix.

$$T^2 = \begin{array}{c} \begin{array}{cccc} \text{XL} & \text{L} & \text{S} & \text{N} \end{array} \\ \begin{bmatrix} 0.0112 & _____ & _____ & _____ \\ 0.0139 & _____ & _____ & _____ \\ 0.0145 & 0.4912 & 0.4272 & 0.0671 \\ 0.0165 & 0.4910 & 0.4289 & 0.0636 \end{bmatrix} \end{array} \begin{array}{l} \text{XL} \\ \text{L} \\ \text{S} \\ \text{N} \end{array}$$

(2 marks)

(ii) If a hen lays an extra large egg today, what it is the probability that she will lay an extra large egg in 2 days' time?

(1 mark)

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(iii) Today two of the fifty hens laid an extra large egg, twenty-one laid a large egg, and twenty-four laid a standard egg.

Express this as an appropriately labelled row matrix, N.

(2 marks)

(iv) Using matrix methods, calculate how many of the hens that laid a standard egg today could be expected to lay a standard egg in 2 days' time.

(2 marks)

(c) (i) Using matrix methods, calculate the percentage of eggs in each category that will be laid per day in the long term.

(2 marks)

(ii) Calculate the number of large eggs that will be laid per day in the long term.

(1 mark)

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(d) After a certain time the hens' laying pattern changes slightly. When the hens lay a large egg, the probability that they will lay a large egg the next day decreases from 55% to 45%, and the probability that they will lay a standard egg increases correspondingly. The probability that the hens will lay a large egg or a standard egg the day after laying a standard egg also changes, as shown in transition matrix T_{new} below:

$$T_{\text{new}} = \begin{array}{c} \\ \text{Size today} \end{array} \begin{array}{c} \begin{array}{cccc} \text{XL} & \text{L} & \text{S} & \text{N} \end{array} \\ \text{Size tomorrow} \end{array} \begin{array}{c} \text{XL} \\ \text{L} \\ \text{S} \\ \text{N} \end{array} \begin{bmatrix} 0.01 & 0.07 & 0.12 & 0.80 \\ 0.01 & ____ & ____ & 0.05 \\ 0.02 & 0.41 & 0.51 & 0.06 \\ 0.01 & 0.33 & 0.65 & 0.01 \end{bmatrix}$$

(i) Complete transition matrix T_{new} above.

(2 marks)

(ii) The new steady state for the categories of eggs laid is now 1.5% extra large, 41.6% large, 50.5% standard, and 6.4% none.

Comment on the reasonableness of assuming that the same percentage of eggs in each category will be laid per day in the long term.

(1 mark)

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You may write on this page if you need more space to finish your answers to Topic 4. Make sure to label each answer carefully (e.g. 'Question 2(b)(iii) continued').

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QUESTION 3

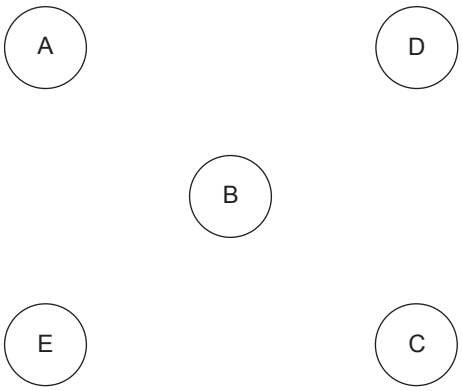
Adam (A), Belinda (B), Carla (C), Dom (D), and Emma (E) are a group of school friends. They text one another about most things but they are more selective about passing on gossip. The connectivity matrix G for this situation is shown below:

$$G = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(a) What does the third row of matrix G indicate about the way gossip is passed on?

(1 mark)

(b) Using the nodes in the diagram below, draw a directed network diagram to illustrate the gossip passing between the friends.



(2 marks)

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(c) (i) Calculate matrix G^2 and write it below.

(2 marks)

(ii) Using the network diagram in part (b), explain the meaning of the entry in the third row of the first column of matrix G^2.

(1 mark)

(d) A sociologist who is studying the group decides to use the formula $P = G + G^2$ to investigate the connectedness of the friends in the group.

(i) Calculate matrix P and write it below.

(1 mark)

(ii) Using matrix P, state which one of the friends seems to be the least connected in the group.

(1 mark)

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(e) The sociologist now considers another way of ranking the connectedness of the friends, using the formula $R = \left(G + G^2 + \frac{1}{2}G^3\right) \times K$, where K is a column matrix of ones.

(i) Comment on the implications of using the coefficient $\frac{1}{2}$ in this formula.

(1 mark)

(ii) Calculate matrix R and write it below.

(2 marks)

(iii) Using your answer to part (e)(ii), rank the friends from most to least connected in the group.

(1 mark)

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(f) The sociologist decides that the best way of ranking the connectedness of the friends in the group is to use the sum of each column in matrix G:

$$G = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

How reasonable is it to use this method?

(1 mark)

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