Quantitative Reasoning

The best way to study quantitative for the test is to make a list of rules and principles given in the review of quantitative section and explanation of the answers to the practice tests found later in this book. Keep a sheet of paper handy, as you are reviewing these tests; jot down any principles, rules, definitions, or formulas that are unfamiliar to you. Then take time to memorize those rules. It is critical that you be able to recall automatically a formula, a short –cut tip or suggestion so you do not spend valuable time on the exam working out a lengthy problem or **reinventing the wheel** each time. Use the review section for added reinforcement.

FOUR STEP STRATEGY

FIRST:

Read the entire problem. Do not start doing calculations until you have read the problem from start to finish; you may find that you do not need to do all the work.

For example, if the problem states to find the value of the expression

$$(n - 13)(n - 52)(n - 33)(n - 4), \text{ for } n = 4$$

You know automatically that the answer is 0. This is because in the fourth parentheses the value of n - 4 for n = 4 is 0 and anything multiplied by 0 is 0. You saved all the calculations involved.

SECOND:

Read the answer choices. Often you can narrow your answer down by estimating or by looking carefully at the units or values.

For example, you may be asked the number of soldiers in a platoon as a captain asks to stand all his soldiers in the form of a square. Looking at your answer choices, you see 108, 196, 224, and 87. Only the second answer choice could be correct, since it is a complete square.

THIRD:

Make absolutely certain that you know exactly what the question is calling for. Many careless errors are made when a person solves for x, not noting that the question asks for y, or for 2x. You should take at least as much time reading and thinking about the problem as you actually do on calculations.

EXAMPLE:

A car goes from city M to city K, Which are 200 miles apart, at an average speed of 50 miles per hour. How many miles from city K, Will the car be after 3 hours.

Answer Choices:

(A) 100 miles (B) 20 miles

(C) 50 miles (D) 150 miles

The car travels 150 miles in 3 hours (50 ×3). So it will be 150 miles from city M, but the question asks the distance from city K. The right choice is C.

FOURTH:

If none of the known methods works, try grabbing an answer from the four options and plugging it into the question. This will often lead you to the right answer quickly.

EXAMPLE:

During the first month after the opening of a new shopping home, sales were $72 million. Each subsequent month, sales declined by the same fraction. If the sales during the third month after the opening totaled $18 million, what is that fraction?

Answer Choices:

A·	1/4	B	1/4
C	1/4	D	2/3

The fastest way to a solution is to plug in an answer. Try choice A. If sales in the second month after opening are declined by 1/4 , it gives the sales of $18 million but this figure is for third month after the opening, so this is not the right choice. Drop it. Only the answer choice C gives the right sale figure during the third month. Therefore, the right choice is C.

BASIC ARITHMETICS

Many math questions in NAT are solved by using signed numbers, algebraic manipulation, equations and inequalities. Important rules and concepts will now be reviewed.

RECALL FROM YOUR MEMORY

Some questions involve arithmetic calculations and require algebraic simplification. Recall some basics from your memory.

7 – 5 = 2, 5 – 7 = -2

5x – 7x = -2x, 7x – 5x = 2x

3x2 – 5x2 = -2x2,

6x3 – 2x2 = 6x3 – 2x2

cannot be added as the exponents are different

- A number is positive (+) when it is greater than zero.
- A number is negative (-) when it is less than zero.
- Any number written without a preceding sign is positive.
- Zero is neither positive nor negative.
-

NUMBER SETS

INTEGERS:

All positive and negative whole numbers including zero are integers.

EXAMPLE:

-349 , -1 , 0 , 4 , 77 , 183

Odd Numbers:

Any number that cannot be divided by 2.

EXAMPLE:

3, 5, 7, 11, 33, 45 etc

- The difference between two consecutive odd numbers is 2.
- If x is an odd number then next odd number is x + 2. Next to x + 2 is x + 4.

EVEN NUMBERS:

Any number that can be divided by 2.

EXAMPLE:

2, 4, 6, 80, 96, 110 etc.

- The difference between two even numbers is 2.
- If x is an even number then next even number is x + 2. Next to x + 2 is x + 4.

PRIME NUMBERS:

A number that hass only two uniwue factors: The number is self and " 1"

EXAMPLE:

2, 3, 5, 7, 11, 13, 17, 19 etc.

2 is the only even Prime number.

1 is not a prime number, become it only has one unique factor.

PRODUCT (MULTIPLICATION)

Product of two prime numbers can never be a prime number. 3 × 7 = 21, which is not a prime number as it is divisible by 3 and 7.

Sum (Addition) of two prime numbers may or may not be a prime number. 2 + 3 = 5

2, 3, and 5 all are prime numbers.

2 + 7 = 9, 2 and 3 are prime numbers but their sum 9 is not a prime number.

The absolute value of a quantity is the quantity with only positive value. The symbol for absolute value is two enclosing vertical segments. The absolute value of -11 and +11 is written as |-11| and |+11| and is equal to 11.

ADDITION AND SUBTRACTION

If you add two quantities that have the same signs, simply add them and retain the sign.

$2 + 7 = 9$ and $-2 + (-7) = -9$

If you add two quantities of different signs, simply add them and the result will bear the sign of the quantity of greater absolute value.

$$2 - 7 = 2 + (-7) = -5$$
$$7-2 = 7 + (-2) = 5$$
$$3A + 4A = 7A$$
$$3A - 4A = -A$$
$$7a^2 + 2a^2 = 9a^2$$

3a2 – 2a can't be added since they are not like terms.

3x + 5 can't be added since they are not like terms.

If three or more quantities are to be added, add like positives, then like negatives, then combine

like terms by subtracting absolute values.

$$8 + 7 - 13 + 12 = +27 - 13 = +14$$
$$6c - 5d - 4 - 8c + 7d - 6 = -2c + 2d - 10$$

MULTIPLICATION

If two quantities having the same sign are multiplied, the answer is positive (+). If two quantities having the different signs are multiplied, the answer is negative (-). Two parentheses with no sign between them indicates multiplication. No sign between a quantity and a parenthesis also indicates multiplication. A raised dot between two quantities indicates multiplication as well.

$(+6)(+5) = 30$

$(-9)(-3) = +27$

$-7(8) = -56$

$4 \cdot - 4 = -16$

You can only add or subtract like terms. However, all terms, whether alike or different, can be multiplied. When like letters are multiplied, add exponents.

$3(-2Y) = -6Y$

$(7G)(-3K) = -21GK$

$A^3 \times A^4 = A^7$

DIVISION

Dividing two quantities having the same sign, the answer will be positive.
Dividing two quantities having different signs will give negative answer.

-18/-3 = +6, -18/+3 = -6
+18/+3 = +6, +18/-3 = -6

Dividing like letters, subtract their exponents.
If a letter such as Y bears no exponent it has exponent of 1.

$Y^4/ Y^3 = Y$, $Y/Y = 1$, $8Y^8/2Y^2 = 4Y^6$

When a quantity divides another quantity that contains two or more terms, divide each of term by the first quantity.

$(6P-10)/(-2) = -3P+5$
$(12Y+6)/6 = 2Y+1$
$(5X-2)/(-3) = [(-5x)/3] + (2/3)$
$(5b^2+10b)/5b = b+2$

FINDING SQUARE ROOT

$\sqrt{4} = 2\ 2 = 2$, $\sqrt{x^2} = x, x = x$

WHICH OPERATION FIRST?

Many problems involve multiple operations. The operations must be performed in a particular order. Occasionally test makers like to see whether you know what that order is. Here's an easy way to remember the order of operations:

Please Excuse My Dear Aunt Salma.

This stands for

P	for	Parentheses
E	for	Exponents
M	for	Multiplication
D	for	Division
A	for	Addition
S	for	Subtraction

Do operations enclosed in parentheses first; then take care of exponents; then you multiply, divide, add, and subtract in the sequence. Moving from left to right

FRACTIONS

A fraction is just another way of expressing division. The expression 12 /17 is exactly the same thing as 12 divided by 17. a/b is nothing more than "a" divided by "b".
In the fraction x/y, x is known as numerator, and y is known as the denominator.
The other important way to think of a fraction is as

Part/Whole

The fraction 7/10 can be thought as 7 parts out of a total of ten parts of an item (number value).

PROPER FRACTIONS:

If the Numerator is less than Denominator in a fraction, the fraction is called a proper fraction.

Improper Fractions:

If the Numerator is greater than Denominator in a fraction, the fraction is called an improper fraction.

MIXED FRACTIONS:

Mixed fraction is a combination of a whole number and a fraction. Like in 7*(2/5) , 7 is wholenumber and 2/5 is the fraction.

ADDING AND SUBTRACTING FRACTIONS

SAME DENOMINATOR

To add two or more fractions that have the same denominator, simply add up the numerators and put the sum over the common denominator.

EXAMPLE:

$$\frac{1}{7}+\frac{5}{7}=\frac{(1+5)}{7}=\frac{6}{7}$$

Subtraction works exactly the same way

$$\frac{6}{7}+\frac{2}{7}=\frac{(6-2)}{7}=\frac{4}{7}$$

DIFFERENT DENOMINATORS

Before you add or subtract two or more fractions with different denominators, you must give all of them the same denominator. To do this, multiply each fraction by a number that will give it a denominator in common with the others.

EXAMPLE:

If you wanted to change 2/1 into sixths, you could do the following:

$$1/2*3/3 = 3/6$$

We haven't actually changed the value of the fraction, because we multiplied it by 1. The new fraction reduces to 2/1 .

If we wanted to add

$$1/2+2/3 = (1/2*3/3)+(2/3*2/2) = \frac{3}{6}+\frac{4}{6}=\frac{7}{6}$$

MULTIPLYING FRACTIONS

To multiply fractions, just and put the product of the numerators over the product of the denominators.

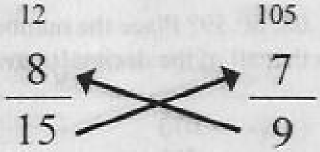
EXAMPLE:

$$2/3 * 4/5 = 8/15$$

REDUCING FRACTIONS

EXAMPLE:

Compare 3/4, 7/8, 9/11, which one is the greatest fraction
Fir st take two fractions 3/4, 7/8 multiply numerator of the first with the denominator of the other (3×8 = 24), similarly multiply numerator of the second with the denominator of the first(7 × 4 = 28).Since 28 is greater than 24 so 7/8 is greater than 3/4. Now take 7/8, 9/11to compare. Multiply numerator of the first with denominator of the second (7 × 11 = 77), similarly multiply numerator of the second to the denominator of the first (9 × 8 = 72). Since 77 is greater than 72, so 7/8 is the greatest of the three given fractions.
To compare 8/15 and 7/9, multiply 8 with 9 and 15 with 7, Since the product of 15 and 7 (105) is greater than the product of 8 and 9 (72). so 7/9 is greater than 8/15.



Tips

If the difference between numerators and denominator of one fraction is the same as the difference between the numerator and denominator of the other fraction, then there are two possible scenarios.
1). It the fractions are greater with the smaller numbers is greater. For intance, 6/5 and 9/8. since 6 is smaller then a9 and 5 is smaller them 8, therefore 6/5 is greater rhen 9/8.
2). It the fractions are smaller then 1, then the fraction with greater numerator is greater. In case of 3/4 and 7/8, the difference between 3 and 4 is the same as the difference between 7 and 8. Since 7 is greater than 3, so 7/8 is greater fraction.

DECIMALS

When decimals are added or subtracted, the decimal points must be placed one under the other.
Every integer has its decimal after unit digit (45 is the same as 45. and $45 is the same as $45.00).

EXAMPLE:

4.9 + .73 + 7. Line up the decimal points.

4 . 90
00 . 73
7 . 00
12 . 63

To add them, fill the empty spaces zeroes, add as usual and place the decimal point in the line of the decimal points of the numbers to be added.

EXAMPLE:

Which is the largest, .073, .5, .586, .08, or .59? Place the numbers under one another, lining up the decimal points. Fill in zeroes so that all of the decimals have the same number of decimal places.

.073
.500
.586
.080
.590

.590 is the largest three-place decimal. Answer: .59.

When you multiply decimal numbers, the decimals do not have to be under one another. The product (answer) must contain as many numbers after its decimal as the total of the decimal places in the two numbers being multiplied. For example, find the product of .28 and .3. 28 times 3 is 84, but where should the point be placed? .28 has two numbers after its point and .3 has one number after its point, making a total of three decimal places. Count three places to the left from the end of 84. Since 84 have two places, a zero must be placed in front of 8.

Answer: 0.084

SOLVED EXAMPLES

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

Answer to each question has been given at the end of each question.

1. Arrange in descending order: 3/8, 4/9, 2/7

EXAMPLE:

Make the comparisons of two fractions each

$$\overset{27}{\frac{3}{8}} \times \overset{32}{\frac{4}{9}} \qquad \overset{28}{\frac{4}{9}} \times \overset{18}{\frac{2}{7}} \qquad \overset{21}{\frac{3}{8}} \times \overset{16}{\frac{2}{7}}$$

4/9 is greater than 3/8, and 4/9 is greater than 2/7 , so 4/9 is the largest fraction. 3/8 is greater than 2/7, so the answer is 4/9, 3/8, 2/7

2. 40 +80/.4= what number?

EXAMPLE:

First simplify the fraction:

80/.4 =80/.4 * 10/10 = 800/4 = 200

Then add: 40 + 200 = 240.

3. If a bushel of apples weighs from 48 to 54 pounds and a bushel of melons weighs from 80 to 90 pounds, what is the smallest ratio between the weight of a bushel of apples and a bushel of melons?

EXAMPLE:

"Smallest ratio" means smallest fraction, which will contain the smallest numerator but the largest denominator (1/10 is smaller than 1/3). The answer is 48/90, or 8/15.

4. Simplify this fraction

EXAMPLE:

Add the fractions in the denominator and rewrite the expression

$$=\frac{10}{3}\div\frac{34}{3}=\frac{10}{3}\times\frac{3}{34}=\frac{10}{34}=\frac{5}{17}$$

5. If r is greater than 0 and b =1/r, does b increase or decrease as r increases?

EXAMPLE:

If the numerators are the same, the smaller fraction has the larger denominator. Therefore, if r increases and the numerator remains 1, the fractions get smaller and b decreases.

6. Reduce $12c^2 / 15c$

EXAMPLE:

Reduce 12 and 15 by canceling both by 3. Treat $c^2 / c = c^{2-1}$. The answer is $\frac{4}{5}$ c.

7. Add $\frac{m}{2} + \frac{m}{3}$

EXAMPLE:

Find L.C.D., which is 6. Convert each fraction to sixths and add:

$$\frac{m}{2} + \frac{m}{3} = \left(\frac{m}{2} \text{ x } \frac{3}{3}\right) + \left(\frac{m}{3} \text{ x } \frac{2}{3}\right)$$

$$= \frac{3m}{6} + \frac{2m}{6} = \frac{5m}{6}$$

8. Subtract $\frac{2}{5x^2}$ from $\frac{3}{4x}$

EXAMPLE:

In a subtraction example, the quantity after the word “from” goes first:

$\frac{3}{4x} - \frac{2}{5x}$ L.C.D. is 20x. $\frac{15-8}{20x}$.15-8 = 7. So the answer is $\frac{7}{20x}$

9. A woman owned $\frac{2}{3}$ of a store and sold $\frac{1}{5}$ of her share. What part of the store did she still own?

EXAMPLE:

“$\frac{1}{5}$ of her share” means $\frac{1}{5}$ times her share. Since $\frac{1}{5}$ of $\frac{2}{3} = \frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$ was sold. If you subtract $\frac{2}{15}$ from $\frac{2}{3}$, you'll know that what part of the store she still owned:

$\frac{2}{3} - \frac{2}{15} = \frac{10\text{-}2}{15} = \frac{8}{15}$ is the answer.

10. Change .68 to a fraction.

EXAMPLE:

$$.68 = \frac{68}{100} = \frac{17}{25}$$

11. Change $\frac{3}{16}$ to a decimal correct to the nearest thousandth.

EXAMPLE:

If your answer is to be rounded to the nearest thousandth, carry the division to one place past the thousandths (the ten-thousandths place):

$$\frac{3}{16} = 16\overline{)\,3.0000}\quad .1875$$

Since the extra place is a 5, we round up. The answer is .188.

12. $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = ?\left(\frac{4}{9}\right)$

EXAMPLE:

$$\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = ?\left(\frac{4}{9}\right) \Rightarrow \frac{5}{36} = \text{x times } \frac{4}{9}$$

Since the missing quantity is multiplied by $\frac{4}{9}$, do the "opposite" on the other side

and divide by $\frac{4}{9} \Rightarrow \frac{5}{36} \div \frac{4}{9} = \frac{5}{36} \times \frac{9}{4} = \frac{5}{16}$

13. If 5x = 28, what does 3x equal?

EXAMPLE:

$5x = 28 \Rightarrow \frac{5x}{5} = \frac{28}{5} \Rightarrow x = \frac{28}{5}$

To find 3x, multiply both sides of the equation $x = \frac{28}{5}$ by 3.

$(3)\ x = \frac{28}{5} \Rightarrow 3x = \frac{84}{5}$ or $16\frac{4}{5}$

14. If 4y + 12 = - 30, what does (y + 2) equal?

EXAMPLE:

$4y + 12 = -30 \qquad \Rightarrow 4y + 12 - 12 = -30 - 12 \Rightarrow 4y = -42 \Rightarrow \frac{4y}{4} = \frac{-42}{4}$

$y = \frac{-42}{4} = \frac{-21}{2} \qquad \Rightarrow$ Now add 2 to both sides $y+2 = \frac{-21}{2} + 2$

$y+2 = \frac{-21}{2} + \frac{4}{2} \qquad \Rightarrow y+2 = \frac{-17}{2}$

15. If r = 3b, what does $\frac{3}{4}$ r equal

$r = 3b$

$$\frac{4}{2} r = \left(\frac{3}{4}\right)\left(3b\right)$$

$$\frac{3}{4} r = \frac{9b}{4}$$

SOLVED EXAMPLES

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

Answer of each question has been given at the end of each question.

Use the space at the right of the options for scratch work.

1. A team played 30 games of which it won . What part of the games played did it lose?

A. 1/5 B. 5/6

C. 6/5 D. 5

2. If a man's weekly salary is \$X and he saves \$Y, what part of his weekly salary does he spend?

A. $\frac{X}{Y}$ B. $\frac{X-Y}{X}$

C. $\frac{Y-X}{X}$ D. $x - y$

3. What part of an hour elapses between 10: 45 a.m. and 11:09 a.m.?

A. 2/5 B. 3/5

C. 11/12 D. 2

4. One half of the employees of Al Karim Co. earn salaries above Rs.18,000 monthly. One third of the remainder earn salaries between Rs.15,000 and Rs.18,000. What part of the staff earns below Rs.15,000?

A. 2/3 B. 1/3

C. 2/5 D. 3/5

5. David receives his allowance on Sunday. He spends ¼ of his allowance on Monday and 2/3 of the remainder on Tuesday. What part of his allowance is left for the rest of the week?

A. 2/3 B. 4/5

C. 6/7 D. 1/4

6. 12 is ¾ of what number?

A. 20 B.24

C.16 D.8

7. A piece of fabric is cut into three sections so that the first is three times as long as the second and the second is three times as long as the third. What part of the entire piece is the smallest section?

A.2/5 B.3/7

C.2/3 D.1/13

8. A factory employs M men and W women. What part of its employees are women?

A.W/(W+M) B.W/M

C.(W + M)/M D.M/W

9. A motion was passed by a vote of 5 : 3. What part of the votes cast was is favor of the motion?

A.3/5 B.5/8

C.3/8 D.5/3

10. If the ratio of x : y is 9 : 7, then x + y is

A.16 B.2

C.1 D.None of the above

ANSWERS

1	A	2	B	3	A	4	B	5	D
6	C	7	D	8	A	9	B	10	C

EXAMPLE:

1. The games lost = 30 – 24 = 6 . Now the part is 6 and whole is 30. Take part over whole.

2. Spending = x – y. Now the part is x – y and whole is x. Take part over whole.

3. The difference between 10: 45 a.m. and 11:09 a. m. is 24 minutes. Now the part is 24 minutes and whole is one hour, which is 60 minutes. Take part over whole.

4. Suppose there are 120 employees in Al Karim Co. Half of these earn salaries above 18000. so the remaining 60 employees are below 18000. Out of these remaining 60 employees. 1/3 0f 60 that is 20 employees earn salaries between 15000 and 18000. Now the rest of the 40 employees earn salaries below 15000 which is our part for this question. Now the part is 40 and whole is 120. Take part over whole 40/120 = 1/3

5. Suppose David's allowance is 120. Spending on Monday is 120 × ¼ = 30. The remaining is 90. 2/3 of 90 = 60 is spent on Tuesday. The rest is 30. Now our part in this question is 30 and whole is 120. Take part over whole that is 30/120 = ¼.

6. Read the question as 12 is ¾ of x. This gives us the equation 12 = ¾ × x.

7. The smallest section of the fabric in this case is the third one. Suppose third section is x.

Now according to conditions

1st	2nd	3rd	Total
3(3x)	3x	x	13x

Now the part is x and whole is 13x. Take part over whole.

8. Part is W and whole is M + W.

9. As the motion was passed the vote in favor were 5 which is our part in this question. The whole is 3 + 5 = 8.

10. since ratios are always in the most simplified form, we do not know the exect values of x and y.

This is not what the question asked. $\frac{x}{x+y} + \frac{y}{x+y} = 1$

RATIOS AND PROPORTIONS

WWThe problems in this topic involve something new, common sense. If you are logical and make sure your answer makes sense, you'll do fine.

COMPARE LOGICALLY

A ratio is the comparison of two things.

What is the ratio of the value of a dime to that of a quarter? This can be shown fractionally 10/25

or by using a colon (10:25). Reducing the fraction to $\frac{2}{5}$ shows that a dime is worth $\frac{2}{5}$ as much

as a quarter (Dime is a coin of 10 cents, quarter is a coin of 25 cents). What is the ratio of the value of a quarter to that of a dime? Since the quarter is written first, it goes on top of the

fraction $\frac{25}{10}$ or before the colon (25:10). Reduction shows that a quarter is worth $\frac{5}{2}$ or

$2\frac{1}{2}$

times of a dime. You can put any two things in a fraction numerically. It's logical to use only

comparable things. What is the ratio of 8 inches to 1 yard? You answer $\frac{8}{1}$, but does it make sense?

Is 8 inches 8 times as large as 1 yard? Of course not! Common sense will make you change 1

yard to 36 inches. Now the ratio becomes $\frac{8}{36}$ or $\frac{2}{9}$. Compare only like things.

EXAMPLE:

What is the ratio of 30 minutes to 2 hours?

$\frac{30}{120}$ (2 hours = 120 minutes) = $\frac{1}{4}$

DIRECT AND INVERSE PROPORTION

Many question in the NAT Mathematics can be done by using proportions.
Proportions are of two types Direct and Inverse.

DIRECT PROPORTIONS

EXAMPLE:

If a car can travel 80 miles in 3 hours, how long will it take to travel 100 miles?
When something varies directly with something else, if you increase one of the things involved,
the other will also increase. In the example given, if you increase the miles from 80 to 100, common sense tells us that the number of hours will also increase. The more miles you drive, the longer it should take. Therefore, let's make a proportion using a ratio of the miles and a ratio of the hours. The first number of miles mentioned is 80, so use the ratio (miles over miles).
As the first number of hours mentioned is 3, so use the ratio (hours over hours). X represents the second amount of hours since that's the unknown. Put the two ratios together to form the proportion. Cross- multiply and solve: $\frac{80}{100} = \frac{3}{x}$

$$80X = 300 \Rightarrow X = \frac{30}{8} = \quad 3\frac{3}{4} \text{ hours}$$

INVERSE PROPORTIONS

In an inverse proportion problem, if one thing is increased, the other will decrease.
Example: If 8 people take 6 hours to build a fence, how long would it take 10 people to build the same fence (assuming they work at the same rate as those in the first group)?
Common sense tells you that if you use more people, it will take less time to finish the job. To write the proportion needed to solve the example, first write the first ratio:

$$\frac{8 \text{ People}}{10 \text{ People}}$$

Since it is not a direct proportion, invert the second ratio ($\frac{6}{x}$ becomes $\frac{x}{6}$), the proportion is

$$\frac{8}{10} = \frac{x}{6} \Rightarrow 10x = 48$$

$$\Rightarrow x = 4 \quad \frac{4}{5} \text{ hours}$$

SOLVED EXAMPLES

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

Answer of each question has been given at the end of each question.

1. Find the ratio of 18 inches to 2 yards.

A. $\frac{3}{4}$ B. $\frac{1}{4}$

C. $\frac{1}{5}$ D. $\frac{2}{5}$

2. If a train travels $\frac{5}{6}$ miles in $1\frac{1}{4}$ minutes, how many miles will it travel in 1 hour?

A. 20 miles B. 50 miles

C. 40 miles D. 30 miles

3. Find the value of x if 3 : b = x : c.

A. $\frac{3b}{c}$ B. $\frac{c}{3b}$

C. $\frac{2c}{3b}$ D. $\frac{3c}{b}$

4. How many miles are there between two cities if the distance is represented by a 2.4-inch line on a map having a scale of 1 inch to 8 miles?

A. 19.2 B. 12.8

C. 8.5 D. 38

5. How many cents will r books cost if t books cost m dollars?

A. $\frac{100mr}{t}$ B. $\frac{mr}{100t}$

C. $\frac{100t}{mr}$ D. $\frac{m}{100t}$

6. If apples cost 3 for 37 cents, find the cost of $1\frac{3}{4}$ dozen apples.

A. 111 cents B. 159 cents

C. 259 cents D. 211 cents

7. If 10 tractors are needed to plow a field in 4 hours, how many tractors are needed to plow the field in 5 hours?

A. 32 B. 4 C. 16 D. 8

8. A car that gets 15 miles per gallon of gasoline can travel 250 miles on a full tank. If the same car got 20 miles per gallon, how many miles could it travel on a full tank?

A. 300 B. 750

C. $250\frac{3}{5}$ D. $333\frac{1}{3}$

9. A candy recipe calls for 5 parts milk, 4 parts cocoa, 4 parts syrup, 2 parts sugar, and 1 part butter. If you use 8 ounces of milk, how many ounces of candy mixture can you make?

A. $25\frac{3}{5}$ B. $5\frac{3}{5}$

C. 20 D. 128

10. If it takes 10 minutes to walk $\frac{3}{7}$ mile, how many minutes will it take to walk the rest of the mile?

A. $2\frac{1}{3}$ B. $13\frac{1}{3}$

C. $4\frac{2}{7}$ D. 30

ANSWERS

1	B	2	C	3	D	4	A	5	A
6	C	7	D	8	D	9	A	10	B

EXAMPLE:

1. First convert yards to inches that is 2 yards = 2 ×3 × 12 = 72 inches. Now the ratio is 18 to72 = 1 to 4 = 1/4.

2. The train travels $\frac{5}{6} \div 1\frac{1}{4} = \frac{5}{6} \div \frac{5}{4} = \frac{5}{6} \times \frac{4}{5} = \frac{2}{3}$ miles in one minute. The distance traveled in 60 minutes (1 hour) = $\frac{2}{3} \times 60 = 40$ miles.
   Note: 12 inches = 1 foot and 3 feet = 1 yard

3. 3c = bx and x = 3c/b

4. The question is to find x from 1 : 8 = 2.4 : x . 1 × x = 8 × 2.4 hence x = 19.2

5. Solve x 100m r t
   Note: 1 dollar = 100 cents.

6. $1\frac{3}{4}$ dozens apples = 7/4 × 12 = 21 apples. Now apply the ratios method $\frac{37}{3} = \frac{x}{21}$

   $X = \frac{21 \times 37}{2}$ or x = 7 × 37 = 259 cents.

7. $\frac{10}{x} = \frac{5}{4}$ the second ratio is reversed, as the relation is inverse. 10×4x = 5x = 3x =8.

8. $\frac{x}{250} = \frac{20}{15}$

9. 5 parts of milk gives (5+4+4+2+1 = 16) parts of candy mixture.

$$\frac{5}{8} = \frac{16}{x} \Rightarrow 5x = 128 \Rightarrow x = 25\frac{3}{5}$$

10. Rest of the mile $= 1 - \frac{3}{7} = \frac{4}{7}$ mile. $\frac{10}{x} = \frac{3/7}{4/7}$

AVERAGE AND COMBINED RATES

Average $= \frac{\text{Sum}}{n}$. The average of 7 and 9 is $\frac{16}{2} = 8$ and average of 3, 5 and 7 is $\frac{16}{2} = 5$.

Sum of entities = (Number of entities) × Average. The average of two numbers is 5, the sum of the numbers is $5 \times 2 = 10$.

Average is also called as mean or arithmetic mean. It gives the central value of data.

MODE:

Modeis the highest frequency entity in a set of data. The mod of 1,1,5,1,6,2,3,5,5,1,3,2,1 is 1 as 1 is repeated 6 times.

MEDIAN:

Median is the middle point of an arranged set of data in ascending or descending order. The median of 1,2,5,8,9 is 5 as it is located at the middle point.

Combined Rates:

One and the same job is completed by two or more agencies in different times, the time taken to

complete the job working together = Tc. Tc is obtained by the relation $\frac{1}{T_c} = \frac{1}{T_1} + \frac{1}{T_2}$

Where T1 and T2 are times taken by the agencies as to complete the job working alone.

SOLVED EXAMPLES

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

Answer of each question has been given at the end of each question.

Use the space at the right of the options for scratch work.

1. Mr. Kashif got an average of 50 in 6 tests. What should he get in the next test to attain the average of 60?

A. 120 B. 60
C. 100 D. 70

2. If a pipe can fill a tank in 2 hours and another pipe can fill the same tank in 40 minutes.
How much time in minutes is needed to fill the tank if both the pipes are working together?

A. 90 B. 90
C. 60 D. 30

3. A clerk filed 73 forms on Monday, 85 forms on Tuesday, 54 on Wednesday, 92 on Thursday and 66 on Friday. What was the average number of forms filed per day?

A. 50 B. 95
C. 84 D. 74

4. Find the arithmetic mean of 25.2, 13.5, 18.5, and 34.8

A. 13 B. 23
C. 27 D. 5

5. The average of x, y, z and 40 is 10. What is the average of x, y, and z.

A. 10 B. 0
C. 2 D. 15

6. A and B can do a job in 6 days. If A does the job alone he takes 10 days. What will be the time required by B to complete the job alone?

A. 8 B. 6
C. 15 D. 3

7. The average height of a class of 14 boys is 5.3 feet. After new boy is admitted to the class, the new average height now becomes 5.25. What is the height of the new boy?

A. 4.55 B. 5.0
C. 6.0 D. 3.5

8. A man bought 27 packets of Chilli Milli at $280 each, 9 packets of Chilli Milli at $320 each and 6 packets of Chilli Milli at $360 each. Find the average price per packet of Chilli Milli.

A. $250 B. $300
C. $400 D. $380

9. Out of the 44 boys in a class 9 are of the age of 10, 15 at the age of 9, and the rest are at the age of 8. Find their average age.

A. 7.85 B. 8.75
C. 12.2 D. 14.35

10. The population of 8 villages is 900, 750, 1100, 1050, 835, 1250, 555, and 630. Find the population of Ninth village if the average population of Nine villages is 900.

A. 1200 B. 1050
C. 1030 D. 7070

ANSWERS

1	A	2	D	3	D	4	B	5	B
6	C	7	A	8	B	9	B	10	C

EXAMPLE:

1. Total marks in 6 tests = 50 × 6 = 300. Total marks required in 7 tests (including next test) = 60 × 7 = 420. The marks in 7th test = Total marks in 7 tests - Total marks in 6 tests = 420 – 300 = 120

2. $\frac{1}{120} + \frac{1}{40} = \frac{1}{x}$ (2 hours = 120 minutes)

$\frac{1}{x} = \frac{1+3}{120}$, $x = \frac{120}{4} = 30$

3. $\frac{73+85+54+92+66}{5} = 74$

4. $\frac{25.2+13.5+18.5+34.8}{4} = 23$

5. You are given $\frac{x+y+z+40}{4} = 10$ and you are to find $\frac{x+y+x}{3}$. Solve the given x + y + z = 0 and hence $\frac{x+y+x}{3} = 0$

6. $\frac{1}{10} + \frac{1}{x} = \frac{1}{6}$

7. (15 × 5.25) - (14 × 5.3) = 4.55 Sum of heights of 15 boys – sum of heights of 14 boys
= height of 15th boy.

8.

No of Packets	Price	Value
27	× 280	= 7560
9	320	2880
6	360	2160
Total: 42		**12600**

Average Price = 12600 ÷ 42 = 300

9. Adopt same procedure as in question 8.

10. Sum of 8 villages = 900 + 750 + 1100 + 1050 + 835 + 1250 + 555 + 630 = 7070 Sum of 9 villages (by applying Sum = Total values × Average) 9 × 900 = 8100
Population of 9th village = 8100 – 7070 = 1030

PERCENTAGE

A percentage is a part of 100. If one of your test score was 85 out of a total of 100, it means your score was 85%. The word 'percent' means 'per hundred' or 'out of 100', and %is the percent sign. Percent is a way of representing a part ofsomething in terms of hundredths (i.e., 1/100).

For example, $100\% = 100 \text{ hundredths} = 100 \text{ x } \frac{1}{100} = 1$

and $75\% = 75 \text{ hundredths} = 75 \text{ x } (\frac{1}{100}) = \frac{3}{4}$. Percent can be expressed as a fraction (with a denominator of 100) or a decimal.

Tips

When you do calculations with percents, change percents to fractions or decimals.

It is advisable to convert a mixed number to an improper fraction during calculations.

For example, $29\% = \frac{29}{100} = 0.29$

Percents are often used to compare fractions with equal denominators of 100. To convert a percent to a fraction, drop the percent sign and divide the number by 100.

For example, $80\% = 80/100 = \frac{4}{5}$ and $125\% =$

$125/100 = \frac{4}{5} = 1\frac{1}{4}$

As a more complicated example, consider the following:
To convert a fraction to a percent, multiply the number by 100 and insert the percent sign.

Example:

$$\frac{3}{5} = \frac{3}{5} \times \frac{100}{1} \% = 60\%$$

$$\text{and } \frac{1}{16} = \frac{100}{16} \% = \frac{25}{4} \%$$

$$= 6\frac{1}{4} \% = 6.25\%$$

To convert a percent to a decimal, drop the percent sign and divide the number by 100 (i.e., move the decimal point two places to the left, inserting zeros to the left if necessary).

EXAMPLE:

13% = 0.13 and 2% = 0.02

To convert a decimal to a percent, multiply the decimal by 100 (i.e., move the decimal point two places to the right) and insert the percent sign.

EXAMPLE:

0.2576 = 25.76% and 0.002 = 0.2%

To find a certain percent of a number, multiply the number by the percent expressed as a fraction or decimal.

For example to find 45% of 900, two possible ways are as follows.

Changing percent to fraction gives $45\% = \frac{45}{100} = \frac{9}{20}$. So, $\frac{9}{20} \times 900 = 405$. Alternatively,

changing percent to decimal gives $45\% = \frac{45}{100} = 0.45$. So, 0.45 x 900 = 405.

The following fraction and decimal equivalents of percents are worth noting:

1% = 1/100 = 0.01	60% = 3/5 = 0.6
2% = 1/50 = 0.02	80% = 4/5 = 0.8
4% = 1/25 = 0.04	25% = 1/4 = 0.25
5% = 1/20 = 0.05	50% = 1/2 = 0.5
10% = 1/10 = 0.1	75% = 3/4 = 0.75
20% = 1/5 = 0.2	120% = 6/5 = 1.2
40% = 2/5 = 0.4	125% = 5/4 = 1.25

PERCENT CHANGE

Percent change (increase or decrease) from an original value to a new value frequently occurs. To find the percent change, first find the amount of the change, then divide this amount by the original value, and finally express this quotient as a percent.

For example if the price of an item changes from \$32 to \$40, the amount of the increase is \$(40

− 32) = \$8 and the percent increase is $\frac{8}{32} = \frac{1}{4} = 0.25 = 25\%$.

On the other hand, if the price of the item changes from \$40 to \$32, the amount of the decrease is \$(40 − 32) = \$8 and the percent decrease is $\frac{8}{40} = \frac{1}{5} = 0.2 = 20\%$.

Interestingly, the percent increase from 32 to 40 is different from the percent decrease from 40 to 32.

SOLVED EXAMPLES

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

Answer of each question has been given at the end of each question.

1. Convert $1\frac{2}{5}$ % to a fraction.

EXAMPLE:

Since a percent is part of 100. $1\frac{2}{5}$% becomes $\frac{1\frac{2}{5}}{100}$, which is written $1\frac{2}{5} \div 100 \to \frac{7}{5} \div \frac{100}{1} \to \frac{7}{5} \cdot \frac{1}{1} = \frac{7}{500}$

2. $\frac{7}{3}$ is what percent?

EXAMPLE:

Write an equation. "What percent" means what part of hundred. Thus, $\frac{7}{3} \cdot \frac{x}{100}$. Cross-multiply and solve:$3x = 700 \to \frac{3x}{3} \to \frac{700}{3}\% \to 233\frac{1}{3}\%$

3. $83\frac{1}{3}$ % is how many sixteenths?

EXAMPLE:

Save time and remember that$83\frac{1}{3}\% = \frac{5}{6}$. You could write $\frac{83\frac{1}{3}}{100}$ and simplify, but the time saved by memorizing the equivalents is significant. To continue, $\frac{5}{6} = \frac{x}{16} \rightarrow 6x=5\times16 \rightarrow 6x = 80 \rightarrow$

$\frac{6x}{6} = \frac{80}{6} \rightarrow x = \frac{40}{3}$ or $13\frac{1}{3}$

4. Change .076 to a percent.

EXAMPLE:

A quick way is to slide the decimal point two places to the right (to change a decimal to a percent):
.076 = 7.6%. However, the answer may appear in fraction form, so .6 must be converted to $\frac{6}{10}$ or $\frac{3}{5}$.

The answer is $7\frac{3}{5}\%$.

5. Change $8\frac{1}{2}\%$ to a decimal.

EXAMPLE:

Change $\frac{1}{2}$ to a decimal: $2\overline{)1.0} = .5$ So $8\frac{1}{2}\% = 8.5\%$. Next, slide the decimal point two places to the left (to change a percent to a decimal) and drop the % sign: 8.5% = .085.

6. 4b is what percent of 30a?

EXAMPLE:

Write an equation, but be careful of the extra letters:

$4b = \frac{x}{100} \cdot 30a \rightarrow \frac{4b}{1} = \frac{30ax}{100} \rightarrow 30ax \cdot 1 = 4b \cdot 100 \rightarrow 300ax\ 400b \rightarrow \frac{30ax}{30a} = \frac{400b}{30a} \rightarrow$

$x = \frac{400b}{30a} \rightarrow x = \frac{40b}{3a}$

7. 7b is y % of what number?

EXAMPLE:

Let x be the missing number.

$$7b = \frac{y}{100} \cdot \frac{x}{1} \rightarrow xy = 700b \rightarrow \frac{xy}{y} = \frac{700b}{y} \rightarrow x = \frac{700b}{y}$$

8. Find the missing number: $\frac{?}{.24} = 12\%$

EXAMPLE:

Put a letter where the question mark is, change the percent to a fraction, and solve:

$\frac{x}{.24} = \frac{12}{100} \rightarrow \frac{x}{.24} \cdot \frac{100}{100} = \frac{100x}{24} \rightarrow \frac{100x}{24} = \frac{12}{100}$ Reduce to: $\frac{25x}{6} = \frac{3}{25}$ Cross-multiply and finish:

$$25x \cdot 25 = 6 \cdot 3 \rightarrow 625x = 18 \rightarrow \frac{625x}{625} = \frac{18}{625} \rightarrow x = \frac{18}{625}$$

9. $\frac{1}{8}$ of 22 is what percent of 4?

EXAMPLE:

Write an equation, and it's easy:

$$\frac{1}{8} \cdot 22 = \frac{x}{100} \cdot 4 \rightarrow \frac{1}{8} \cdot \frac{22}{1} = \frac{x}{100} \cdot \frac{4}{1} \rightarrow \frac{22}{8} = \frac{4x}{100} \rightarrow \frac{11}{4} = \frac{x}{25} \rightarrow 4x = 275$$

$$\frac{4x}{4} = \frac{275}{4} \rightarrow x = \frac{275}{4} \text{ or } 68\frac{3}{4}$$

10. Steve paid \$7.50 to repair a toaster rather than buy a new one for \$30. What percent of the cost of the new toaster did he save?

EXAMPLE:

Use the question to write an equation: "What percent of the cost did he save?" $\frac{x}{100} \cdot \$30 = \$22.50$

(To find out what he saved, subtract \$7.50 from \$30.) Simplify and solve: x = 75%

PRACTICE EXERCISE

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

The answers of the questions have been given at the next page after exercise.

Use the space at the right of the options for scratch work.

1. What is the number, 5% of which is 10?

A.200 B.100

C.50 D.10

2. What is the sum of money, 6 % of which is 18 dollars?

A.600 B.200

C.300 D.10

3. Find the sum of money, 11% of which is Rs. 1650.

A.150 B.3300

C.25000 D.15000

4. A man has Rs. 2000 and spends 18% of it. What money has he left now?

A.3600 B.820

C.1640 D.4000

5. In a school there are 400 students, of whom 70% are boys What is the number of girls?

A.120 B.200

C.280 D.2800

6. The population of a city increased in two years from 25,000 to 30,000: find the percent increase during the time.

A.10% B.20%

C.40% D.5%

7. A man opens a bookstall with a capital of Rs. 25000. In three months his capital amounts to rupees 27500.What is the increase percent?

A. 1% B. 10%

C. 20% D. 7%

8. A man spent 10% of his money. After spending 60% of the remainder he has Rs. 72 left.How much had he in the start?

A. 10 B. 100

C. 200 D. 400

9. A word processing operator typed 44 words per minute. After practice, the operator's speed increased to 55 words per minutes. By what percent did the operator's speed increase?

A. 25% B. 50%

C. 15% D. 20%

10. The annual decrease in the population of a city was 10% and the present number of inhabitants is 1620. What was the population 2 years hence?

A. 20 B. 400

C. 2000 D. 1000

ANSWERS

1	A	2	C	3	D	4	C	5	A
6	B	7	B	8	C	9	A	10	C

Explanations

1. 5% of x is 10. x × $\frac{5}{100}$ = 10. → x = 200

2. x is the sum of money. 6% of x is 18. Solve as in question 1.

3. 11% of x is 1650. Solve as in the above questions.

4. The question takes the form of find the number 18% of which is 2000. Solve as in the above questions.

5. If boys are 70% of 400, then girls are the remaining 30% of 400. So , 400 × $\frac{30}{100}$.= 120

6. The change is 30,000 – 25,000 = 5000. Percent increase = (Increase in value ÷ Original value) × 100 → $\frac{5000}{25000}$ × 100 → 20%

7. The increase in capital is 27500 – 25000 = 2500. Solve as above question.

8. Suppose x is the amount in the start. After spending 10% he is left with 90% of x. That is, he has $\frac{90x}{100}$. Out of this remaining amount he spends 60%.Which means that he has 40% of this amount ($\frac{90x}{100}$) left .

$= \frac{90x}{100} \times \frac{40}{100} = 72$ (given remaining amount). By solving this you have the answer.

9. Same as question 6 and 7.
Suppose the population 2 years ago was x. This population was decreased by 10%. The remaining population is 90% 0f x which is then decreased by 10% in the second year. You are to solve 90% (90% of x) = 1620.

QUANTITATIVE BRIEF REVIEW

HOW TO USE PEMDAS

When you're given an arithmetic equation, it's important to know the order of operations. Just remember PEMDAS (as in —PLEASE EXCUSE MY DEAR AUNT SALMA). What PEMADS means is this: First solve Parentheses; second deal with Exponents; third do the Multiplication fourth do Division, and finally do the Addition and Subtraction together, going from left to right.

EXAMPLE:

$9 - 2 \times (5 - 3)^2 + 6 \div 3 =$

Begin with parentheses: $9 - 2 \times (2)^2 + 6 \div 3$.

Then do the exponent: $9 - 2 \times 4 + 6 \div 3$

Now do the multiplication and division from left to right: $9 - 8 + 2$

Finally, do addition and subtraction from left to ri

$9 - 8 + 2 = 1 + 2 = 3$.

Tips

You'll usually find the part near the word is and the whole near the word of.

Be sure to use the original whole – not the new whole – for the base.

How To Use The Percent Formula

Identify the part, the percent, and the whole.

$$\text{Part} = \text{percent} \times \text{whole}$$

Example

(Find the part) What is 12 percent of 25?

$$\text{Part} = \frac{x}{100} \times 25 = 3$$

OR Say it as —x is 12 percent of 25I so it gives you

$$\frac{x}{100} \times 12 = 25 \rightarrow x = 3$$

Example

(Find the percent) 45 is what percent of 9?

$$45 = \text{Percent} \times 9 = 5 \times 9$$

$$\text{Percent} = 5 \times 100\% = 500\%$$

OR Say it as "45 is x percent of 9". So it gives you $45 = \frac{x}{100} \times 9 \rightarrow x = 500\%$

How To Use The Percent Increase / Decrease Formulas

Identify the original whole and the amount of increase / decrease.

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original whole}} \times 100\%$$

$$\text{Percent decrease} = \frac{\text{amount of decrease}}{\text{original whole}} \times 100\%$$

Example

The price goes up from $80 to $100. What is the percent increase?

$$\text{Percent increase} = \frac{20}{80} \times 100\% = 25\%$$

How to Predict Whether A Sum, Difference, Or Product Will Be Odd Or Even

Don‘t bother memorizing the rules. Just take simple numbers like 1 and 2 and see what happens.

EXAMPLE:

If m is even and n is odd, is the product mn odd or even?
Say m = 2 and n = 1
2 × 1 is even, so mn is even.

How to Find a Common Factor

Break down both numbers to their prime factors to see what they have in common. Then multiply the shared prime factors to find all common factors.

EXAMPLE:

What factors greater than 1 do 135 and 225 have in common?
First find the prime factors of 135 and 225.
135 = 3 × 3 × 3 × 5, and 225 = 3 × 3 ×5 × 5. The number share 3 × 3 × 5 in common. Thus, aside from 3 and 5, the remaining common factors can be found by multiplying 3, 3, and 5 in every possible combination: 3 × 3 = 9, 3 × 5 = 15, and 3 × 3 × 5 = 45.

HOW TO FIND A COMMON MULTIPLE

The product of the numbers is the easiest common multiple to find. If the two numbers have any factors in common, you can divide them out of the product to get a lower common multiple.

EXAMPLE:

What is the least common multiple of 28 and 42?
The product 28 42 = 1,176 is a common multiple, but not the least. 28 = 2 2 7, and 42 = 2 3 7. They share a 2 and a 7, so divide the product by 2 and then by 7.

1,176 ÷ 2 = 588.
588 ÷ 7 = 84.

The least common multiple is 84.

How to find the AVERAGE

$$\text{Average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

EXAMPLE:

What is the average of 3, 4 and 8?

$$\text{Average} = \frac{3+4+8}{3} = \frac{15}{3}$$

How To Use The Average To Find The Sum

$$\text{Sum} = (\text{average}) \times (\text{number of terms})$$

EXAMPLE:

17.5 is the average (arithmetic mean) of 24 numbers. What is the sum?

$$\text{Sum} = 175 \times 24 = 420$$

How to Find the Average of Consecutive Numbers

The average of evenly spaced numbers is simply the average of the smallest number and the largest number. The average of all the integers from 13 to 77, for example, is the same as the average of 13 and 77

$$\frac{13+77}{2} = \frac{90}{2} = 45$$

How To Count Consecutive Numbers

The number of integers from A to B inclusive is B – A + 1.

EXAMPLE:

How many integers are there from 73 through 419, inclusive?

$$419 - 73 + 1 = 347$$

Note: Don't forget to add 1!

How To Find The Sum Of Consecutive Numbers

$$\text{Sum} = (\text{average})\ (\text{number of terms})$$

EXAMPLE:

What is the sum of the integers from 10 through 50, inclusive?

$$\text{Average} = (10 + 50) \div 2 = 30;$$
$$\text{Number of terms} = 50 - 10 + 1 = 41$$

Sum = 30 41 = 1,230

HOW TO FIND THE MEDIAN

Put the numbers in numerical (ascending or descending) order and take the middle number. (If there's an even number of numbers, the average of the two numbers in the middle is the median.)

EXAMPLE:

What is the median of 88, 86, 57, 94, and 73?
Put the numbers in numerical order and take the middle number:

57, 73, 86, 88, 94

The median is 86. (If there's an even number of numbers, take the average of the two in the middle.)

HOW TO FIND THE MODE

Take the number that appears most often. For example, if your test scores were 88, 57, 68, 85, 98, 93, 93, 84, and 81, the mode of the scores is 93 because it appears more often than any other score. (If there's a tie for most often, then there's more than one mode.)

HOW TO FIND THE RANGE

Simply take the difference between the highest and the lowest values. Using the previous example, if your test scores were 88, 57, 68, 85, 98, 93, 84, and 81, the range of the scores is 41, the difference between the highest and lowest values (98 – 57 = 41).

How To Use The Actual Numbers To Determine Ratio

To find a ratio, put the number associated with —of‖ on the top and the word associated with —to‖ on the bottom.

$$\text{Ratio} = \frac{\text{of}}{\text{to}}$$

The ratio of 20 oranges to 12 apples is

$$\frac{20}{12} \text{ or } \frac{5}{3}$$

HOW TO USE RATIO TO DETERMINE AN ACTUAL NUMBER

EXAMPLE:

The ratio of boys to girls is 3 to 4. If there are 135 boys, how many girls are there?

$$\frac{3}{4} = \frac{135}{x}$$

$$3 \times x = 4 \times 135$$
$$x = 180$$

HOW TO USE ACTUAL NUMBERS TO DETERMINE A RATE

Identify the quantities and the units to be compared. Keep the units straight.

EXAMPLE:

Anders typed 9,450 words in $3\frac{1}{2}$ hours. What was his rate in words per minute?

First convert $3\frac{1}{2}$ hours to 210 minutes. Then set up the rate with words on top and minutes on bottom:

$$\frac{9{,}450 \text{ words}}{210 \text{ min utes}} = 45 \text{ words per minute.}$$

Note: The unit before per goes on top, and the unit after per goes in the bottom.

HOW TO COUNT THE NUMBER OF POSSIBILITIES

The number of possibilities is generally so small that the best approach is just to write them out systematically and count them.

EXAMPLE:

How many three-digit numbers can be formed with the digits 1, 3 and 5?

Write them out. Be systematic so you don't miss any: 135, 153, 315, 351, 513, and 531. Count them: six possibilities.

How To Calculate A Simple Probability

$$\text{Probability} = \frac{\text{number of favorable/outcomes}}{\text{total number of possible/outcomes}}$$

EXAMPLE:

What is the probability of throwing a 5 on a fair six-sided die?

There is one favorable outcome – throwing a 5. There are six possible outcomes – one of each side of the die.

$$\text{Probability} = \frac{1}{6}$$

HOW TO WORK WITH NEW SYMBOLS

If you see a symbol you've never seen before, don't freak out: It's a made-up symbol. Everything you need to know is in the question stem. Just follow the instructions.

HOW TO SIMPLIFY POLYNOMIALS

First multiply to eliminate all parentheses. Each term inside one parenthesis is multiplied by term inside the other parentheses. All like terms are then combined.

Example

$(3x^2 – 5x)(x –1)$

$3x^2(x – 1) + 5x (x – 1)$

$(3x^3 – 3x^2 + 5x^2 – 5x)$ $(3x^2+2x^2 – 5x)$

HOW TO FACTOR CERTAIN POLYNOMIALS

Learn to spot these classic factorables:

$ab + ac = a(b + c)$

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 – 2ab + b^2 = (a – b)^2$

$a^2 – b^2 = (a – b)(a + b)$

HOW TO SOLVE FOR ONE VARIABLE IN TERMS OF ANOTHER

To find x —in terms ofl y: isolate x on one side, leaving y as the only variable on the other.

HOW TO SOLVE AN INEQUALITY

Treat it much like an equation – adding, subtracting, multiplying, and dividing both sides by the same thing. Just remember to reverse the inequality sign if you multiply or divide by a negative.

EXAMPLE:

Rewrite $7 – 3x > 2$ in its simplest form:

$7 – 3x > 2$ Subtract 7 from both sides.

$7 – 3x – 7 > 2 – 7$

So $– 3x > -5$. Now divide both sides by -3, and remember to reverse the inequality sign:

$$x < \frac{5}{3}$$

HOW TO HANDLE ABSOLUTE VALUES

The absolute value of any heal number n, is defined as n (without any positive or negative sign) 0. It's also referred to as the distance from zero to the number on the number line:

$|-5| = 5$

If $|x| = 3$, the x could be 3 or -3

EXAMPLE:

If $|x- 3| < 2$, what is the range of possible values for x?

$|x- 3| < 2$ so $(x - 3) < 2$ and $-(x-3) < 2$.

So $x - 3 < 2$ and $-x +3 < 2$.

So $x < 2 + 3$ and $x > - 2 + 3$.

So $x < 5$ and $x > 1$.

So $1 < x < 5$.

HOW TO TRANSLATE ENGLISH INTO ALGEBRA

Look for the key words and systematically turn phrases into algebraic expressions and sentences into equations. Here's a table of key words that you may have to translate into mathematical equations.

Operation	Key words
Addition	Sum, plus, and, added to, more than, increase by, combined with, exceeds, total, greater than
Subtraction	Difference between, minus, subtracted from, decreased by, diminished by, less than, reduced by
Multiplication	Of, product, times, multiplied by, twice, thrice, double, triple, half
Division	Quotient, divided by, per, of, ratio of to
Equals	Equals, is, was, will be, the result is, adds up to, costs, is the same as

PRACTICE EXERCISE

DIRECTIONS:

You are given following questions from the topic, with four choices A through D. Select the choice that will answer the question best.

The answers and explanations of the questions have been given at the next page after exercise.

Use the space at the right of the options for scratch work.

1. One-sixth of a day is what part of the time between 3 p.m. Monday and 3 a.m. Thursday of the same week?

A. $\frac{1}{10}$ B. $\frac{1}{18}$

C. $\frac{1}{15}$ D. $\frac{1}{12}$

2. If you have 50 green, 50 orange, and 50 ye low jelly beans, how many bags can you fill for Halloween each containing 2 green, 3 orange, and 4 yellow jelly beans?

A. 12 B. 13

C. 16 D. 17

3. If abc = 2 and a = c then b =

A. a^2 B. $\frac{1}{2a}$

C. $\frac{2}{a^2}$ D. 2 – a2

4. t is an integer greater than 5. The expression that must represent an odd integer is

A. t(t + 1) B. 3t – 1

C. t2 D. 2t – 3

5. Which of the following is the sum of two consecutive prime numbers?

A. 66 B. 52

C. 41 D. 29

6. If Myra had bowling scores of b + 6, b -2, b + 4, and b -5, what must she score in the next game to get an overall average of b +2?

A. b + 7 B. b – 3

C. b + 3 D. b - 7

7. A clock gain 8 minutes every x hours. How many hours will the clock gain in 3 days?

A. $\frac{576}{x}$ B. $\frac{48}{5x}$

C. $\frac{24}{x}$ D. $\frac{576}{5x}$

8. How many integers from 28 to 98, both exclusive are exactly divisible by 7?

A. 9 B. 11

C. 12 D. 8

9. Four people are asked to stand in a straight line. In how many different orders can they line up?

A. 12 B. 16

C. 24 D. 6

10. If (p – 3)(p + 5) > (p – 3)(p + 8), what is

the best description of p?

A. p = 3 B. -8 < p < -5

C. p = { } D. p < 3

11. In solving an arithmetic example, Donna, by mistake multiplied by 6 instead of dividing by 6. If her answer was 13 $\frac{1}{5}$, what should be the correct answer to the example?

A. $2\frac{8}{11}$ B. $\frac{5}{66}$

C. $2\frac{1}{5}$ D. $\frac{11}{30}$

12. If (36)(?)(7) = 21, then ? equals

A. $\frac{21}{43}$ B. $\frac{1}{42}$

C. $\frac{1}{12}$ D. $\frac{1}{11}$

13. How many tens are equal to the number whose hundreds, tens, and units digits are a, b, and c, respectively?

A. b B. $a + \frac{1}{10} b + \frac{1}{100} c$

C. 10a + b + c D. $10a + b + \frac{c}{10}$

14. If a machine can place a cap on a bottle of soda every 0.8 seconds, how many bottles can be capped in 2 hours?

A. 8000 B. 9000

C. 300 D. 900

15. The death rates for three diseases are

Disease R	**2 people out of 10,000**
Disease S	**13 people out of 1,000,000**
Disease T	**9 people out of 100,000**

What is the combined death rate for the three diseases?

A. 123 out of 1,000,000 B. 42 out of 10,000

C. 42 out of 1000,000 D. 303 out of 1,000,000

16. If 7 apples cost y cents, how many apples will x dollars buy?

A. $\frac{x}{7y}$ B. $\frac{7x}{y}$

C. $\frac{7x}{100y}$ D. $\frac{700x}{y}$

17. Dave is twice as old as Bob, who is 3 years older than Steve. If Steve is 4a years old, Dave's age is

A. 8a B. 22a

C. 14a D. 8a + 6

18. If $3\frac{1}{5}c = 2\frac{1}{2}b$ and $c \neq 0$, then $\frac{b}{c}$

A. $\frac{25}{32}$ B. $\frac{7}{8}$

C. $\frac{32}{25}$ D. $\frac{11}{10}$

19. The average height of five men is 68 inches. If one man is 70 inches tall and three others have an average of 67 inches, the height of the fifth man, in inches, is

A. 68 B. 69

C. 70 D. 71

20. If p is a negative integer and $p^2 + 11p = t$, a value of t could be

A. 12 B. 18

C. -18 D. 11

ANSWERS

1	C	2	A	3	C	4	D	5	B
6	A	7	B	8	A	9	C	10	D
11	D	12	C	13	D	14	B	15	D
16	D	17	D	18	C	19	B	20	C

EXPLANATIONS

1. One-sixth of a day is $\frac{1}{6}(24) = 4$ hours. Find the number of hours from 3 p.m. Monday to 3 a.m. Thursday. There are 9 hours remaining on Monday plus 24 hours on Thursday plus 24 hours on Wednesday plus 3 hours to begin Thursday or $9 + 24 + 24 + 3 = 60$ hours. 4 hours is $\frac{4}{60}$ or $\frac{1}{15}$ of 60 hours.

2. Concentrate on the yellow jelly beans because the largest number of jelly beans placed in one bag will be that color. Divide 4 into 50, giving 12 and a remainder.

3. The five choices all contain the letter a, so eliminate c by substituting a for it.

$abc = 2aba = 2$

$a^2b = 2$

$b = \frac{2}{a^2}$

4. You are not told whether t is odd or even, so assume it can be either.
Choice A.: If t is odd, then t + 1 will be even. If t is even, then t + 1 will be odd. The product of an even and an odd is always even. Choice B.: 3 times an even number will be even, and 1 less than an even will be odd. However, 3 times an odd number is odd, and 1 less than an odd is even.
Choice C.: An even integer raised to the second power is even.
Choice D. 2 times either an odd number or an even number will have an even result. Subtracting 3, an odd number, from an even will always give an odd result.

5. With the exception of 2, every prime number is odd. The sum of two consecutive odd primes is an even integer. Therefore, choices C., D., and (E) can't be correct. If you run out of time at this time moment, guess either A. or B.. The odds would be in your favor to take this educated guess. The primes needed are 23, 29, 31, and 37.23 + 29 = 52, and neither 29 + 31 nor 31 + 37 equals 66.

6. Let x = Myra's score in the next game.

$b+2$

$$\text{Average} = \frac{\text{sum of the scores}}{\text{number of games}} = \frac{b+6+b-2+b+4+b-5+x}{5}$$

$$5b + 10 = 4b + 3 + x$$
$$x = b + 7$$

7. You want to compare the time gained and also the time to do the gaining. Write a proportion, but be sure to use compatible units. Compare 8 minutes with —how many hours.‖ Use any letter other than x (because x is already being used). Change y hours to 60y minutes. Next, compare x hours with 3 days, but change 3 days to 3(24), or 72, hours.

$$\frac{8 \text{ min utes}}{60y \text{ min utes}} = \frac{x \text{ hours}}{72 \text{ hours}}$$ Solve for y: $60xy = 576$

$$y = \frac{576}{60x} = \frac{48}{5x}$$

8. The word —between‖ means that 28 and 98 cannot be considered. The first number greater than 28 that is divisible by 7 is 35. The first number less than 98 that is divisible by 7 is 91. How many are divisible by 7 from 35 to 91? Subtract, divide by 7, then add 1 to count the first number that is not counted when you subtract, $\frac{56}{7} + 1 = 8 + 1 = 9$.

9. The people can stand in any one of four positions: first in line, second in line, third in line, or fourth in line. In the first position, any of the four can stand. After one stays in that position, the second position can be filled only by one of the three still not in line. After one stays in the second position, the third position can be filled by only one of the two people still not in line. After the third position is filled, there is only one person remaining to fill the fourth position. Therefore, the possibilities for people to fill the positions in line are: 4 in the first, 3 in the second, 2 in the third, and 1 in the fourth. The number of different ways they can arrange themselves is the product of the possibilities. (4)(3)(2)(1) =24 ways.

10. Multiply on each side of the inequality using the FOIL method, giving

$p^2 + 5p - 3p - 15 > p^2 + 8p - 3p - 24p^2 + 2p - 15 > p^2 + 5p - 24$
Add –p2 – 2p + 24 to each side, leaving $9 > 3p$. Divide by 3: $3 > p$, which means that p is less than 3.

11. To find the number Donna mistakenly multiplied by 6, do the opposite and divide $13\frac{1}{5}$ by 6.

$13\frac{1}{5} \div 6 = \frac{66}{5} \cdot \frac{1}{6} = \frac{11}{5}$ Now do what she should have done, divide by 6.

$$\frac{11}{5} \div 6 = \frac{11}{5} \cdot \frac{1}{6} = \frac{11}{30}$$

12. Put x in place of the question mark and solve: $(36)(x)(7) = 21$

$$x = \frac{21}{36(7)} = \frac{3}{36} = \frac{1}{12}$$

13. The number is represented as 100a + 10b + c. To find the number of tens, divided by 10.

$$\frac{100a+10b+c}{10}=10a+b+\frac{c}{10}$$

14. Write a proportion, but first change 2 hours to seconds. There are 3600 seconds in an hour, and 7200 seconds in 2 hours.

$$\frac{1\text{ bottle}}{x\text{ bottles}}=\frac{8\text{second}}{7200\text{seconds}}.$$

.8x = 7200Multiply both sides by 10 to remove the decimal point, giving
8x = 72000
x = 9000

15. The words —out ofl indicate a ratio (fraction). —Combinedl means to add. Express the rates fractionally, then add.

$$\frac{2}{10,000}\,\frac{100}{100}=\frac{200}{1,000,000}\rightarrow$$

$$\frac{9}{100,000}\,\frac{10}{10}=\frac{90}{1,000,000}\rightarrow$$

$$\frac{\frac{13}{100,000}=\frac{13}{1,000,000}}{\frac{303}{1,000,000}}$$

16. A proportion should be written, but first pick a letter to represent the missing number pf apples. Don't use x or y because these letters are used to stand for money. Also, change x dollars to 100x cents.

$$\frac{7\text{ apples}}{a\text{ apples}}=\frac{y\text{ apple}}{100x\text{ cent}}100x\text{ cents.}$$

Cross-multiply and solve for a. ay = 700x

$$a=\frac{700x}{y}$$

17. Start with Stave's age.

Steve = 4a.
Bob's age = 4a + 3
Dave's age = 2(4a + 3) = 8a + 6

18. Remember that it is not necessary to find the exact values of letters if you are comparing their relative sizes. Multiply both sides of the given equation by 10, the LCD, giving $32c = 25b$. Divide by

$$25c.\ \frac{32}{25} = \frac{b}{c}$$

19. Find the sum of the heights and divide by 5. let x = the height of the fifth man

$$68 = \frac{70+3(67)+x}{5}\ 340 = 70 + 201 + x$$

$340 = 271 + x$
$x = 69$ inches

20. Add $-t$ to both sides, resulting in $p^2 + 11p - t = 0$, a quadratic equation. If the values of p have to be negative, as stated, the two factors of the trinomial $p^2 + 11p - t$ must each be p plus some number (because p plus a number = 0 will become p = a negative number). Therefore, the third term, $-t$, must represent a positive number, which means that t is negative (the negative of a negative is positive). There are two choices that are negative. If t were -11, the trinomial would be $p^2 + 11p + 11$, which cannot be factored into integers. If t equals -18, the trinomial would be $p^2 + 11p + 18$, which factors to $(p + 2)(p + 9)$. When each factor is set equal to 0, p will be -2 and -9.