

SECTION II

Analytical Reasoning

Each analytical reasoning question is designed to test the analytical skills. Usually, each logical reasoning question is a logical puzzle, based on given conditions. These questions have only one correct answer, which is asked to be selected.

Analytical reasoning questions consist of groups of four or five questions. At begging, a short passage followed by a set of conditions is given. Sometimes, graphs and tables are given instead of passage. To understand this type of questions, we define some logical terms and their applications with examples.

Preposition:

A declarative statement which may be true or false but not both is called a preposition. For example, the statement $x = y$ can be either true or false and there could not be any other possibility.

Symbols Used in Logic:

Capital letters are used to represent specific statements.

For Example:

A: A triangle has four angles.

B: $\sqrt{17}$ is an irrational number.

C: $20 + 18 = 36$

Solution: B is true; A and C are false.

Lower case letters, such as p, q are used to refer the preposition that are not specific. The following table is a brief list of the symbols which can be used:

Symbol	Meaning	Expression	How to read
\sim	not	$\sim p$	not p ; negation of p
\wedge	and	$p \wedge q$	p and q
\vee	or	$p \vee q$	p or q
\rightarrow	If...then, implies	$p \rightarrow q$	If p then q p implies q
\leftrightarrow	If and only if is equivalent to	$p \leftrightarrow q$	p if and only if q p is equivalent to q

Explanation:**1. Negation:**

The negation of a statement has the opposite truth value of the statement. The symbol for negation is \sim , thus, $\sim p$ is read: "not p ", "the negation of p " or "it is false that p ". The adjoining table called the truth table, gives the possible truth values of p and $\sim p$. Thus

p	$\sim p$
T	F
F	T

The negation of true statement is a false statement.

The negation of false statement is a true statement.

2. Conjunction:

Conjunction of two statements p and q is denoted by $p \wedge q$ (p and q) and it is considered to be true only if both of its components are true.

3. Disjunction:

Disjunction of p and q is p or q . It is symbolically written $p \vee q$. The disjunction $p \vee q$ is considered to be true when at least one of the components p and q is true, and false if both components are false.

4. Implication or Conditional:

A compound statement of the form if p then q , also written p implies q , is called an implication or conditional. In the conditional statement below, p is called the antecedent or hypothesis, and q is called the consequent.

$$p \rightarrow q$$

Examples:

- (i) If he works hard, then he will get through.
- (ii) If he wastes time, then he will fail.
- (iii) If $a = 3$, then $a^2 = 9$.
- (iv) If $a = -3$, then $a^2 = 9$.

5. Biconditional:

If $p \rightarrow q$ and $q \rightarrow p$, then the proposition p and q are said to be biconditional and shortly written as " p if and only if q ". Symbolically, it is written as:

$$(p \rightarrow q) \wedge (q \rightarrow p) = p \leftrightarrow q$$

Examples:

- (i) If an triangle is an isosceles triangle then, its two angles are congruent.
- (ii) If $x = y$, then $ax = ay$, where a , x and y are real numbers and $a \neq 0$.

Note: $p \leftrightarrow q$ is true only when both p and q are true or both p and q are false.

Model Example:

At All-Pakistan Weightlifting Championship, seven college athletes... M , N , O , P , Q , R and S ... are being weighted. In order to make categories, the coach has given the following information:

- (i) Each athlete has not exactly the same weight as another athlete.
- (ii) R is heavier than S , but lighter than O .
- (iii) P is heavier than S .
- (iv) Both M and N are heavier than O .

1. Which of the following could not be true?

- (A) M is the heaviest.
- (B) N is the heaviest.
- (C) P is the heaviest.
- (D) More than three athletes are heavier than R .
- (E) More than three athletes are lighter than R .

2. Which of the following, if true, would be sufficient to determine the lightest athlete?

- (A) P is the heaviest.
- (B) P is lighter than R .
- (C) R is heavier than Q .
- (D) Q is heavier than R .

(E) Five candidates are lighter than M .

3. If Q is heavier than M , how many different ranking by weight, of the athletes are possible?

- (A) 1 (B) 2
(C) 3 (D) 5 (E) 7

4. If O is heavier than P , which of the following cannot be true?

- (A) P 's weight is equal to the average of R 's weight and S 's weight.
(B) Q 's weight is equal to the average of R 's weight and S 's weight.
(C) P 's weight is equal to the average of M 's weight and N 's weight.
(D) Q is the second lightest.

Solution: First of all, we decompose the given information, symbolically:

- (i) $M \neq N \neq O \neq P \neq Q \neq R \neq S$
(ii) $R > S, R < O$
(iii) $P > S$
(iv) $M > O \wedge N > O$

1. (E) From above, we find the exact answer.

Take false option, " M is the heaviest"

From the given information, $M > O$ and $N > O$, also $R < O$. Thus option A must be true.

Now, take second option, " N is the heaviest"

From above conclusion, it may be possible that N is the heaviest.

Third option is, " P is the heaviest". We solve it symbolically:

$$\therefore R > S \wedge R < O \Rightarrow S < R < O$$

$$\text{also } P > S \Rightarrow P \wedge R > S$$

$$\text{But } M \wedge N > O \Rightarrow R < M \wedge N$$

From above, we cannot deduct that option " P is the heaviest" is wrong.

In third option, more than three athletes are heavier than R . Since, from the given information, $R > S$ But $R < O$ also $M > O$ and $N > O \Rightarrow M, N > O$.

$\Rightarrow M, N, O > R$. Because the information about Q not given, therefore, Q may be greater than R . This option may be true. Now we take the option E, according to this option, more than three athletes are lighter than R . Here we analyze it. Information (i), (ii) and (iv) can be written symbolically, as

$$M, N > O > R > S$$

Since, from above the three circled athletes are heavier than athlete R , and athlete R is heavier than only S . Suppose if the remaining two athletes P and Q are lighter than R . In this condition only three athletes are lighter than athlete R . Thus it is impossible that more than three athletes are lighter than R .

Thus the answer is "Choice E".

2. (D) The first option is " P is the heaviest". Suppose P is the heaviest, then $P > S$, also $R > S$

$$\Rightarrow P > R, \text{ But } R < O \Rightarrow P > O \text{ (given)}$$

$$\Rightarrow P > M, N, \text{ from this evaluation we cannot determine the lightest.}$$

Take a look at the second option, " P is lighter than R ", Suppose P is lighter than R , the $R > P \wedge R > S$

$$\Rightarrow R > P \wedge S.$$

This is also insufficient to find the lightest weight.

In option C, R is heavier than Q , if R is heavier than Q , then, symbolically

$$R > Q \wedge R > S. \therefore O > R \Rightarrow O > R, Q, S$$

Hence it is not possible to find the lightest weight. In option D, Q is heavier than R . If this is true, then

$$Q > R \wedge R > S \Rightarrow Q > R > S$$

But $R < O$, therefore, $Q > O > R > S$, also

$$\begin{aligned} M, N > O &\Rightarrow Q > M, N > O > R > S \\ &\Rightarrow Q > M, N > O > R, P > S \end{aligned}$$

Thus, we can find the lightest weight after accepting this option.

Hence, the correct choice is choice D.

3. (C) If Q is heavier than M , then

$$Q > M \Rightarrow Q > O$$

$$O > R \Rightarrow Q > O > R$$

$$R > S \Rightarrow Q > O > R > S$$

Thus, the three categories are possible, which are

$$(i) \quad Q > O \quad (ii) \quad O > R \quad (iii) \quad R > S$$

4. If O is heavier than P , then according to the first option which says that, P 's weight is equal to the average of R 's weight and S 's weight.

$$P > S \wedge O > P \Rightarrow O > S, \text{ i.e., } O > P > S \text{ also } R < O. \text{ But } R > S \Rightarrow O > R > S$$

Multiple Choice Questions (MCQs)

Questions 1-3:

A chemist is preparing a nutriment using eight different vitamins and minerals... A, B, C, D, E, H, F (Ferric), and Z (Zinc). According to the recipes, the following requirements apply to the use of ingredients:

- (i) If B is used, both C and Z must also be used.
- (ii) E and H must always be used together.
- (iii) If C is used, at least two of A, B and F must also be used.
- (iv) C and H cannot be used together.
- (v) E, F and Z cannot all be used in a same nutriment.
- (vi) A, D and Z cannot all be used in the same nutriment.

Question 1:

1. Which of the following is a suitable combination of vitamins and minerals for a nutriment?

(A) A, B, C, F	(B) D, E, H, Z
(C) A, D, E, Z	(D) C, D, E, F (E) E, H, F, Z
2. Which of the following cannot be included in a nutriment that contains E ?

(A) B	(B) D
(C) H	(D) F (E) Z
3. By the addition of exactly one more mineral, which of the following could make an acceptable combination of vitamins and minerals?

(A) A, D, Z	(B) B, H, E
(C) C, D, H	(D) C, E, Z (E) E, H, F

Questions 4-6: A railway track from Lahore to Islamabad consists of six main stations, P, Q, R, X, Y and Z . Trains run only according to the following condition:

- (i) From P to Q
- (ii) From Q to P and from Q to R
- (iii) From R to X
- (iv) From X to Q and from X to Y
- (v) From Z to P , from Z to Y and from Z to R .
- (vi) From Y to X .
- (vii) It is possible to transfer a station for another train.

4. The complete and accurate listing of the stations from which it is possible to reach R with exactly one transfer, is:
 (A) P and Q (B) P and X
 (C) X and Y (D) X and Z (E) X, P and Z
5. The greatest number of stations that can be visited without visiting any station more than once, is:
 (A) 4 (B) 5
 (C) 6 (D) 3 (E) 2
6. The trip which requires greatest number of transfers, is:
 (A) P to R (B) Q to Y
 (C) Z to Q (D) Z to R (E) Z to Y
7. A group of dog lovers declare that the principal virtue of the dog is its general friendliness towards all people. But, another group of cat lovers declare that the principal virtue of the cat is its peculiar friendliness towards its provider.
 Which of the following is true of the claims of both dogs and cats lovers?
 (A) Animals have not a sense to understand human behaviour.
 (B) Groups of animal lovers are friendly.
 (C) Friendliness is a virtue.
 (D) They apply the same standard.
 (E) Uncommon virtue of friendliness.
8. Rizwan was born in 1956, and so in 1965 he was nine years old. If we per use this example, it is clear that the last two digits of a person's birth will be the same as the last two digits of the year of that person's ninth birthday, except that the position of the digits will be reversed.
 Identify, which is the best criticism of the assertions made above.
 (A) The generalization is valid only for those, in which last digit of their birth years is greater than four.
 (B) The generalization is applicable only for those birth years that do not end in two zeroes.
 (C) This example is not best illustration of the fact.
 (D) The generalization is valid only for those birth years in which the difference of the last two digits is one.
 (E) The generalization is valid only for those birth years that ends with 6.

Questions 9-10: At IOWA University, Students of Economics must complete a total of twelve courses selected from three different parts of the syllabus...comparative economics, environmental economics, and regional economics...in order to graduate, the students must meet the following course distribution requirements:

- (i) At least six of the required twelve courses must be from the environmental economics.
 (ii) At least five of the required twelve courses must be from comparative economics and regional economics, with at least one, but no more than three, selected from comparative economics.
9. The minimum number of regional economics courses required to fulfil the course distribution requirements is
 (A) 1 (B) 3
 (C) 5 (D) 2 (E) 4
10. If the student has completed six environmental economics courses and one regional economics course, the possible groups of courses to fulfil the course distribution requirements must include at least:
 (A) One Environmental Economics Course
 (B) Three Regional Economics Courses
 (C) One Regional Economics Course

- (D) Two Comparative Economics Courses
 (E) Three Comparative Economics Courses

Questions 11-15: Six candidates.....Ali, Amin, Omer, Hamza, Saleem and Osama are being interviewed for a job. The interview will take place over four consecutive days, starting on Thursday. Each candidate will have only one interview. The day on which the different candidates will interview must conform to the following conditions:

- (i) At least one interview will take place each day.
 (ii) No more than two interviews will take place on any day.
 (iii) No more than three interviews will take place on any two consecutive days.
 (iv) Ali's interview must take place on Saturday.
 (v) Amin's interview must take place on the same day with another interview.
 (vi) Saleem's interview must take place on the day before Osama's interview.
 (vii) Omer's interview must take place on a day after Hamza's interview.

11. *If only one interview takes place on Thursday which candidate could have that interview?*

- (A) Ali (B) Amin
 (C) Omer (D) Saleem
 (E) Osama

12. *If the director decides to take two interviews on Thursday and two on Sunday, how many candidates would be eligible to interview on Friday?*

- (A) 1 (B) 2
 (C) 3 (D) 4
 (E) 5

13. *If Hamza and Osama have their interviews on the same day which of the following must be true?*

- (A) Hamza's interview will take place on Thursday.
 (B) Saleem's interview will take place on Friday.
 (C) Amin's interview will take place on Saturday.
 (D) Osama's interview will take place on Saturday.
 (E) Amin's interview will take place on Sunday.

Explanatory Answers

Q1. (B) Take first option, which is A, B, C, B. By the first condition, $B \rightarrow (C \wedge Z)$. But D has not given, so this combination is not suitable. Now take the second combination, which is D, E, H, Z. This option is a correct choice, since it satisfies all the given conditions.

The third option is rejected due to the sixth condition. According to this condition, A, D and Z cannot be used altogether in a nutriment. Since H is not present with E, thus we reject option D due to second condition. The fourth condition is rejected due to fifth condition. So the correct answer is choice B.

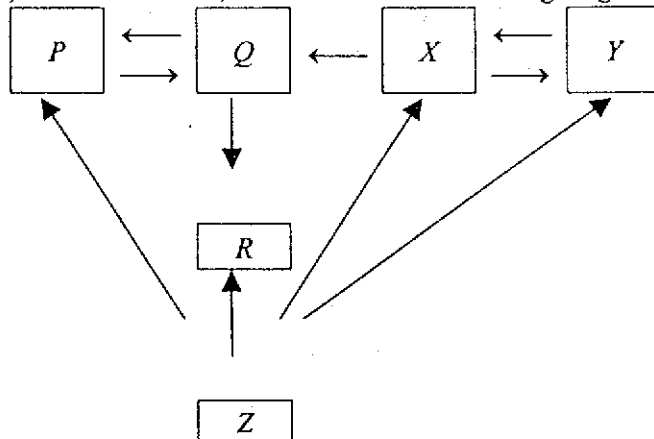
Q2. (A) If nutriment contains E, then according to second condition, $E \leftrightarrow H$. But according to fourth condition $H \rightarrow \sim C$.

If we include B in the nutriment, then according to first condition $B \rightarrow (C \wedge Z)$ i.e., C and Z must also be used. But if we use C then H must be absent in the nutriment. But E and H must always be used together. Thus we cannot include B in the presence of E. The correct choice is option A.

Q3. (E) Since A, D and Z cannot all be used in a nutriment, so we reject option A. The option B is, B, H, E, Since, C and Z must also be used with B (first condition) so we reject option B. Since C and H cannot

be used together, so we reject option C. Since *E* and *H* must always be used together, so we reject option D. So the correct choice is option E. Which satisfies all the given conditions.

Q4. (B) For convenience, here we draw the following diagram:



It is clear from the diagram that from *P* to *R*, there is one transfer, but from *Q* to *R*, there is not any transfer, the track is direct. Thus, the option A is not possible. Now, from *P* to *R* there is only one transfer and same is the case from *X* to *R*. Thus option B is the correct choice.

Q5. (C) Here, the following list shows the track of trains that can be visited without visiting any station more than once:

$R \rightarrow X$

$Q \rightarrow R$

$Z \rightarrow R$

$Z \rightarrow P$

$Z \rightarrow Y$

$Q \rightarrow X$

The stations used in above tracks on

R, X, Q, Z, Y, P

Thus option (C) is the correct choice.

Q6. (B) From $P \rightarrow R$, there is only one transfer, i.e., $P \rightarrow Q$. From $Q \rightarrow Y$, there are two transfers, i.e., $Q \rightarrow R$ and $R \rightarrow X$. From *Z* to *Q*, there is only one transfer i.e., $Z \rightarrow P$. From *Z* to *R*, there is not any transfer. At last, *Z* to *Y* there is not any transfer. Thus the correct choice is option B, i.e., 2.

Q7. (C) This question asks you to identify the main point of both dog and cat lovers. According to first option, animals have not a sense to understand human behaviour. Since animals have a great sense to understand human behaviour, like, love, anger, etc., so we reject this option. It is not sufficient that only the pet lovers should be friendly, the animals (cats, dogs etc.) should also be friendly. Thus, option B is not a the best choice. The best choice is option C, since it focusses upon the characteristic of animals and their lovers, which they require.

Q8. (D) The given generalization is true only if the difference of the last two digits of birth year is 1. Suppose a man was born in 1959 and so in 1995, he was 36 years old. This is a same example as given, the only difference is the, difference of the last two digits of birth year which is not 1. Now, suppose that a man was born in 1956 and so in 1965 he was 9 years old. This is so because the difference of the last two digits of the birth year is 1 i.e., $(6 - 5 = 1)$. Thus the correct answer is the option D.

Q9. (D) Since at least five courses must be from comparative and regional economics, with at least one, but no more than three selected from comparative economics. Thus if we select three courses from comparative economics, we must select two courses from regional economics group. Thus the correct choice is option D.

Q10(C)

<u>C</u>	<u>E</u>	<u>R</u>	
	6	1	
③	①	1	
3	+ 7	+ 2	= 12

From above, we took 1 regional economics which is least. Thus if we take 3 from comparative and one from environmental economic, it will fulfil the requirement.

Q(11).D

Here, first, we express the given conditions symbolically. Name of candidates Ali, Amin, Omer, Hamza, Saleem and Osama.

Days: Thu, Fri, Sat, Sun

Each days condition: 1 or 2 interviews

2 consecutive days: 2 or 3 interviews

Ali's interview = Saturday

According to condition (iv), Ali's interview will take place on Saturday. According to condition (v), Amin's interview must take place on the same day, an another interview. Thus choices C and E are impossible. Since Omer's interview must take place on a day after Hamza's interview and Osama's interview must take place on a day after Saleem's interview, cannot take place on Thursday. Thus the correct choice is choice D.

Q(12). According to third condition, no more than three interviews will on any two consecutive days. Thus only one interview can take place on Friday. Therefore according to fifth condition, it cannot be Amin. Since Ali's interview is on Saturday. Thus, it cannot be Ali. Any of the other four candidates could be interviewed on Friday as indicated in the following points:

(i)	Thu Hamza/Saleem	Fri Omer	Sat Ali	Sun Amin/Osama
(ii)	Thu Amin/Saleem	Fri Hamza	Sat Ali	Sun Omer/Osama
(iii)	Thu Amin/Hamza	Fri Saleem	Sat Ali	Sun Omer/Osama
(iv)	Thu Hamza/Saleem	Fri Osama	Sat Ali	Sun Amin/Omer

Thus the correct choice is D.

Q13. (E) The possible schedule to fulfil this condition is:

Thu	Fri	Sat	Sun
Saleem	Hamza/Osama	Ali	Amin/Omer

Thus the correct choice is choice (E).
