Part 2 Algebra

Chapter 1

POLYNOMIALS

Polynomial:

A sum of finite number of monomials is called a polynomial. Each monomial is called a term of the polynomial.

Monomial:

A monomial is a variable, or a constant, or a product of constant and one or more variables, with the variables having only non-negative integer in exponents.

Example:

$3x^2y$, $-5xy$, and $-7xy^3$ are monomials.

The algebraic expression

$$4y^{-3} \text{ and } \frac{3}{y}$$

are not monomials, because these expressions have not non-negative integer in exponent, and cannot be written as a product of a constant and a variable with a non-negative integer exponent.

Degree of Monomial:

In any monomial the sum of the exponents of the variables is called the degree of monomial.

Example:

What are the degrees of the monomials

$$-3x^2y,\ 7x^3y,\ -18xy^2$$

Solution:

In algebraic expression $-3x^2y$, the degree of the monomial is 3, because the exponents of x and y are 2 and 1 respectively therefore their sum is $(2 + 1 = 3)$. Similarly the degree of the expressions $7x^3y$ and $-18xy^2$ are 4 and 3 respectively.

Note:

In monomial, the constant is called the numerical coefficient or simply the coefficient of the monomial.
$-3x^2y$, $7x^3y$ and $-18xy^2$ are monomials of coefficient -3, 7, and -18 respectively.

Multiplication of Monomials:

The process of multiplication is illustrated in the following example:

Example:

What is the value of $-5xy^2$, when $x = -2$ and $y = -3$

Solution:

First of all write the coefficient of the monomial, then substitute the value of x and y in monomial. Then evaluate:

$$-5(-2)(-3)^2 = -5(-2)(9) = 90$$

Polynomial:

A sum of a finite number of monomials is called a polynomial. Each monomial in a polynomial is called a term of the polynomial.

Degree of a Polynomial:

The degree of a polynomial is the largest degree of the terms in the polynomial.

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What are like terms in a Polynomial?

Terms of polynomial that have exactly the same variables raised to the same powers are called like terms.

Example:

Each of the following is a polynomial:

$3x^2 + 5, 3x^2, 2x^2 + 9x - 12, -4x^2, 7x^2y, 9x^2 - 8$

Explanation:

In above lists of polynomials; $3x^2 + 5$, and $9x^2 - 8$ are called binomial because each polynomial has two terms; the polynomials $3x^2$, $-4x^2$ and $7x^2y$ are monomials; the polynomial $2x^2 + 9x - 12$ is called trinomial because it has three terms. In above list of polynomials $3x^2$ and $-4x^2$ are like terms, similarly $3x^2 + 5$ and $9x^2 -8$ are like terms, because they have exactly the same variables raised to the same power.

General Form of a Polynomial:

In a single variable x, the general form of a polynomial of degree n is

$a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots\ldots + a_2x^2 + a_1x + a_0$ where n is a non negative integer, and $a_n \neq 0$

Combination of Like Terms in a Polynomial:

The polynomial $4x^2 + 3x + 6x + x^2 + x$ is equivalent to the binomial $5x^2 + 10x$, because like terms are combined in a polynomial. The process of combination is illustrated as:

$4x^2 + x^2 = 5x^2$ and $3x + 6x + x = 10x$

$\therefore \quad 4x^2 + 3x + 6x + x^2 + x = 5x^2 + 10x$

Remember:

Only like terms in a polynomial can be combined.

Arithmetic Operations on Polynomials:

We use usual law of arithmetic, to add subtract, multiply and divide polynomials.

Addition and Subtraction:

Polynomials are added or subtracted by combining like terms.

Example:

$(2x^3 + 3x^2 + 7x + 6) + (4x^2 + 3x - 2) - (5x^2 + 4x)$

$= 2x^3 + (3x^2 + 4x^2 - 5x^2) + (7x + 3x - 4x) + (6 - 2)$

$= 2x^3 + 2x^2 + 6x + 4$

The rules for adding like terms are:

Rule 1:

If all the terms are positive in a polynomial, then add their coefficients.

Example:

Find the value of $8x^2 + 2x^2 + 7x^2$

Solution:

Here we have to increase 8 like things by 2 and 7 like things of the same kind, and aggregate is 17 of each thing.

Rule 2:

If all the terms in a polynomial are negative add the coefficient numerically and prefix the minus sign to the sum.

Example:

What is the sum of $-4x$, $-x$, $-3x$ and $-7x$

Solution:

In this example the word sum indicates the aggregate of 4 subtractive quantities of like terms. In this case we have to take away successively 4, 1, 3 and 7 like things, therefore the result is the same as taking away 15(4 +

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1 + 3 + 7) such things in the aggregate.

$\therefore$ The sum of $-4x, -x, -3x, -7x$ is $-15x$.

Rule 3:

If all the terms have not same sign, add together separately the coefficient of all the negative terms and the coefficient of all the positive terms: Then find the difference of those two results, preceded by the sign of the greater, will give the coefficient of the sum required.

Example:

Find the sum of $12x^2 - 3x^2 + 15x^2 - 17x^2$

Solution:

The sum of the coefficient of positive terms is $12 + 15 = 27$

The sum of the coefficient of negative terms is $3 + 17 = 20$

The difference of these is 7, and the sign of the greater is positive: hence the required sum is $7x^2$.

Multiplication of Monomials:

To multiply two simple monomials together, first multiply their coefficients together and prefix their product to the product of the different letters, giving to each letter on index equal to the sum of the indices that letter has in the separate factors.

Example:

What is the product of $5x^2y^3$ and $-3xy^2$

Solution:

$$(5x^2y^3)(-3xy^2) = (5)(-3)(x^2 \times x)(y^3 \times y^2)$$
$$= -15x^3y^5$$

Note:

The product of a monomial by any polynomial is the algebraic sum of the partial products of each term of the polynomial by that monomial.

Example:

Find the product of $2xy^2$ and $(4x^2 + 3y + 7xy)$

Solution:

$$2xy^2(4x + 3y + 7xy) = 8x^3y^2 + 6xy^3 + 14x^2y^3$$

Multiplication of two Binomials:

The procedure of multiplication of two binomials is illustrated as:

1. Multiply each term of the first binomial by each term of the second.
2. When the terms multiplied together have like signs, prefix to the product the sign +, when unlike prefix –.
3. The algebraical sum of the partial products so formed gives the complete product.

Example:

Multiply $(x + 3)$ by $(x - 5)$

Solution: $(x + 3)(x - 5) = x(x - 5) + 3(x - 5)$

$$= x^2 - 5x + 3x - 15$$
$$= x^2 - 2x - 15$$

Example:

Find the value of $(x + 2)(x - 3) - (x + 4)(x - 5)$

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Solution:

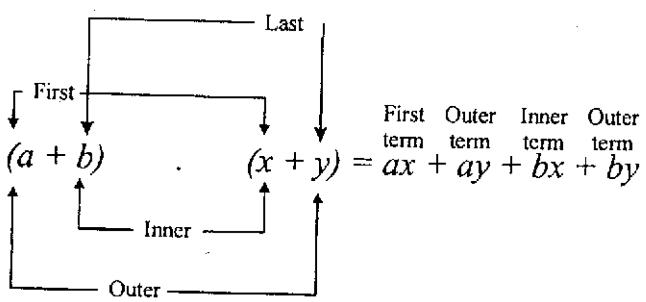
First of all, multiply both pairs of binomials separately, then subtract the second result from the first.

$$(x+2)(x-3) = x(x-3)+2(x-3) = x^2-3x+2x-6$$
$$= x^2-x-6$$
$$(x+4)(x-5) = x(x-5)+4(x-5) = x^2-5x+4x-20$$
$$= x^2-x-20$$

Suotracting: $(x^2-x-6)-(x^2-x-20) = x^2-x-6-x^2+x+20$
$$= 14$$

FOIL Method:

The product of the two binomials can be computed by the FOIL method. This method is illustrated in the following example



Example:

Find the product of $(2x-3)$ and $(4x+2)$ using FOIL Method

$$(2x-3)(4x+2) = \overset{\text{First}}{(2x)(4x)} + \overset{\text{Outer}}{(2x)(2)} + \overset{\text{Inner}}{(-3)(4x)} + \overset{\text{Last}}{(-3)(2)}$$
$$= 8x^2+4x-12x-6$$
$$= 8x^2-8x-6$$

Important Binomial Formulas:

Following are most important binomial products, those occur frequently in algebra.

1. $(x+y)(x-y) = x^2-y^2$
2. $(x+y)^2 = x^2+2xy+y^2$
3. $(x-y)^2 = x^2-2xy+y^2$

Example:

Find each of the following products:

a) $(2a+3)(2a-3)$

b) $(a-5b)^2$

Solution:

a) Using formula $(x+y)(x-y) = x^2-y^2$

Here $(2a+3)(2a-3) = (2a)^2-(3)^2 = 4a^2-9$

b) Using formula $(x-y)^2 = x^2-2xy+y^2$

$(a-5b)^2 = (a)^2-2(a)(5b)+(5b)^2 = a^2-10ab+25b^2$

Example:

Given $x+y=5$, and $x^2-y^2=10$, what is the value of $x-y$?

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Solution:

Using the fact $(x + y)(x - y) = x^2 - y^2$

$$(5)(x - y) = 10$$

$$x - y = \frac{10}{5}$$

$$\Rightarrow x - y = 2$$

Example:

Find the value of xy, when $(x + y)^2 = 25$ and $x^2 + y^2 = 3$

Solution:

We know $(x + y)^2 = x^2 + 2xy + y^2$

We can write $(x + y)^2 = (x^2 + y^2) + 2xy$

Substituting the value of $(x + y)^2$, and $x^2 + y^2$ in above

$$25 = (3) + 2xy$$

$$22 = 2xy$$

$$xy = \frac{22}{2}$$

$$xy = 11$$

Division of Polynomial by Monomial:

Division is the inverse of multiplication. The object of division is to find out the quantity called quotient.

Thus $\frac{\text{divided}}{\text{divisor}} = \text{quotient}$

To divide a monomial by a monomial, use distributive law, the index of each letter in the quotient is obtained by subtracting the index of that letter in the divisor from that in the divided. To the result so obtained prefix its proper sign the quotient of the divided by that of divisor.

To divide a polynomial by a monomial, divide each term separately by that monomial, and take the algebraic sum of the partial quotient so obtained.

Example 1:

What is the quotient when $-4x^2y$ is divided by $2x$.

Solution:

The quotient $= \frac{-4x^2y}{2x} = -2xy$

Example 2:

Divide $12x^3 - 6x^2 - 9x$ by $3x$

Solution:

$$\frac{12x^3}{3x} - \frac{6x^2}{3x} - \frac{9x}{3x} = 4x^2 - 2x - 3$$

Evaluating a Polynomial:

To evaluate a polynomial, substitute the given value(s) for the variable(s) and then perform the given operation.

Example:

If $x = 3$, $y = -7$ and $z = -2$, find the value of $x^2 - 26y + 17z$

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Solution:

$$x^2 - 26y + 17z = (3)^2 - (26)(-7) + 17(-2)$$
$$= 9 + 182 - 34 = 157$$

Factorising Polynomials:

Writing a polynomial as a product of polynomials of lower degree is called factoring.

When each of the terms which compose a polynomial is divisible by a common factor, the polynomial may be simplified by dividing each term separately by this factor, and enclosing the quotient within brackets; the common factor being placed outside as a coefficient.

Example 1:

Resolve into factors $4x^2 - 20x$

Solution:

The terms of the polynomials $4x^2 - 20x$ have a common factor $4x$;

$$\therefore \quad 4x^2 - 20x = 4x(x - 5)$$

Example 2:

Resolve into factors $x^2 - sx + tx - st$

Solution:

We see that the first two terms contain a common factor x, and the last two terms a common factor t, we enclose the first two terms in one bracket, and the last in another. Thus,

$$x^2 - sx + tx - st = (x^2 - sx) + (tx - st)$$
$$= x(x - s) + t(x - s);\text{ take } (x - s) \text{ common}$$
$$= (x - s)(x + t)$$

Factorising Quadratic Trinomials:

Some trinomials of the form $x^2 + bx + c$ can be factorized by trial and error procedure. This method is the reverse of the FOIL method. This is illustrated in the following example.

Example:

Consider the following binomial expansion:

$$(x + 5)(x + 6) = x(x + 6) + 5(x + 6)$$
$$= x^2 + 6x + 5x + 30$$
$$\therefore \quad x^2 + 11x + 30 = (x + 5)(x + 6)$$

Notice at $11 = 5 + 6$ and $30 = 5 \times 6$

This result can be used to factorize trinomials? For example, to factorize the trinomial $x^2 + 7x + 12$ we need to find two numbers so that:

Product $= 12$ and sum $= 7$

The two numbers are 4 and 3

$4 \times 3 = 12$ and $4 + 3 = 7$

$\therefore x^2 + 7x + 12 = (x + 4)(x + 3)$

Example:

Factorize *i)* $x^2 + 7x - 18$

ii) $m^2 - 9m + 14$

Solution:

i) $x^2 + 7x - 18$

Product $= 9 \times (-2) = -18$

Sum $= 9 + (-2) = 7$

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$\therefore \quad x^2 + 7x - 18 \quad = (x+9)(x-2)$

ii) $m^2 - 9m + 14$

Product $= (-7)(-2) = 14$

Sum $= (-7) + (-2) = -9$

$\therefore \quad m^2 - 9m + 14 \quad = (x-7)(x-2)$

Example:

Find the value of $(10001)^2$

Solution:

$$
\begin{aligned}
(10001)^2 &= (10000+1)^2 \\
&= (10000)^2 + 2(10000)(1) + (1)^2 \\
&= 100000000 + 20000 + 1 \\
&= 100020001
\end{aligned}
$$

Example:

What is the value of $(9999)^2$

Solution:

$$
\begin{aligned}
(9999)^2 &= (10000-1)^2 \\
&= (10000)^2 - 2(10000)(1) + (1)^2 \\
&= 100000000 - 20000 + 1 \\
&= 99980001
\end{aligned}
$$

Algebraic Fraction:

An expression which has a variable in the denominator, is called an algebraic expression. Algebraic fractions are added and subtracted using the same method as for arithmetic fractions. The denominator must be the same before these operations can be carried out.

Example:

Simply *i)* $\frac{3x}{4} + \frac{x}{6}$

ii) $\frac{x+3}{8} - \frac{x-4}{4}$

Solution:

i) $\frac{3x}{4} + \frac{x}{6}$

$= \frac{9x+2x}{12}$ (lowest common denominator is 12)

$= \frac{11x}{12}$

ii) $\frac{x+3}{8} - \frac{x-4}{4}$

$= \frac{(x+3)-2(x-4)}{8} = \frac{(x+3)-(2x-8)}{8} = \frac{x+3-2x+8}{8}$

$= \frac{11-2x}{8}$

Multiplication and Division of Algebraic Fractions:

Algebraic fractions are multiplied and divided using the same method as for arithmetic fractions.

Example:

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Simply *i)* $\frac{x}{15}\times\frac{9}{y}$

ii) $\frac{a^2}{2}\div\frac{a^3}{4}$

Solution:

i) $\frac{x}{15}\times\frac{9}{y}=\frac{3x}{5y}$

ii) $\frac{a^2}{2}\div\frac{a^3}{4}$

$=\frac{a^2}{2}\times\frac{4}{a^3}$

$=\frac{2}{a}$

Example:

Simplify $\frac{9x^3-x}{(3x-1)(9x-3)}$, also find the value when $x=39$

Solution:

$$\frac{9x^3-x}{(3x-1)(9x-3)}=\frac{x(9x^2-1)}{(3x-1)3(3x+1)}=\frac{x(3x-1)(3x+1)}{3(3x-1)(3x+1)}$$

using $a^2-b^2=(a+b)(a-b)$

$$=\frac{x}{3}$$

Now when $x=39$

$$=\frac{39}{3}=13$$

Example:

Find the value of $\frac{a^2-b^2}{(a-b)}$, when $a=2.9$ and $b=9.1$

Solution:

$\frac{a^2-b^2}{a-b}$

$=\frac{(a-b)(a+b)}{(a-b)}=a+b$

$=(2.9+9.1) \quad =12$

Example:

Simply $\frac{e}{4c}\div\frac{eb}{ac}$

Solution:

$\frac{e}{4c}\times\frac{ac}{eb}=\frac{a}{4b}$

Example:

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What is the value of a and b, If $a^2 - b^2 = 36$ and $a + b = 6$?

Solution:

$$a^2 - b^2 = 36 \Rightarrow (a-b)(a+b) = 36$$

$\Rightarrow$ $(a-b)(6) = 36$ as $a + b = 6$

$\therefore$ $a - b = 6$ ....(1)

adding $a + b = 6$ ....(2)

$2a = 12$

$\Rightarrow$ $a = 6$

Substituting $a = 6$ in (2) we have

$6 + b = 6,\quad b = 0$

$\therefore$ $a = 6,\quad b = 0$

Example:

Find the value of xy, when $x^2 + y^2 = 58$ and $x^2 - y^2 = 42$

Solution:

adding

$$(x^2 + y^2) + (x^2 - y^2) = 2x^2$$

$$(58) + (42) = 2x^2 \Rightarrow 2x^2 = 50 \Rightarrow x = \pm 5$$

Substituting $x = \pm 5$ in $x^2 + y^2 = 29$

$$(5)^2 + y^2 = 29 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$$

$$\Rightarrow \boxed{xy = (5)(2) = 10}$$

Multiple Choice Questions (MCQs)

Q1. **If $x = 235$ and $y = 117$, then $\dfrac{x^2 - y^2}{x - y} = ?$**

(A) 118 (B) 100
(C) 115 (D) 352

Q2. **If $x^2 - y^2 = 16$ and $x^2 + y^2 = 34$, which of the following could be the value of xy?**

I 15 II −15 III 45

(A) only I (B) only II
(C) I and II only (D) III only

Q3. **The average of the polynomials, $2x^2 + 5x - 6$, $5x^2 - 5x - 6$ and $30 - 7x^2$ is:**

(A) 14 (B) 18
(C) 6 (D) $5x$

Q4. **What is the value of $x^2 + 14x + 24$, when $x = 854$?**

(A) 1000 (B) 100,000
(C) 741,296 (D) 742,398

Q5. **If $x^2 + y^2 = 9$ and $(x - y)^2 = 3$, what is the value of xy?**

(A) 16 (B) 9
(C) 6 (D) 3

Q6. **The value of $(5x + 6)(x + 12) - (5x - 6)(x + 3)$ is:**

(A) $2(5x^2 + 9x)$ (B) 14
(C) 4 (D) 22

Q7. **If $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$ and $xy = z$, what is the average of x and y?**

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(A) $\frac{1}{2}$ (B) 1

(C) $\frac{x+y+z}{3}$ (D) $\frac{x+y+z}{2}$

Q8. **If $p^2 - q^2 = 48$ and $p - q = 12$, what is the average of p and q?**

(A) 4 (B) 6

(C) 2 (D) 12

Q9. $\left(\frac{1}{x}+x\right)^2 - \left(\frac{1}{x}-x\right)^2 = ?$

(A) 4 (B) 2

(C) $2\left(\frac{1}{x^2}+x^2\right)$ (D) $2\left(\frac{1+x^2}{x^2}\right)$

Q10. **If $\left(x+\frac{1}{x}\right) = 81$, then $x^2+\frac{1}{x^2} = ?$**

(A) 6563 (B) 6561

(C) 6559 (D) 79

Q11. **If $x < 0$, then $-3x^2$ is:**

(A) less than $(-3x)^2$ (B) greater than $(-3x)^2$

(C) equal to $(-3x)^2$

(D) greater than or equal to $(-3x)^2$

Q12. **If $x > y$, then $(x-y)(x+y)$ is:**

(A) equal to $(x-y)(x-y)$ (B) less than $(x-y)(x-y)$

(C) greater than $(x-y)(x-y)$ (D) options A and C

Q13. **If $a = -5$ and $b = 3$ then $-a^2b^3$ is:**

(A) less than 0 (B) equal to 0

(C) greater than 0 (D) options B and C

Q14. $(a+b)(a-b) =$

(A) $a(a-b) - b(a-b)$ (B) $a(b-a) + b(a-b)$

(C) $a(a+b) - b(b+a)$ (D) $a(a-b) + b(b-a)$

Q15. $\frac{3x^2-27}{x-3}$ **and $(x > 0)$, is:**

(A) less than $2x+9$ (B) equal to $2x+9$

(C) greater than $2x+9$ (D) cannot find

Q16. **The sum of the polynomials, $6x^2+9x-8$ and $2x^2-5x+3$ is:**

(A) $4x^2-14x-5$ (B) $8x^2+14x+11$

(C) $8x^2-4x+5$ (D) $8x^2+4x-5$

Q17. $(6x^2+9x-8)-(4x^2-5x+3) = ?$

(A) $2x^2-14x-11$ (B) $2x^2+14x-11$

(C) $10x^2+4x-11$ (D) $2x^2+14x+11$

Q18. **The product of $-3x^2y$ and $2x^2y^2z$ is:**

(A) $-6x^4y^3z^2$ (B) $-6x^4y^3z$

(C) $-6x^3y^3z$ (D) $6x^2y^2z^2$

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Q19. **What is the product of $2x$ and $6x^2 - 3xy^2 + 4$?**

(A) $3x^2 - xy^2 + 2$ (B) $3x - \frac{3}{2}y^2 + \frac{2}{x}$

(C) $12x^3 - 6x^2y^2 + 8x$ (D) $6x^3 - 6x^2y^2 - 8x$

Q20. **What is the product of $(2x + y)$ and $(4x^2 - 6xy^2)$?**

(A) $8x^3 + 12x^2y^2 + 4x^2y - 6xy^3$ (B) $8x^3 - 12x^2y^2 + 4x^2y + 6xy^2$

(C) $8x^3 + 12x^2y^2 - 4x^2y + 6xy^3$ (D) $8x^3 - 12x^2y^2 + 4x^2y - 6xy^3$

Q21. **What is quotient if $36x^2y + 21xy^3z$ is divided by $9xy$?**

(A) $4x - \frac{7}{3}yz^2$ (B) $4y - \frac{3}{7}y^2z$

(C) $4x + \frac{3}{7}y^2z^2$ (D) $4x + \frac{7}{3}y^2z$

Q22. **If $p = 3q - s$, then what is the value of q in terms of p and s?**

(A) $\frac{p+s}{3}$ (B) $\frac{p-s}{3}$

(C) $\frac{s-p}{3}$ (D) $\frac{3}{p-s}$

Q23. **If $x - 3 = 11$, what is the value of $x - 6$?**

(A) 14 (B) 8

(C) 22 (D) 19

Q24. $\frac{x^2y^2 - 1}{xy - 1} = ?$

(A) $xy - 1$ (B) $(xy + 1)$

(C) $(xy + 1)^2$ (D) $(1 - xy)$

Q25. **If $y = \dfrac{1}{\frac{1}{a} + \frac{1}{b}}$, when $a = 1$ and $b = \frac{1}{3}$, then $y =$**

(A) $\frac{1}{3}$ (B) 1

(C) 3 (D) $\frac{1}{4}$

Q26. **If $\dfrac{1}{1 + \frac{x}{1+x}} = 1$, then $x =$**

(A) 0 (B) 1

(C) 2 (D) $\frac{1}{1+x}$

Q27. **If $a^2 + b^2 = 16$ and $(a - b)^2 = 4$, then ab is equal to:**

(A) -6 (B) 20

(C) -20 (D) 6

Q28. **If $a^2 - b^2 = 27$ and $a^2 + b^2 = 13$ then the value of ab is equal to:**

(A) $2\sqrt{5}$ (B) $2i\sqrt{35}$

(C) 14 (D) 20

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Q29. What is the value of $\left[\frac{1}{x}+x\right]^2-\left[\frac{1}{x}-x\right]^2$?

(A) 4 (B) 0

(C) $2x^2$ (D) $\frac{2}{x^2}$

Q30. What is the arithmetic mean of x and y if $\frac{1}{x}=\frac{1}{z}-\frac{1}{y}$ and $xy = z$?

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{x+y+z}{3}$ (D) $\frac{x+y}{2}$

Q31. If $x^2 - y^2 = 25$ and $x - y = 5$, then the average of x and y is:

(A) 2.5 (B) 5

(C) 15 (D) 7.5

Q32. What is the value of $\frac{1}{x^2}+x^2$, when $\left(x-\frac{1}{x}\right)^2 = 36$?

(A) 6 (B) 8

(C) 38 (D) 34

Q33. What is the value of $\frac{m^2-m-6}{m^2-6m+9}$, when m = 6666?

(A) 6666 (B) –1

(C) 0 (D) 1

Q34. If 7 + 4p = q – kp, what is value of p?

(A) $\frac{4+k}{q-7}$ (B) $\frac{4-k}{q+7}$

(C) $\frac{q-7}{4+k}$ (D) $\frac{k-4}{7-q}$

Q35. If $F = C + \frac{bv^2}{K}$, then v is terms of F, C, K and b is:

(A) $\pm\sqrt{\frac{K}{C}(F-b)}$ (B) $\pm\sqrt{\frac{C}{b}(FK-C)}$

(C) $\pm\sqrt{\frac{K}{b}(F-C)}$ (D) $\pm\sqrt{\frac{KF+C}{K}}$

Q36. If $x = 7$, what is the value of $x^{5/2} \div x^{1/2}$?

(A) $\sqrt{2}$ (B) $\sqrt{7}$

(C) 49 (D) 4

Q37. What is the value of $m^2 + 7m - 18$ when m = 91?

(A) 8882 (B) 8900

(C) 1260 (D) 8918

Q38. What is the value of ab, when $a^2 + b^2 = 9$ and $(a - b)^2 = 7$?

(A) 16 (B) 2

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(C) 1 (D) 8

Q39. What is the difference of the reciprocals of x^2 and y^2?

(A) $\frac{y^2 - x^2}{x^2 y^2}$ (B) $\frac{-y^2 - x^2}{x^2 y^2}$

(C) $\frac{x^2 y^2}{y^2 - x^2}$ (D) $x^2 + y^2$

Q40. What is the value of $a^2 - b^2$, when $a + b = 2.95$ and $a - b = 1000$?

(A) .000295 (B) .00000295

(C) 295 (D) 2950

Q41. What is the value of $(x - 7)(x + 8) - (x - 9)(x + 10)$?

(A) 34 (B) 146

(C) −146 (D) $-14x + 34$

Explanatory Answers

Q1. (D) $\frac{x^2 - y^2}{x - y} = \frac{(x - y)(x + y)}{x - y} = x + y = 235 + 117 = 352$

Q2. (C) Adding both equations *i.e.,*

$$x^2 + y^2 = 34$$
$$\underline{x^2 - y^2 = 16}$$
$$2x^2 = 50 \quad \Rightarrow x^2 = 25$$
$$\Rightarrow \quad x = \pm 25$$

Now, $x^2 + y^2 = 34 \Rightarrow 25 + y^2 = 34 \Rightarrow y^2 = 9$

$$\Rightarrow y = \pm 3$$

Hence, $xy = (-5)(-3) = 15 = (5)(3)$

and $xy = (-5)(3) = -15 = (5)(-3)$

So correct answer is *C*.

Q3. (C) First of all we find the sum of the three polynomials, then divide the answer by 3.

Sum of the three polynomials

$$2x^2 + 5x - 6$$
$$5x^2 - 5x - 6$$
$$\underline{-7x^2 \qquad +30}$$
$$18$$

Now, Average= $\frac{\text{Sum of the three polynomials}}{3}$

$$= \frac{18}{3} = 6$$

Q4. (C) To avoid time consuming calculation, factorize the given polynomial

$x^2 + 14x + 24 = x^2 + 12x + 2x + 24 = x(x + 12) + 2(x + 12)$

$= (x + 2)(x + 12)$

Substituting the value of x in above

$= (854 + 2)(854 + 12) = (856)(866)$

$= 741,296$

Q5. (D) Solving $(x - y)^2 = 3 \Rightarrow x^2 + y^2 - 2xy = 3$

Substituting the value of $x^2 + y^2$ in above

$(x^2 + y^2) - 2xy = 3 \Rightarrow 9 - 2xy = 3$

$\Rightarrow 9 - 3 = 2xy \Rightarrow 6 = 2xy$

$\Rightarrow xy = 3$

Q6. (D) $(5x + 4)(x + 1) - (5x - 6)(x + 3) =$

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$(5x^2 + 5x + 4x + 4) - (5x^2 + 15x - 6x - 18)$
$(5x^2 + 9x + 4) - (5x^2 + 9x - 18) = 4 + 18 = 22$

Q7. (A) $\frac{1}{z} = \frac{1}{x} + \frac{1}{y} \Rightarrow \frac{1}{z} = \frac{x+y}{xy} \Rightarrow \frac{1}{z} = \frac{x+y}{z} (\because z = xy)$

$$\Rightarrow 1 = x + y \Rightarrow \frac{1}{2} = \frac{x+y}{2}$$

Hence $\frac{x+y}{2} = \frac{1}{2}$

Q8. (C) $p^2 - q^2 = 38 \Rightarrow (p+q)(p-q) = 48$ ...(i)
Now, given that $p - q = 12$ ...(ii)
Dividing equation (i) by (ii), we have

$$\frac{(p+q)(p-q)}{(p-q)} = \frac{48}{12}$$

$$p + q = 4 \quad \text{...(iii)}$$

Dividing both sides of equation (iii) by 2, we get

$$\frac{p+q}{2} = \frac{4}{2} = 2$$

Q9. (A) Expanding each square of the polynomial, we get

$$\left(\frac{1}{x^2} + x^2 + 2\right) - \left(\frac{1}{x^2} + x^2 - 2\right)$$

$$\Rightarrow \left(\frac{1}{x^2} + x^2 + 2 - \frac{1}{x^2} - x^2 + 2\right) = 4$$

Q10.(D) Given $\left(x + \frac{1}{x}\right)^2 = 81 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 81$

$$\Rightarrow x^2 + \frac{1}{x^2} = 81 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 79$$

Q11.(A) Since x is negative ($\because x < 0$), therefore x^2 is positive, implies that $-(+3)(+x^2) = -3x^2$ is negative
Now, we take $(-3x)^2$, because x is negative.
$\therefore [-3(-x)]^2 = (-3)^2(-x)^2 = 9x^2$,
which is positive, hence $-3x^2$ is less than $(-3x)^2$.

Q12.(C) Since $x > y$, therefore, $x - y$ is positive. Thus dividing $(x - y)(x + y)$ and $(x - y)(x - y)$ by $(x - y)$, we have, $\frac{(x-y)(x+y)}{x-y}$ $\frac{(x-y)(x-y)}{x-y}$

$x + y$ $x - y$

Because both quantities are positive, but L.H.S is greater than R.H.S.

Q13.(A) As $a = -5$ and $b = 3$ $\Rightarrow -a^2b^3 = -(-5)^2(3)^3$
$\Rightarrow -a^2b^3 = -(25)(27)$
$= -675$
which is clearly less than 0.

Q14.(C) If we multiply $(a + b)$ by $(a - b)$, we proceed as
$a(a + b) - b(a + b)$ so option C is the correct answer.

Q15.(C) $\frac{3x^2 - 27}{x - 3} = \frac{3(x^2 - 9)}{x - 3} = \frac{3(x-3)(x+3)}{(x-3)} = 3(x + 3) = 3x + 9$
which is clearly greater than $2x + 9$.

Q16.(D) $(6x^2 + 9x - 8) + (2x^2 - 5x + 3)$
$= (6x^2 + 2x^2) + (9x - 5x) + (-8 + 3)$
$= 8x^2 + 4x - 5$

Q17.(B) $(6x^2 + 9x - 8) - (4x^2 - 5x + 3)$
$= (6x^2 - 4x^2) + (9x - (-5x)) + (-8 - 3)$
$= 2x^2 + (9x + 5x) + (-11)$

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$= 2x^2 + 14x - 11$

Q18.(B)
$$\begin{array}{r} 2x^2y^2z \\ \underline{\times -3x^2y} \\ -6x^4y^3z \end{array}$$

Q19.(C)
$$\begin{array}{r} 6x^2 - 3xy^2 + 4 \\ \underline{\times 2x} \\ 12x^3 - 6x^2y^2 + 8x \end{array}$$

Q20.(D) $(2x + y) \times (4x^2 - 6xy^2) = 2x(4x^2 - 6xy^2) + y(4x^2 - 6xy^2)$
$= 8x^3 - 12x^2y^2 + 4x^2y - 6xy^3$

Q21.(D) $(36x^2y + 21xy^3z) \div 9xy$
$$= \frac{36x^2y + 21xy^3z}{9xy} = \frac{36x^2y}{9xy} + \frac{21xy^3z}{9xy}$$
$$= 4x + \frac{7}{3}y^2z$$

Q22.(A) $p = 3q - s$
$\Rightarrow p + s = 3q \ \Rightarrow q = \dfrac{p+s}{3}$

Q23.(B) Given, $x - 3 = 11$
Subtracting 3 both sides of the equation
$x - 3 - 3 = 11 - 3$
$\Rightarrow \boxed{x - 6 = 8}$

Q24.(B) Factorizing the numerator
$$\frac{x^2y^2 - 1}{xy - 1} = \frac{(xy+1)(xy-1)}{(xy-1)} = xy + 1$$

Q25.(D) $y = \dfrac{1}{\frac{1}{a} + \frac{1}{b}}$, putting $a = 1$ and $b = \frac{1}{3}$, we get
$$y = \frac{1}{\frac{1}{1} + \frac{1}{1/3}} \Rightarrow y = \frac{1}{1+3} \Rightarrow y = \frac{1}{4}$$

Q26. A $\dfrac{1}{1 + \frac{x}{1+x}} = 1$, Solving for x

$\dfrac{1}{\frac{1+x+x}{1+x}} = 1$ (Taking "$1 + x$" L.C.M in denominator)

$\Rightarrow \dfrac{1 \times (1+x)}{\frac{1+2x}{(1+x)} \times (1+x)} = 1$ (Multiplying denominator and numerator by $(1 + x)$)

$\Rightarrow \dfrac{1+x}{1+2x} = 1 \quad \Rightarrow 1 + x = 1 + 2x \Rightarrow 1 - 1 = 2x - x$

$\Rightarrow x = 0$

Q27. d) $a^2 + b^2 = 16$ and $(a - b)^2 = 4$
$(a - b)^2 = 4 \Rightarrow a^2 + b^2 - 2ab = 4$
$\Rightarrow a^2 + b^2 - 4 = 2ab$
but $a^2 + b^2 = 16$ substituting above equation
$12 = 16 - 4 = 2ab$
$\Rightarrow \boxed{ab = 6}$

Q28. b) Given $a^2 - b^2 = 27$ and $a^2 + b^2 = 13$

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Adding $a^2 + b^2 + a^2 - b^2 = 27 + 13$

$2a^2 = 40 \Rightarrow a^2 = 20$

Which gives $20 + b^2 = 13 \Rightarrow b^2 = -7$

$a^2b^2 = -140 = 140i^2$

$ab = 2\sqrt{35}\, i \Rightarrow 2i\sqrt{35}$

Q29. a) $\left[\frac{1}{x}+x\right]^2 - \left[\frac{1}{x}-x\right]^2$

Expand each square

$$\left[\frac{1}{x}+x\right]^2 = \frac{1}{x^2}+x^2+2$$

$$\left[\frac{1}{x}-x\right]^2 = \frac{1}{x^2}+x^2-2$$

$$\left[\frac{1}{x^2}+x^2+2\right]-\left[\frac{1}{x^2}+x^2-2\right] = \frac{1}{x^2}+x^2+2-\frac{1}{x^2}-x^2+2$$

$$= 4$$

Q30. d) $\frac{1}{x}=\frac{1}{z}-\frac{1}{y} \Rightarrow \frac{1}{z}=\frac{1}{x}+\frac{1}{y} \Rightarrow \frac{1}{z}=\frac{x+y}{xy}$

$$\Rightarrow \frac{xy}{z} = x+y$$

but $xy = z$ which gives

$\frac{z}{z} = x + y$

$\Rightarrow 1 = x + y$

$\Rightarrow \boxed{\frac{x+y}{2} = \frac{1}{2}}$

Q31. a) Given $x^2 - y^2 = 25$ and $x - y = 5$

$x^2 - y^2 = 25 \Rightarrow (x-y)(x+y) = 25$

but $x - y = 5$ (given)

$\Rightarrow 5(x+y) = 25$

$\Rightarrow (x+y) = 5$

$\Rightarrow \boxed{\frac{x+y}{2} = \frac{5}{2} = 2.5}$

Q32. c) $\left(x-\frac{1}{x}\right)^2 = 36,$ Expanding the square

$x^2 + \frac{1}{x^2} - 2 = 36$

$\Rightarrow x^2 + \frac{1}{x^2} = 38$

Q33. d) $\frac{m^2 - m - 6}{m^2 - 6m + 9}$

$$= \frac{m^2 - 3m + 2m - 6}{(m-3)^2} = \frac{m(m-3)+2(m-3)}{(m-3)^2}$$

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$$= \frac{(m+2)(m-3)}{(m-3)^2} = \frac{m+2}{m-3} = \frac{6666+2}{6666-3}$$

$$= \frac{6668}{6663} = 1 \text{ (approx)}$$

Q34. c)

$$7 + 4p = q - kp$$
$$4p + kp = q - 7$$
$$p(4+k) = q - 7$$
$$p = \frac{q-7}{4+k}$$

Q35. (C)

$$F = C + \frac{bv^2}{K}$$

$$FK - CK = bv^2 \Rightarrow v^2 = \frac{K(F-C)}{b}$$

$$v = \pm\sqrt{\frac{K}{b}(F-C)}$$

Q36. (C) $x^{5/2} \div x^{1/2}$ and $x = 7$

$$7^{5/2} \div 7^{1/2} \Rightarrow \frac{7^{5/2}}{7^{1/2}} = 7^{5/2} \times 7^{-1/2}$$
$$= 7^{5/2 - 1/2} = 7^2 = 49$$

Q37. (B) Given $m^2 + 7m - 18$ and $m = 91$

$m^2 + 9m - 2m - 18$

$\Rightarrow m(m+9) - 2(m+9) \Rightarrow (m-2)(m+9)$

$\Rightarrow (91-2)(91+9) = (89)(100)$

$= 8900$

Q38. (C) Given $(a-b)^2 = 7$ and $a^2 + b^2 = 9$

$a^2 + b^2 - 2ab = 7 \Rightarrow 9 - 2ab = 7$

$\Rightarrow -2ab = -2$

$\Rightarrow \boxed{ab = 1}$

Q39. (A) Given x^2 and y^2

reciprocals of x^2 and y^2 are $\frac{1}{x^2}$ and $\frac{1}{y^2}$ and their difference is

$$\frac{1}{x^2} - \frac{1}{y^2} \Rightarrow \frac{y^2 - x^2}{x^2y^2}$$

Q40. (D) Given $a + b = 2.95$ $a - b = 1000$

$(a+b)(a-b) = (2.95)(1000)$

$a^2 - b^2 = 2950$

Q41. (A) Given $(x-7)(x+8) - (x-9)(x+10)$

$(x^2 + x - 56) - (x^2 + x - 90)$

$x^2 + x - 56 - x^2 - x + 90 = 34$

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Chapter 2

EQUATION

An equation is a statement that has an equal sign. The parts of an equation to the right and left of the sign of equality are called sides of the equation and are distinguished as the right side and left side.

Highest power of the variable determines the degree of the equation. The letters used for variables in an equation are called unknown quantity. The process of finding the values of variables is called solving the equation. The value so found is called the root or solution of the equation.

Linear Equation:

The equation in which the highest power of the variable is one, is called a simple or linear equation of the first degree.

Example:

$3x = 9, 2x + 5 = 7, x - 7 = 9 \quad \frac{x}{2} - \frac{2}{3} = 5$ are linear equations

Axioms of Solving Linear Equation:

The process of solving linear equation depends only upon the following axioms:

1. If we add equals in an equation on both sides, the sums are equal.
2. If from equals we take equals the remainders are equal.
3. If equals are multiplied to both sides of an equation the products are equal.
4. If equals are divided by equals then the quotients are equal.

Rules of Solving Linear Equation:

We use following rules to solving a linear equation.

Rule 1:

In a linear equation, any term may be transposed from one side of the equation to the other by changing sign.

Example 1:

Consider a equation

$$-7x + 14 = -3x - 18$$

Transposing $3x + 14 = 7x - 18$

or $18 + 14 = 7x - 3x$

which is the original equation with the sign of some terms are changed.

Example 2:

Solve $3x - 8 = 16$

Solution:

The variable x is multiplied by 3 and then 8 has been subtracted

$$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-8} \boxed{3x - 8}$$

Transposing the operations of "×", "–" in other words "undo" or backtrack these two operations, first add 8, and then divide by 3.

$$\boxed{x} \xleftarrow[\div]{} \boxed{3x} \xleftarrow[+8]{} \boxed{3x - 8}$$

To keep this equation balance, the same operation must be carried out on both sides of the equation. The process of solving above equation is illustrated simply in two steps as fellows:

$$3x - 8 = 16$$
$$3x - 8 + 8 = 16 + 8$$
$$3x \div 3 = 24 \div 3$$

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$$\boxed{x = 8}$$

Steps for Solving Linear Equations:

1. If the equation involves a fraction, first, if necessary, clear the fractions.
2. Transpose all the terms containing the unknown quantity to one side of the equation, and the known quantity to the other side of the equation.
3. Collect the terms on each side.
4. Divide both sides of the coefficient of the unknown variable.
5. Compute for the result.

Example:

Solve *(i)* $7x - 12 = 3x$ *(ii)* $\frac{4}{x} = \frac{-1}{3}$

(iii) $\frac{3}{1+x} = \frac{1}{2}$ *(iv)* $\frac{4}{3+a} + 1 = \frac{1}{3}$

Solution:

Check

(i)
$$7x - 12 = 3x$$
$$7x - 3x = 12$$
$$4x = 12$$
$$x = 3$$

Substituting $x = 3$ in equation $7x - 12 = 3x$

$$7(3) - 12 = 3(3)$$
$$21 - 12 = 9$$
$$9 = 9$$

Solution is correct

(ii) $\frac{4}{x} = \frac{-1}{3}$ **Check**

Multiplying both sides by $3x$

$$\frac{4}{x} \times 3x = \frac{-1}{3} \times 3x$$
$$12 = -x$$
$$(-1)(x) \div (-1) = 12 \div (-1)$$
$$x = -12$$

Note: $3x$ is LCD

Substituting $x = -12$ in

$$\frac{4}{x} = \frac{-1}{3}$$
$$\frac{4}{-12} = \frac{-1}{3}$$
$$\frac{-1}{3} = \frac{-1}{3}$$

Solution is correct

(iii) $\frac{3}{1+x} = \frac{1}{2}$ **Check**

Multiplying both sides by $2(1+x)$

$$\frac{3}{1+x} \times 2(1+x) = \frac{1}{2} \times 2(1+x)$$
$$6 = 1 + x$$
$$6 - 1 = 1 + x - 1$$
$$x = 5$$

Substituting $x = 5$ in

$$\frac{3}{1+5} = \frac{1}{2}$$
$$\frac{3}{6} = \frac{1}{2}$$
$$\frac{1}{2} = \frac{1}{2}$$

Solution is correct

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Check

(iv) $\frac{4}{3+a}+1=\frac{1}{3}$

Multiplying both sides by 3(3 + a)

$$\left(\frac{4}{3+a}\right)\times 3(3+a)+1\times 3(3+a)=\frac{1}{3}\times 3(3+a)$$

$$(4\times 3)+3(3+a)=3+a$$

Removing Brackets

$$12+9+3a = 3+a$$
$$21+3a = 3+a$$
$$21+3a-21-a = 3+a-21-a$$
$$2a = -18$$
$$a = -9$$

Check

Substituting a = –9 in given equation

$$\frac{4}{3+(-9)}+1=\frac{1}{3}$$

$$\frac{4}{-6}+1 = \frac{4-6}{-6}$$

$$= \frac{-2}{-6}=\frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{3}$$

$\boxed{\text{Solution is correct}}$

Example:

If $\frac{1}{x}=\frac{1}{y}+\frac{1}{z}$, what is the value of x?

Solution:

$$\frac{1}{x} = \frac{1}{y}+\frac{1}{z}$$

$$\frac{1}{x} = \frac{z+y}{yz}$$

Multiplying both sides by (xyz)

$$\frac{1}{x}\times xyz = \frac{(z+y)}{yz}\times xyz$$

$$\frac{yz}{y+z} = \frac{x(y+z)}{(y+z)}$$

$$x = \frac{yz}{y+z}$$

Example:

If $x = y(a+b)$, find a in terms of x, y and b.

Solution:

$$x = y(a+b)$$

$$\frac{x}{y} = \frac{y(a+b)}{y}$$

$$\frac{x}{y}-b = a+b-b$$

$$\boxed{a = \frac{x}{y}-b}$$

Solving Second-Degree Equation:

A second-degree equation involving the variable x has the generalized form

$$ax^2+bx+c = 0$$

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where a, b, and c are constants with $a \neq 0$. Second-degree equations are usually called quadratic equations. A quadratic equation in which the term containing x is missing is called a pure quadratic equation. Examples of second-degree equations are

$$\begin{aligned} 2x^2 - 5x + 12 &= 0 \\ 4x^2 &= 16 \\ 7x^2 - 12 &= 3x + 5 \end{aligned}$$

Example:

If $z^2 = x^2 + y^2$ and $x > 0$, what is y in terms of x and z.

Solution:

$$\begin{aligned} z^2 &= x^2 + y^2 \\ z^2 - x^2 &= x^2 + y^2 - x^2 \\ \Rightarrow \quad y^2 &= z^2 - x^2 \end{aligned}$$

Taking square root

$$\sqrt{y^2} = \sqrt{z^2 - x^2}$$

The value of $\sqrt{\ }$ is $\frac{1}{2}$

$$\therefore \quad (y^2)^{1/2} = \sqrt{z^2 - x^2}$$

$$\boxed{y = \sqrt{z^2 - x^2}}$$

Example:

If x is positive number and $x^2 - 25 = 56$, what is the value of x.

Solution:

$$\begin{aligned} x^2 - 25 &= 56 \\ x^2 - 25 + 25 &= 56 + 25 \\ x^2 &= 81 \end{aligned}$$

Taking square root

$$\begin{aligned} \sqrt{x^2} &= \sqrt{81} \\ x &= \pm 9 \end{aligned}$$

But x is +ive (given)

$$\therefore \quad x = 9$$

Example:

What is the value of 2^{x+3}, when $3^{x+2} = 81$?

$$3^{x+2} = 81 \quad \Rightarrow \quad 3^{x+2} = 3^4 \Rightarrow x + 2 = 4$$

$$\Rightarrow \quad x = 2$$

$$\text{Now} \quad 2^{x+3} = 2^{2+3} = 2^5 = 32$$

$$\therefore \quad \boxed{2^{x+3} = 32}$$

The Index Laws:

For multiplying and dividing powers, we use some rules. These rules are called index laws.

These rules are summarized below:

Multiplying powers	$x^a x^b = x^{a+b}$
Dividing powers	$\frac{x^a}{x^b} = x^{a-b}$

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Power of a power	$(x^a)^b = x^{ab}$
Power of a quotient	$\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$
Power of a product	$(xy)^a = x^a y^a$
Special Index:	
Zero Index	$x^o = 1$
Index in fraction	$x^{1/a} = \sqrt[a]{x}$
Index in negative form	$x^{-a} = \dfrac{1}{x^a}$

Example:

Find the value of x when $27^{-2x+1} = 729^{-2x+3}$

Solution:

$$27^{-2x+1} = 729^{-x+3}$$

Take L.H.S. $27^{-2x+1} = (3^3)^{-2x+1} = 3^{3(-2x+1)} = 3^{-6x+3}$ ......(1)

by Power of a power in Index law

Now take R.H.S. $729^{-x+3} = (3^6)^{-2x+3} = 3^{6(-2x+3)} = 3^{-12x+18}$ ......(2)

comparing (1) and (2)

$$3^{-6x+3} = 3^{-12x+18}$$

$$\Rightarrow \quad -6x + 3 = -12x + 18$$

$$\Rightarrow \quad -6x + 12x = 18 - 3$$

$$6x = 15 \Rightarrow x = \frac{15}{6}$$

$$\Rightarrow \quad \boxed{x = \frac{5}{2}}$$

Check:

Substitute $x = \dfrac{5}{2}$ in given equation

$$27^{-2\times 5/2+1} = 729^{-2\times 5/2+3}$$

$$27^{-4} = 729^{-2}$$

$$(3^3)^{-4} = (3^6)^{-2}$$

$$3^{-12} = 3^{-12}$$

Hence the solution is correct.

Systems of Linear Equations:

A system of equation is two or more equations considered together. If the equations in a system are linear, then it is called linear system of equations. The following system of the equations is a linear system of equations in two variables

$$\begin{cases} x + y = 7 \\ x - y = 3 \end{cases}$$

Simultaneous Equations:

A pair of equation which has two unknown, and are solved together, are called simultaneous equation. In simultaneous equations the values of unknown quantities satisfied both the given equations.

Example:

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$$3x + 5y = 9$$
$$3x + 7y = -19$$

Solution of a System of Equations:

The solution of a system of equation is an order pair that is a solution of both equations.

This system of equations can be solved by following two method

(1) Substitution Method

(2) Elimination Method

(1) Substitution Method:

This method is illustrated in the following example:

Example:

Solving the following system of equations using substituting method

$$3x - 4y = 2$$
$$4x + 3y = 14$$

Solution:

$$3x - 4y = 2 \quad ....(1)$$
$$4x + 3y = 14 \quad ....(2)$$

Solving equation (1) for x in terms of y

$$3x - 4y = 2 \Rightarrow 3x = 4y + 2 \Rightarrow x = \frac{4y+2}{3}$$

Substituting the value of $x = \frac{4y+2}{3}$ in (2)

$$4\left(\frac{4y+2}{3}\right) + 3y = 14 \quad ....(3)$$

To get rid of friction multiply both sides of the equation (3) by 3

$$4(4y+2) + 9y = 42$$
$$\Rightarrow 16y + 8 + 9y = 42$$
$$\Rightarrow 25y = 34$$
$$\Rightarrow \boxed{y = \frac{34}{25}}$$

To find the value of "x" substitute $y = \frac{34}{25}$, in equation (1) or (2). Here we substitute it in equation (1)

$$3x - 4\left(\frac{34}{25}\right) = 2$$
$$75x - 136 = 50$$
$$75x = 186$$
$$x = \frac{186}{75}$$
$$\boxed{x = \frac{62}{25}}$$

The solution of the equation in the form of order pair is $\left(\frac{62}{25}, \frac{34}{25}\right)$.

Elimination Method:

The process by which we get rid of either of the unknown quantities is called elimination. In this method one of the unknown is eliminated by adding or subtracting one equation from the other.

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Note: Since multiplying each side of an equation by the same non-zero constant does not change the solution of the equation. Therefore, if the coefficient of the unknown are not the same size, one or both equations are first multiplied by an appropriate number.

Example 1:

Solve

$$x + 2y = 22 \quad \ldots\ldots\ldots(1)$$
$$x - 2y = 2 \ldots\ldots\ldots(2)$$

Solution:

Since y terms have equal but opposite coefficient, eliminate by adding

$$x + \cancel{2y} = 22 \quad \ldots\ldots\ldots(1)$$
$$x - \cancel{2y} = 2 \quad \ldots\ldots\ldots(2)$$
$$\text{(by adding) } 2x = 24$$
$$\boxed{x = 12}$$

Substitute $x = 12$ in (1)

$$12 + 2y = 22 \quad \Rightarrow \quad 2y = 10$$
$$\boxed{y = 5}$$

Solution set is (12, 5)

Example 2:

Solve

$$3x + 6y = 11 \quad \ldots\ldots\ldots(1)$$
$$2x + 4y = 9 \ldots\ldots\ldots(2)$$

Solution:

In above system of equations, to eliminate the x variable. Multiplying equation (1) by 2 and equation (2) by -3. Then add the resultant equation and solve for y

$$6x + 18y = 11 \quad \ldots\ldots\ldots(1)$$
$$-6x - 12y = -27 \quad \ldots\ldots\ldots(2)$$
$$6y = -5$$
$$\boxed{y = -\frac{5}{6}}$$

Note: The multipliers we chosen so that the coefficient of the variables we want to eliminate are additive inverses.

Substitute $y = -\frac{5}{6}$ in (1)

$$3x + 6\left(\frac{-5}{6}\right) = 11 \quad \Rightarrow \quad 3x = 16 \Rightarrow \boxed{x = \frac{16}{3}}$$

Solution is $\left(\frac{16}{3}, -\frac{5}{6}\right)$

Example 3:

What is the arithmetic mean (average) of x and y, when $3x + 4y = 21$, and $4x + 3y = 35$

Solution:

$$3x + 4y = 21 \quad \ldots\ldots\ldots(1)$$
$$4x + 3y = 35 \quad \ldots\ldots\ldots(2)$$
$$(1) + (2) \qquad 7x + 7y = 56$$
$$7x + 7y = 56 \Rightarrow 7(x + y) = 56 \Rightarrow x + y = 8$$

Arithmetic mean of x and y is $\frac{x + y}{2} = \frac{8}{2} = \boxed{4}$

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Multiple Choice Questions (MCQs)

Q1. If $3x + 9 = 18$, what is the value of $x + 3$?

(A) 3 (B) 6
(C) −3 (D) 36

Q2. If $5x + 12 = 44$, what is the value of $5x - 12$?

(A) 24 (B) 32
(C) 20 (D) 22

Q3. If $3x + 17 = 9 - x$, what is the value of x?

(A) 2 (B) 3
(C) −2 (D) −3

Q4. If $x - 5 = 9$, what is the value of $x^2 - 5$?

(A) 196 (B) 191
(C) 16 (D) 11

Q5. If $at - b = c - dt$, what is the value of t in terms of a, b, c and d?

(A) $\frac{b-c}{a-d}$ (B) $\frac{a}{b}$
(C) $\frac{c}{d}$ (D) $\frac{b+c}{a+d}$

Q6. If $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x = 22$, what is the value of x?

(A) 88 (B) 44
(C) 1 (D) 24

Q7. If $2x - 3 = 15$, what is the value of $(2x - 3)^2$?

(A) 81 (B) 227
(C) ± 225 (D) 225

Q8. If $81^{10} = 3^{x-7}$, what is the value of x?

(A) 47 (B) 27
(C) 51 (D) 14

Q9. If $\frac{1}{x-y} = 7$, then $x =$

(A) $x + \frac{1}{7}$ (B) $x - \frac{1}{7}$
(C) $\frac{1}{7} - x$ (D) $\frac{x}{7} - 1$

Q10. If $x = 2t + 5$, and $y = 4t^2$, what is y in terms of x?

(A) $\left(\frac{x-5}{2}\right)^2$ (B) $\frac{x+5}{2}$
(C) $(x-5)^2$ (D) $\frac{x-5}{4}$

Q11. If x is a positive number and $x^2 + 36 = 100$, what is the value of x?

(A) 6 (B) 8
(C) 14 (D) 64

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Q12. If $4^{x+5} = 8^{x-1}$, what is the value of x?

(A) $\frac{3}{5}$ (B) $-\frac{5}{3}$

(C) $-\frac{3}{5}$ (D) 4

Q13. If $\sqrt{x} = 9$, then $x^2 - \sqrt{x}$ equals:

(A) 6561 (B) 2530

(C) 6552 (D) $\sqrt{6} - 9$

Q14. If $\frac{a+3}{6} = \frac{12}{a+4}$, then positive value of x equals:

(A) 5 (B) 12

(C) 15 (D) 18

Q15. If x and y are positive integers and $x^2 + 2y^2 = 41$, $2x^2 + y^2 = 34$, then $x^2 =$

(A) 6 (B) 8

(C) 75 (D) 9

Q16. For any positive integer p, $\$p = \frac{p^2}{3}$ and $£p = \frac{9}{p}$, which of the following is an expression for the product of $\$p$ and $£p$?

(A) $\frac{2}{p}$ (B) $\frac{1}{p}$

(C) p (D) $3p$

Q17. If a, b, and c are different positive odd integers and $a + b + c = 11$, what is the greatest positive value of c?

(A) 9 (B) 7

(C) 3 (D) 1

Q18. If $n + 5 = n \times 5$, then $n =$

(A) 1.25 (B) 1.5

(C) 0.5 (D) 5

Q19. If $\frac{a}{b} = .75$, then $4a - 3b =$

(A) 1 (B) 2

(C) 0 (D) 3

Q20. Let $ab = c$, where a, b and c are non zero numbers. If a is multiplied by 3 and c is divided by 3, this is equivalent to multiply b by:

(A) $\frac{1}{3}$ (B) 3

(C) $\frac{1}{9}$ (D) 9

Q21. If $5a = 3$, then $(5a + 3)^2 = ?$

(A) 9 (B) 36

(C) 4 (D) 25

Q22. If $a = \frac{1}{3}$, then $a^3 = ?$

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$

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(C) $\frac{3}{27}$ (D) $\frac{1}{27}$

Q23. If $3a - 5 = 7$, what is the value of a?

(A) $\frac{2}{3}$ (B) 4

(C) 12 (D) 2

Q24. If $4 + \frac{5W}{2} = 19$, what is the value of W?

(A) 6 (B) 30

(C) 15 (D) 75

Q25. What is the value of $(11 - y)$ when $121 - 11y = 77$?

(A) 11 (B) 44

(C) 7 (D) 4

Q26. What is the value of $x^2 - 4$, when $x^6 - 4x^4 = 64$ and $x^2 = 4$?

(A) 16 (B) 20

(C) 12 (D) 8

Q27. One factor of $8x^3 - 27y^3$ is $(2x - 3y)$, what is the other factor?

(A) $(2x + 3y)$ (B) $(4x^2 + 9y^2)$

(C) $(4x^2 + 6xy + 9y^2)$ (D) $(4x^2 + 12xy + 9y^2)$

Q28. If $32^{x+y} = 16^{x+2y}$, then $x =$

(A) y (B) 5y

(C) $\frac{y}{3}$ (D) 3y

Q29. If $px - q = r - sx$, what is the value of x?

(A) $\frac{p+s}{r+q}$ (B) $\frac{r+q}{p+s}$

(C) $\frac{r-q}{p+s}$ (D) $\frac{r-q}{p-s}$

Q30. If one factor of $a^2 - b^2 + am + bm$ is $a + b$, then the other factor is

(A) $(a + b - m)$ (B) $(a - m)$

(C) $(b - m)$ (D) $(a - b + m)$

Explanatory Answers

Q1. (B) $3x + 9 = 18 \Rightarrow 3(x + 3) = 18$(Taking 3 common from L.H.S)

$\Rightarrow \frac{3(x+3)}{3} = \frac{18}{3} \Rightarrow x + 3 = 6$ (Dividing both sides by 3)

Q2. (C) Given that $5x + 12 = 44$, subtracting -24 on both sides of the given equation, we have $5x + 12 - 24 = 44 - 24$

$\Rightarrow 5x - 12 = 20$

Q3. (C) $3x + 17 = 9 - x \Rightarrow 3x + x = 9 - 17$

$\Rightarrow 4x = -8$

$\Rightarrow x = \frac{-8}{4} = -2$

Q4. (B) $x - 5 = 9 \Rightarrow x - 5 + 5 = 9 + 5$

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(adding 5 both sides of the equation)

$$\Rightarrow x = 14 \Rightarrow x^2 = (14)^2 = 196$$

Now $x^2 - 5 = 196 - 5 \Rightarrow x^2 - 5 = 191$

Q5. (D) $at - b = c - dt \quad \Rightarrow \quad at + dt = b + c$

$$\Rightarrow t(a + d) = b + c$$

$$\Rightarrow \quad t = \frac{b + c}{a + d}$$

Q6. (D) $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x = 21$, (taking L.C.M, 8)

$\frac{4x + 2x + x}{8} = 21$, Multiplying both sides by 8, we have

$\frac{7x}{8} \times 8 = 21 \times 8 \quad \Rightarrow 7x = 21 \times 8$

$$\Rightarrow \quad x = \frac{21 \times 8}{7}$$

$$\Rightarrow \quad x = 3 \times 8$$

$$\Rightarrow \quad x = 24$$

Q7. (D) Given that $2x - 3 = 15$

Taking square both sides of the equation, we get

$$(2x + 3)^2 = (15)^2$$

$$\Rightarrow \quad (2x + 3)^2 = 225$$

Q8. (A)

$$\begin{aligned} 81^{10} &= 3^{x-7} \\ (3 \times 3 \times 3 \times 3)^{10} &= 3^{x-7} \\ (3^4)^{10} &= 3^{x-7} \\ 3^{40} &= 3^{x-7} \\ \Rightarrow \quad 40 &= x - 7 \\ \Rightarrow \quad 40 + 7 &= x - 7 + 7 \\ \Rightarrow \quad & \boxed{x = 47} \end{aligned}$$

Q9. (B) $\frac{1}{x - y} = 7$, Multiplying both sides of the equation by $(x - y)$, we have

$$\begin{aligned} \frac{1}{x - y} \times (x - y) &= 7 \times (x - y) \\ 1 &= 7x - 7y \\ \Rightarrow \quad 7y &= 7x - 1 \qquad \Rightarrow y = \frac{7x - 1}{7} \end{aligned}$$

$$\Rightarrow y = x - \frac{1}{7}$$

Q10.(C) Let $x = 2t + 5 \ldots$(i) and

$y = 4t^2 \qquad \ldots$(ii)

Solving (i), for t

$x = 2t + 5 \Rightarrow x - 5 = 2t + 5 - 5$

$$\Rightarrow x - 5 = 2t \quad \Rightarrow \frac{x - 5}{2} = \frac{2t}{2}$$

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$$\Rightarrow \frac{x-5}{2} = t$$

Putting the value of t in (ii), we have

$$y = 4\left(\frac{x-5}{2}\right)^2$$

$$y = \frac{4}{4}(x-5)^2 \Rightarrow y = (x-5)^2$$

Q11.(D) $x^2 + 36 = 100 \quad \Rightarrow x^2 + 36 - 36 = 100 - 36$

$\Rightarrow \quad x^2 = 64$

$\Rightarrow \quad \sqrt{x^2} = \sqrt{64}$

$\Rightarrow \quad x = \pm 8$

Since x is positive $\Rightarrow \quad x = 8$

Q12.(B) $4^{x+5} = 8^{x-1}$

$\Rightarrow \quad (2 \times 2)^{x+5} = (2 \times 2 \times 2)^{x-1}$

$\Rightarrow \quad (2^2)^{x+5} = (2^3)^{x+5}$

$\Rightarrow \quad 2^{2 \times (x+5)} = 2^{3(x+5)}$

$\Rightarrow \quad 2^{2x+10} = 2^{3x+15}$

$\Rightarrow \quad 2x + 10 = 3x + 15$

$\Rightarrow \quad 2x - 3x = 15 - 10$

$\Rightarrow \quad -3x = 5$

$\Rightarrow \quad \frac{-3x}{-3} = \frac{5}{-3}$

$\Rightarrow \quad x = -\frac{5}{3}$

Q13.(C) $\sqrt{x} = 9 \Rightarrow x = 81 \Rightarrow x^2 = 6561$

Now $x^2 - \sqrt{x} = 6561 - 9$

$\Rightarrow x^2 - \sqrt{x} = 6552$

Q14.(A) $\frac{a+3}{6} = \frac{12}{a+4} \Rightarrow \quad (a+3)(a+4) = 12 \times 6$

$\Rightarrow \quad a^2 + 7a + 12 = 72$

$\Rightarrow \quad a^2 + 7a + 12 - 72 = 0$

$\Rightarrow \quad a^2 + 7a - 60 = 0$

$\Rightarrow \quad a^2 + 12a - 5a - 60 = 0$

$\Rightarrow a(a+12) - 5(a+12) = 0$

$\Rightarrow \quad (a-5)(a+12) = 0$

$\Rightarrow \quad \boxed{a = 5, -12}$

Q15.(D) Given $2x^2 + y^2 = 34$

$\Rightarrow \quad y^2 = 34 - 2x^2$

Substituting the value of y^2 in the first equation

$x^2 + 2y^2 = 41$, gives

$x^2 + 2(34 - 2x^2) = 41$

$\Rightarrow x^2 + 68 - 4x^2 = 41$

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$\Rightarrow -3x^2 = 41 - 68 \Rightarrow -3x^2 = -27$

$\Rightarrow 3x^2 = 27 \Rightarrow x^2 = \frac{27}{3} \Rightarrow \boxed{x^2 = 9}$

Q16.(D) $\$p = \frac{p^2}{3}$ and $£p = \frac{9}{2p}$

$\therefore \quad \$p \,.\, £p = \frac{p^2}{3} \times \frac{9}{p} \Rightarrow \$p \,.\, £p = 3p$

Q17.(B) The set of positive odd integers is $\{1,3,5,7,9......\}$

The sum of the three positive integers should be 11. If we take greatest possible value of c, then there exist least positive integers a and b, the value of least positive integers a and b is 1 and 3, so their sum = 1 + 3 = 4. Thus the greatest positive integer is $1 + 3 + c = 11 \Rightarrow 4 + c = 11$

$\Rightarrow \boxed{c = 7}$

Q18.(A) Given that $n + 5 = n \times 5$

Subtracting n both sides of the equation

$n + 5 - n = 5n - n$

$5 = n(5 - 1)$ (Taking n common)

$5 = 4n$

$\Rightarrow \quad \frac{5}{4} = \frac{4n}{4}$ (Dividing both sides by 4)

$\Rightarrow \quad n = 1.25$

Q19.(C) $\frac{a}{b} = .75 \quad \Rightarrow \frac{a}{b} = \frac{75}{100} \Rightarrow \frac{a}{b} = \frac{3}{4}$

$\Rightarrow 4a = 3b \Rightarrow 4a - 3b = 0$

Q20.(C) $ab = c \qquad ...(1)$

If a is multiplied by 3, and c is divided by 3, the above equation becomes

$3ab = \frac{c}{3} \qquad ...(2)$

The above equation (2), is equivalent to (1), if b is multiplied by $\frac{1}{9}$

$\therefore \quad 3a \times \frac{1}{9}b = \frac{c}{3}$

$\frac{1}{3}ab = \frac{c}{3} \qquad$ As $ab = c$

$\frac{1}{3}c = \frac{c}{3} \Rightarrow c = c$

Q21.(B) $5a = 3 \Rightarrow 5a - 3 = 0 \qquad ...(1)$

Adding 6 both sides of equation (1), we get

$5a - 3 + 6 = 0 + 6 \Rightarrow 5a + 3 = 6 \quad ...(2)$

Squaring both sides of equation (2), we get

$(5a + 3)^2 = (6)^2 = (5a + 3)^2 = 36$

Q22.(D) $a = \frac{1}{3} \qquad \Rightarrow (a)^3 = \left(\frac{1}{3}\right)^3$

$\Rightarrow \quad a^3 = \frac{1 \times 1 \times 1}{3 \times 3 \times 3}$

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$\Rightarrow \; a^3 = \frac{1}{27}$

Q23. (B) $3a - 5 = 7$

$\Rightarrow \; 3a - 5 + 5 = 7 + 5$ (adding 5 both sides)

$\Rightarrow \; 3a = 12$

$\Rightarrow \; \boxed{a = 4}$

Q24. (A) $4 + \frac{5W}{2} = 19$

$\Rightarrow \; 4 + \frac{5W}{2} - 4 = 19 - 4$ (To get rid 4 from L.H.S. subtract 4 both sides)

$\Rightarrow \; \frac{5W}{2} = 15$ (To get rid 2 from L.H.S. multiply 2 both sides)

$\frac{5W}{2} \times 2 = 15 \times 2 \; \Rightarrow \; 5W = 30$

last to get rid 5 from L.H.S. divide both sides by 5

$\frac{5W}{5} = \frac{30}{5} \; \Rightarrow \; \boxed{W = 6}$

Q25. c) Given $121 - 11y = 77$ taking 11 common from L.H.S.

$11(11 - y) = 77$ Dividing 11 both sides

$\frac{11(11 - y)}{11} = \frac{77}{11}$

$\Rightarrow \; 11 - y = 7$

Q26. a) Given $x^6 - 4x^4 = 64$ and $x^4 = 4$

$x^4(x^2 - 4) = 64$ (Taking x^4 common)

$4(x^2 - 4) = 64$ (Substituting the value of x^4)

$\Rightarrow \; x^2 - 4 = 16$ (Dividing both sides by 4)

Q27. (C) $8x^3 - 27y^3$

$\Rightarrow \; (2x)^3 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$ (factorizing)

Q28. (D) $32^{x+y} = 16^{x+2y}$

$(2)^{5(x+y)} = (2)^{4(x+2y)} \; \Rightarrow \; 2^{5x+5y} = 2^{4x+8y}$

$\Rightarrow \; 5x + 5y = 4x + 8y$

$\Rightarrow \; 5x - 4x = 8y - 5y$

$\boxed{x = 3y}$

Q29. (B) $px - q = r - sx$

$px + sx = r + q$

$\Rightarrow \; x(p + s) = r + q$

$x = \frac{r + q}{p + s}$

Q30. (D) $a^2 - b^2 + am + bm$

$(a - b)(a + b) + m(a + b)$ as $a^2 - b^2 = (a + b)(a - b)$

$(a + b)(a - b + m)$

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Chapter 3

INEQUALITY

An inequality, or inequation is a statement which involves one of the sign below:

$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

Examples:

$$6x > 52$$
$$11y \geq -101$$
$$-3x \leq 8$$
$$-52w \leq 9$$

The set of all solutions of an inequality is called the solution set of the inequality. For example the solution of $x + 3 > 5$ is the set of all real numbers greater than 2.

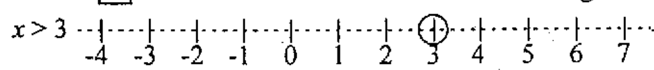
Note:

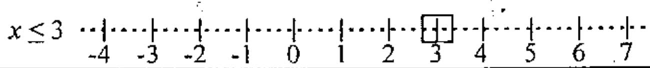
Equivalent Inequalities have the same solution set.

Representation of Inequality on number line:

Inequalities such as $x > 3$ or $x \leq 3$ can be represented on number line.

In following number line a circle " $\square$ " shows that x is included and a rectangle "$\bigcirc$" shows that it is not





Note:

1. Any quantity x is said to be greater than another quantity y when $(x - y)$ is positive.

Example:

If $x = 2$ and $y = -3$, thus $x > y$ because $2 - (-3) = 5$ or positive.

2. y is said to be less than x when $y - x$ is negative.

Example:

If $x = 2$ and $y = -3$, than $y < x$ because $-3 - 2 = -5$ or negative.

Properties of Inequalities:

We apply the following properties to solve in-equalities.

1. An inequality will still hold after each side has been increased, decreased, multiplied or divided by the same positive quantity

If $x > y$

For example: $x + z > y + z$; $x - z > y - z$; $xz > yz$; $\frac{x}{z} > \frac{y}{z}$

2. In an un-equality any term may be transposed from one side to the other if its sign be changed

If $x - y > z$

For example: $x > z + y$

3. If the sides of an inequality is transposed, then the sign of inequality is reversed

Example: If $x > y$, then evidently $y < x$

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4. If both sides of the inequality are multiplied or divided by a negative number, then direction of the inequalities sign is reversed

Example: If $x > y$, then $-x < y$ and

$\therefore \quad -xz < yz$

5. The square of real quantity is positive, therefore it is greater than zero.

Therefore$(x-y)^2$ is always positive

$\therefore (x-y)^2 > 0$

$\therefore x^2+y^2 > 2xy$

6. If x and y are two positive quantities, then their arithmetic mean $\left(\frac{x+y}{2}\right)$ is greater than their geometric mean $\left(\sqrt{xy}\right)$.

$$\therefore \quad \frac{x+y}{2} > \sqrt{xy}$$

Example:

Solve the following inequalities

(i) $3x-11<13$ *(ii)* $\frac{-x}{2}\le 2$

Solution:

(i)

$$3x-11 < 13$$
$$\therefore \quad 3x-11+11 < 13+11 \text{ (using property 1)}$$
$$\therefore \quad 3x < 24$$
$$\therefore \quad \frac{3x}{3} < \frac{24}{3} \text{ (using property 1)}$$
$$\therefore \quad x < 8$$

(ii)

$$\frac{-x}{2} \le 2$$
$$\therefore \quad \frac{-x}{2}\times 2 < 2\times 2 \quad \text{(by property 1)}$$
$$\therefore \quad -x \le 4$$
$$\therefore \quad -x\times -1 \ge 4\times -1 \quad \text{(using property 4)}$$
$$\therefore \quad x \ge -4$$

Example: Find the greatest possible value of x, when the arithmetic mean of 5, 7 and x is less than 24.

Solution:

The arithmetic mean of three numbers 5, 7 and x is

$$\frac{5+7+x}{3}$$

By given condition $\frac{5+7+x}{3} < 24$

Now $\frac{(5+7+x)}{3}\times 3 < 24\times 3$ (using property 1)

$$\therefore \quad 12+x < 72$$
$$\therefore \quad 12+x-12 < 72-12 \text{ (using property 1)}$$
$$\therefore \quad x < 60$$

Thus the greatest possible value of x is 59.

Example:

Solve $\frac{x}{4}-4 > \frac{x}{5}$

Solution:

$$\frac{x}{4}-4 > \frac{x}{5}$$

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$$\frac{x}{4}-\frac{x}{5}-4+4 > \frac{x}{5}-\frac{x}{5}+4 \quad \text{(using property 1)}$$

$$\therefore \quad \frac{x}{4}-\frac{x}{5} > 4$$

$$\therefore \quad \frac{5x-4x}{20} > 4$$

$$\therefore \quad \frac{x}{20} > 4$$

$$\therefore \quad \frac{x}{20}\times 20 > 4\times 20 \quad \text{(using property 1)}$$

$$\therefore \quad x > 80$$

Multiple Choice Questions (MCQs)

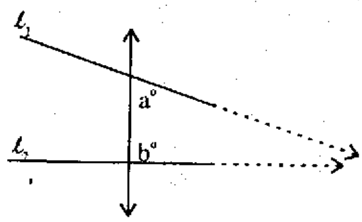
Q1. If $xy > 0$ and $x < 0$, which of the following negative?

(A) $-x$ (B) $-y$
(C) y (D) xy

Q2. If $a > 0$, $b > 0$ and $a - b < 0$, then?

(A) $a < b$ (B) $a + b < 0$
(C) $a > b$ (D) $b - a < 0$

Q3. If lines ℓ_1 and ℓ_2 meet when extended to the right, which inequality best expresses the relationship between a and b?



(A) $a = b$ (B) $a + b < 180$
(C) $a - b > 0$ (D) $b - a > 0$

Q4. If $a + b > 7$ and $a - b > 5$, then which of the following gives all possible values of a and only possible value of a?

(A) $a > 6$ (B) $a < 5$
(C) $a > 4$ (D) $a < 7$

Q5. If $A > B$ an $C < 0$, then which of the following is not true?

(A) $AC < BC$ (B) $A + C > B + C$
(C) $A - C < B - C$ (D) All of the above

Q6. If $a = 1$ and $1 > b > 0$, then which of the following statement is true?

(A) $a = b$ (B) $b > a$
(C) $\frac{1}{b} > a$ (D) $\frac{1}{a} < b$

Q7. If $a < c$ and $a < b$, assume $a \geq 0$ then which of the following statements are always true?

(i) $b < c$ (ii) $a < bc$ (iii) $2a < b + c$

(A) only (i) (B) only (ii)
(C) only (iii) (D) (i) and (ii)

Q8. If $6 - a > 7$, then

(A) $a > 1$ (B) $a > -1$
(C) $a < -1$ (D) $a < 1$

Q9. a has to be a whole number such that $0 \leq a \leq 10$. The solution for $a < 4$ and $a \geq 6$ is:

(A) 5 (B) 7
(C) 3 (D) no solution

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Q10. **If $5x > 2$ and $\frac{1}{2}x \leq 4$, list all the possible integral values of x?**

(A) 2, 3, 4, 5, 6 (B) 1, 2, 3, 4, 5, 6, 7, 8
(C) 2, 3 (D) 1

Q11. **The solution of the inequality $-1 < 5x - 6 \leq 4$ in whole number is**

(A) 1 (B) 2
(C) 4 (D) 5

Q12. **In inequality $y > 3x - 2$ if $a > b$, then which of the following statement is true?**

(A) $x = 1$ (B) $x > 1$
(C) $x < 1$ (D) $x \geq 1$

Q13. **If $\frac{a}{2} - 2 > \frac{a}{3}$, then which of the following statement is true?**

(A) $a < 12$ (B) $a > 12$
(C) $a = 12$ (D) $a \geq 12$

Q14. **Which of the following inequalities is the solution of the inequality $7a - 5 < 2a + 18$?**

(A) $a < 23$ (B) $a > 13$
(C) $a \leq 23$ (D) $a \geq 13$

Q15. **For which values of p is $p^2 - 5p + 6$ negative?**

(A) $p < 0$ (B) $2 < p < 3$
(C) $x > 3$ (D) $x < 2$

Explanatory Answers

Q1. (C) The product of two numbers > 0 is only possible when either both numbers are positive or both are negative. Since $x < 0$, y must also be negative.

Q2. (A) In this case a and b are both positive ($a > 0$, $b > 0$), but $a - b$ is negative, which is only possible when $a < b$.

Q3. (B) When the lines will be extended to the right. They will make a triangle, and the sum of the angles of the triangle is 180°. Therefore, the sum of the two angles in a triangle is less than 180°.

Q4. (A) Since both inequalities have the same direction, therefore the corresponding sides can be added. Thus,

$$\begin{array}{r} a + b > 7 \\ \underline{a - b > 5} \\ 2a > 12 \\ \boxed{a > 6} \end{array}$$

Q5. (C) If $A > B$ and $C < 0$, then multiplication of both sides by C reverses the inequality. Which implies
$AC < BC$. Also adding and subtracting in inequality, gives

$$A + C > B + C \quad \text{and}$$
$$A - C > B - C$$

But $A - C < B - C$ is not possible.

Q6. (C) Since b is a +ve fraction less than 1, therefore $\frac{1}{b}$ is a positive fraction greater than 1. Hence

$$\frac{1}{b} > a$$

Q7. (C) Statements (i) and (ii) are not always true.

Q8. (C) Given $6 - a > 7$
$\Rightarrow$ $-a > 1$
Dividing both sides by -1. This will reverse the inequality sign
$a < -1$

Q9. (D) Given set is {0,1,2,3,4,5,6,7,8,9,10}, the number $a < 4$ are {0,1,2,3} and the numbers $a \geq 6$ are {6,7,8,9,10}. Since there are no common elements between the last two sets. Therefore, there

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is no solution of the inequality.

Q10. (B) $5x > 2$ $\qquad \frac{1}{2}x \le 4$

$x > \frac{2}{5}$ $\qquad \frac{1}{2}x \times 2 \le 4 \times 2$

$x \le 8$

from above the integers greater than $\frac{2}{5}$ and less than and equal to 8 are 1,2,3,4,5,6,7,8

Q11. (B) Given $-1 < 5x - 6 \le 4$, first of all get rid -6 then 5 in the middle term

To get rid -6, add 6 to each part

$-1 + 6 < 5x - 6 + 6 \le 4 + 6$

$\Rightarrow \quad 5 < 5x \le 10$

To get rid of 5, divide each part by 5.

$\frac{5}{5} < \frac{5x}{5} < \frac{10}{5}$

$1 < x \le 2$

$\therefore$ only 2 is a whole number solution

Q12. (C) Since $x > y$ and $y > 3x - 2$, this implies that

$x > 3x - 2 \quad \Rightarrow \quad -2x > -2$

Dividing both sides by -2 will reverse the inequality symbol

$\frac{-2x}{-2} < \frac{-2}{-2}$

$\Rightarrow \quad \boxed{x < 1}$

Q13. b) Given $\frac{a}{2} - 2 > \frac{a}{3}$

adding $\frac{-a}{2}$ both sides of the inequality

$\frac{a}{2} - 2 - \frac{a}{2} > \frac{a}{3} - \frac{a}{2}$

$-2 > \frac{-a}{6} \Rightarrow \frac{-a}{6} < -2$

$-a < -12$

dividing both sides by -1 will reverse the inequality sign, therefore

$a > 12$

Q14. (A) $7a - 5 < 2a + 18$

$7a - 2a < 18 + 5$

$5a < 23$

$\Rightarrow \quad a < 23$

Q15. (B) Given $p^2 - 5p + 6$. The given expression factors into $(p - 3)(p - 2)$. If the expression is negative then the factors must have opposite signs. If $(p - 2)$ is negative and $(p - 3)$ is positive there are no such number. It is only possible when $(p - 3)$ is negative and $(p - 2)$ is positive, then $p > 2$ and $p < 3$. So, $2 < p < 3$ is the correct choice.

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Chapter 4

WORD PROBLEMS

Multiple Choice Questions (MCQs)

Q1. **If 5 is subtracted from a certain number, the result is 7 less than twice the number. What is the number?**

(A) 2 (B) $\frac{1}{2}$
(C) 5 (D) 6

Q2. **Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?**

(A) 11 (B) 12
(C) 15 (D) 13

Q3. **Two-fifth of a certain number is 30. What is the number?**

(A) 75 (B) 25
(C) 90 (D) 150

Q4. **Saira weights 25 pounds more than Umbar. If together they weigh 205 pounds, what is the weight of Saira?**

(A) 90 (B) 105
(C) 115 (D) 135

Q5. **If the sum of two numbers is 36, and the larger is three times as larger as the smaller, what is the larger number?**

(A) 27 (B) 30
(C) 15 (D) 18

Q6. **The sum of integers p and q is 352. The units digits of p is 0. If p is divided by 10, the result is equal to q, what is the value of p?**

(A) 30 (B) 230
(C) 320 (D) 32

Q7. **A soap factory has 30 packers. Each packer can load $\frac{1}{8}$ of a box in 9 minutes. How many boxes can be loaded in $1\frac{1}{2}$ hours by all 20 packers?**

(A) 28 (B) $37\frac{1}{2}$
(C) 35 (D) $35\frac{1}{2}$

Q8. **Uzma is 15 years old. Asma is one-third older then Uzma. How many years ago when Asma was twice as old as Uzma is?**

(A) 5 (B) 12
(C) 15 (D) 10

Q9. **Mohin is now three times Mohsin's age. Four years from now Mohin will be y years old. In terms of y, how old will Mohsin be?**

(A) $\frac{x-4}{3}$ (B) $\frac{x+4}{3}$
(C) $x+4$ (D) $x-4$

Q10. **If the sum of one third of a number and twice the same number is 28, the number is:**

(A) 10 (B) 12
(C) 28 (D) 14

Q11. **A man's present age is x years. If his age in 8 years will be $\frac{4}{5}$ of what it will be in 20 years, then**

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his present age is:

(A) 45 (B) 25
(C) 30 (D) 40

Q12. When 42 is added to twice a number, the result is 346, the number is:

(A) 304 (B) 242
(C) 152 (D) 265

Q13. A man was 26 years old when his daughter was born. Now, he is three times as old as his daughter. How many years old is the daughter now?

(A) 13 years (B) 22 years
(C) 15 years (D) 12 years

Q14. 13 years ago Shabbir's mother was 7 times as old as he was. She is now 48 years old. How many years old is Shabbir now?

(A) 28 (B) 18
(C) 38 (D) 20

Q15. If 5 years are added to a man's present age and that age is tripled, he will be 84. What is his present age?

(A) 18 (B) 23
(C) 32 (D) 54

Explanatory Answers

Q1. (A) Let the required number be x. Then $x - 5 = 2x - 7$

$\Rightarrow$ $\boxed{x = 2}$. Thus the correct answer is 2.

Q2. (C) Let

$$\begin{aligned} x &= \text{first integer} \\ x+2 &= \text{second integer} \\ x+4 &= \text{third integer} \\ 3(x) &= 3 + 2(x+4) \\ 3x &= 3 + 2x + 8 \\ x &= 11 \end{aligned}$$

Third integer is $(x+4) = \boxed{15}$

Q3. (A) Let the number $= x$, then

$$\frac{2}{5}x = 30$$

$$\Rightarrow x = \frac{30 \times 5}{2}$$

$$\Rightarrow x = 75$$

Q4. (C)Let the weight of Saira $= x$

and Umber's weight $= y$

$$\begin{aligned} x - 25 &= y \\ \text{and} \quad x + y &= 205 \\ \Rightarrow \quad x - y &= 25 \\ \underline{x + y} &\underline{= 205} \\ 2x &= 230 \\ x &= \frac{230}{2} = 115 \text{ pound} \end{aligned}$$

Q5. (A)Let the smaller number $= x$

Then the larger number $= 3x$

$$\begin{aligned} \text{Now} \quad 3x + x &= 36 \\ 4x &= 36 \\ x &= 9 \end{aligned}$$

The larger number is $36 - 9 = 27$

Q6. (C) $p + q = 352$ and $\frac{p}{10} = q \Rightarrow p = 10q$

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$$10q + q = 352 \Rightarrow 11q = 352 \Rightarrow q = 32$$
$$\text{Now } p + 32 = 352 \Rightarrow \boxed{p = 320}$$

Q7. **(B)** 30 packers will load $30 \times \frac{1}{8}$ or $\frac{30}{8}$ boxes in 9 minutes. There are 90 minutes in $1\frac{1}{2}$ hours. So the 30 packers will load $10 \times \frac{30}{8}$ or $37\frac{1}{2}$ boxes in $1\frac{1}{2}$ hours.

Q8. **(D)** Asma is one-third older or $\frac{1}{3} \times 15 = 5$ years older. Let x be the age of Uzma and $x + 5$ be Asma's age. When Asma was twice the age of Uzma, $2x = x + 5$ or $x = 5$. Uzma was 5 years old and Asma was $x = 5$ or 10 years old, twice Uzma's age. Since Uzma is 15 years old now, Uzma was 5 years old 10 years ago.

Q9. **(A)** Assume x for Mohin and y for Mohsin

x is three times y $\Rightarrow x = 3y$

x in four years $\Rightarrow x = x + 4$

$$\Rightarrow x = 3y + 4$$
$$\Rightarrow x - 4 = 3y$$
$$\frac{x-4}{3} = y$$

Q10. **(B)** Let x be the required number, then

$$\frac{1}{3}x + 2x = 28$$
$$\Rightarrow x + 6x = 84$$
$$\Rightarrow 7x = 84$$
$$\Rightarrow \boxed{x = 12}$$

Q11. **(D)** Present age $= x$

$$x + 8 = \frac{4}{5}(x + 20)$$
$$5x + 40 = 4x + 80$$
$$5x - 4x = 80 - 40$$
$$\boxed{x = 40}$$

Q12. **(C)** Let x be the required number, then

$$2x + 42 = 346$$
$$\Rightarrow 2x = 304$$
$$\Rightarrow \boxed{x = 152}$$

Q13. **(A)** Let x be the age of man and y be the age of his daughter

$$x - 26 = y \quad ....(1)$$
$$x = 3y \quad ....(2)$$

Substituting the value of x in (1)

$$3y - 26 = y$$
$$2y = 26 \Rightarrow \boxed{y = 13}$$

Q14. **(B)** Let x be the age of Shabbir

$$7(x - 13) = 48 - 13$$
$$7(x - 13) = 35$$
$$x - 13 = 5$$
$$\boxed{x = 18}$$

Q15. **(B)** Let x be the man's present age, then

$$3(x + 5) = 84$$
$$\Rightarrow x + 5 = 28$$
$$\Rightarrow \boxed{x = 23}$$

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