SECTION I

NTS Guide

Quantitative Ability

The basic mathematical skills, understanding of elementary mathematical concepts, and the ability to reason quantitatively and solve problems in a quantitative setting are measured in the quantitative part of the test. The knowledge of **arithmetic, algebra, geometry** and **data analysis**, which are usually essential area of study of the high school level are measured in balanced questions. The questions about quantitative ability can also be asked from:

- Discrete Quantitative Questions
- Quantitative Comparison Questions
- Data Interpretation Questions etc.

This section is discussed and explained in detail in this book. Topic by topic explanation is given to facilitate the candidates. Explanatory answers are also given to avoid complications.

NTS Guide

Part 1 Arithmetic

Chapter-1: NUMBERS

Numbers:

In decimal number system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits, to represent any number.

Note: A group of figures, denoting a number is called numeral.

Types of Numbers

Natural Numbers:

Numbers which we use for counting the objects are known as natural numbers. It is denoted by 'N'.

$$N = \{1, 2, 3, 4 \ldots\ldots\}$$

Whole Numbers:

All Natural Numbers together with zero form the set of all whole numbers. It is denoted by 'W'.

$$W = \{0, 1, 2, 3, \ldots\ldots\}$$

Integers:

The set of numbers which consists of whole numbers and negative numbers is known as integers. It is denoted by Z.

$$Z = \{\ldots\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$$

Positive Integers:

The set $Z^+ = \{1, 2, 3, 4, \ldots\ldots\}$ is the set of all positive integers. It is clear that positive integers and natural numbers are synonyms.

Negative Integers:

The set $Z^- = \{-1, -2, -3, \ldots\ldots\}$ is the set of all negative integers.

Remember: "0" is neither positive nor negative.

Non-Negative Integers:
The set {0, 1, 2, 3, ...} is a set of non-negative integers.
Non-Positive Integers:
The set {0, −1, −2, −3, ...} is the set of non-positive integers.
Even Numbers:
The numbers which are divisible by 2 are called Even Numbers.

$$E = \{2, 4, 6, ...\}$$

Odd Numbers:
The numbers which are not divisible by 2 are called Odd Numbers.

$$O = \{3, 9, 11, 17, 19, ...\}$$

Properties of zero:

1. 0 is neither positive nor negative.
2. 0 is an even integer.
3. 0 is smaller than every positive number.
4. 0 is greater than every negative number.
5. For any integer p; $p \times 0 = 0$.
6. For any integer p (including 0): $p \div 0 = 0$.
7. For any positive integer p; $0 \div p$; $\frac{0}{p}$ = undefined.
8. For every integer p; $p + 0$ and $p - 0 = p$.
9. If the product of two or more numbers is 0, then at least one of them is 0.

Properties of one:

1. For any number p: $p \times 1 = p$ and $\frac{p}{1} = p$.
2. 1 is the divisor of every integer.
3. 1 is an odd integer.
4. 1 is not a prime number, because prime numbers should be greater than 1.
5. 1 is the smallest positive integer.
6. For any number n : $1^n = 1$.

Factors and Multiples:
A number which divides a given number exactly is called a factor of the given number.
Example 1: Find the factors of (i) 64 and (ii) 75.
Solution: (i) $64 = 1 \times 64$
$= 2 \times 32$
$= 4 \times 16$
$= 8 \times 8$
The factors of 64 are 1, 2, 4, 8, 16, 32 and 64.
(ii) $75 = 1 \times 75$
$= 3 \times 25$
$= 5 \times 15$
The factors of 75 are 1, 3, 5, 15, 25 and 75.
Division Algorithm:
Let a and b be two given integers such that $b \neq 0$. On dividing a by b, let q be the quotient and r the remainder, then $a = bq + r$.
Clearly, $0 < r < b$
In general, we have

Dividend = (Divisor × Quotient) + Remainder

Multiple of a Number:
A multiple of any natural number is a number obtained by multiplying that number by any natural number.
Example: Find the multiples of:
(i) 4 less than 30 (ii) 9 less than 60

Solution: (i) $4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
$4 \times 5 = 20$
$4 \times 6 = 24$
$4 \times 7 = 28$ etc.

$\therefore$ The multiples of 4 less than 30 are 4, 8, 12, 16, 20, 24 and 28.

(ii) $9 \times 1 = 9$
$9 \times 2 = 18$
$9 \times 3 = 27$
$9 \times 4 = 36$
$9 \times 5 = 45$
$9 \times 6 = 54$ etc.

$\therefore$ The multiples of 9 less than 60 are 9, 18, 27, 36, 45 and 54.

Divisible of a Number:

If a number divides a second number without leaving any remainder, then we say that the second number is divisible by the first number. For example, since the number 2 divides 14 without leaving any remainder, we say that 14 is divisible by 2.

Multiple Choice Questions (MCQs)

Q1. How many numbers between 200 and 500 are divisible by 13?
(A) 23 (B) 17
(C) 15 (D) 32

Q2. The first five multiples of 17 are:
(A) 0, 1, 17, 34, 51 (B) 17, 34, 51, 68, 85
(C) 38, 57, 76, 95, 114 (D) None of these

Q3. The number which is divisible by 7 but not by 14 is:
(A) 21 (B) 12
(C) 71 (D) None of these

Q4. The total number of even prime numbers is:
(A) 0 (B) 1
(C) 2 (D) None of these

Q5. The least prime number is:
(A) 0 (B) 1
(C) 2 (D) 3

Q6. The smallest member of set W is:
(A) 0 (B) 1
(C) 2 (D) 3

Q7. The smallest even number of three digits is:
(A) 98 (B) 102
(C) 998 (D) 100

Q8. The smallest 4-digit number using 7, 0, 8 and 9 is:
(A) 0879 (B) 0789
(C) 0978 (D) 7890

Q9. The cube of $\frac{1}{2}$ is:
(A) $\frac{1}{4}$ (B) $\frac{1}{8}$
(C) $\frac{1}{2}$ (D) $\frac{1}{16}$

Q10. $3 - 7 =$

(A) -7 (B) 7
(C) -4 (D) 4

Q11. If 1 is added to the denominator of a fraction, it becomes $\left(\frac{1}{2}\right)$ and if 1 is added to the numerator, the fraction becomes 1. The fraction is:

(A) $\frac{4}{7}$ (B) $\frac{10}{11}$
(C) $\frac{2}{3}$ (D) $\frac{5}{9}$

Q12. How many two-digit numbers are there which are divisible by 6?

(A) 17 (B) 18
(C) 16 (D) 15

Q13. A number whose fifth part increased by 5 is equal to its fourth part diminished by 5, is:

(A) 160 (B) 180
(C) 200 (D) 220

Q14. If $(5^a)(5^b) = \frac{5^c}{5^d}$, what is d in terms of a, b and c?

(A) $a + b - c$ (B) $a - b + c$
(C) $a + b + c$ (D) $c - a - b$

Q15. Which of the following is equal to $(3^8 \times 3^9)^{10}$?

(A) 3^{720} (B) 3^{170}
(C) 3^{27} (D) 3^{98}

Q16. If $0 < p < 1$, which of the following lists the numbers are in increasing order?

(A) $p, \sqrt{p}, p^2$ (B) $p, p^2, \sqrt{p}$
(C) $\sqrt{p}, p, p^2$ (D) $p^2, p, \sqrt{p}$

Q17. The value of x satisfying $\sqrt{5 + \sqrt[3]{x}} = 3$ is:

(A) 64 (B) 27
(C) 125 (D) 9

Q18. If, $x^{x\sqrt{x}} = (x\sqrt{x})^x$, then $x =$

(A) $\frac{1}{2}$ (B) $\frac{9}{4}$
(C) $\frac{3}{2}$ (D) $\frac{1}{4}$

Q19. $(16)^{7/4}$ is equal to:

(A) 28 (B) 128
(C) 27 (D) None of these

Q20. $\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is:

(A) 30 (B) 60
(C) 75 (D) 90

Explanatory Answers

Q1.(A) Number of numbers up to 200 which are divisible by 13

$$= \frac{200}{13} = 15 + \frac{5}{13}, \text{ i.e., } 15$$

Number of numbers up to 500 which are divisible by 13

$$= \frac{500}{13} = 38 + \frac{6}{13} \text{ i.e., } 38$$

The required numbers = 38 – 15 = 23

Hence, the correct answer is choice A.

Q2.(B) The first five multiples of 17 are

$$17 \times 1 = 17$$
$$17 \times 2 = 34$$
$$17 \times 3 = 51$$
$$17 \times 4 = 68$$
$$17 \times 5 = 85$$

First five multiples of 17 are 17, 34, 51, 68 and 85.

Q3.(A) The number which is divisible by 7 but not by 14 is 21. Hence, the correct answer is choice A.

Q4.(B) There is only one even prime number, namely 2. Hence, the correct answer is choice C.

Q5.(C) The least prime number is 2. Hence, the correct answer is choice C.

Q6.(A) 0 is the smallest member of the set W. Hence, the correct choice is A.

Q7.(D) The smallest even number of three digits is 100. The correct choice is choice D.

Q8.(B) Using 0, 7, 8, 9, the smallest number is 0789. Hence, the correct answer is choice B.

Q9.(B) $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$$= \frac{1 \times 1 \times 1}{2 \times 2 \times 2} = \frac{1}{8}$$

Correct answer is choice B.

Q10.(C) $3 + (-7) = 3 - 7 = -4$

Correct answer is choice C.

Q11.(C) Let the fraction be $\frac{x}{y}$. Then $\frac{x}{y+1} = \frac{1}{2}$ and $\frac{x+1}{y} = 1$

First we solve $\frac{x}{y+1} = \frac{1}{2}$ $\Rightarrow y + 1 = 2x$

$\Rightarrow 2x - y = 1$ ...(i)

Similarly $\frac{x+1}{y} = 1$ $\Rightarrow x + 1 = y$

$\Rightarrow x - y = -1$ ...(ii)

Subtracting (ii) from (i), we have

$$\begin{array}{r} 2x - y = 1 \\ -x + y = -1 \\ \hline x = 2 \end{array}$$

$x = 2, \Rightarrow x - y = -1 \Rightarrow 2 - y = -1 \Rightarrow y = 3$

Hence, the required fraction is $\frac{2}{3}$.

The correct answer is choice C.

Q12.(D) Required numbers are 12, 18, 24, ... 96

Here, $a = 12$ and $d = 6$

$$\begin{aligned} Tn = 96 \quad &\Rightarrow a + (n-1)d = 96 \\ &\Rightarrow 12 + (n-1)6 = 96 \\ &\Rightarrow 12 + 6n - 6 = 96 \\ &\Rightarrow 6(n+1) = 96 \\ &\Rightarrow n + 1 = \frac{96}{6} = 16 \\ &\Rightarrow \boxed{n = 15} \end{aligned}$$

Hence, the correct answer is choice D.

Q13.(C) $\frac{x}{5} + 5 = \frac{x}{4} - 5 \Rightarrow \frac{x}{4} - \frac{x}{5} = 10$

$$\Rightarrow 5x - 4x = 200$$
$$\Rightarrow x = 200$$

Hence, the correct answer is choice C.

Q14.(D) $(5^a)(5^b) = \frac{5^c}{5^d}$

$5^{a+b} = 5^{c-d}$ (By power rule)

$\Rightarrow a + b = c - d$

$\Rightarrow d = c - a - b$

Hence, the correct answer is choice D.

Q15.(B) Given that, $(3^8 \times 3^9)^{10}$

$= (3^{8+9})^{10}$ (By power rule)

$= (3^{17})^{10}$

$= 3^{17 \times 10}$

$= 3^{170}$

Hence, the correct answer is choice B.

Q16.(D) For any number p, between 0 and 1

$p^2 < p$ and $p < \sqrt{p}$

Hence, the correct answer is choice D.

Q17.(A) $\sqrt{5 + \sqrt[3]{x}} = 3$

$5 + \sqrt[3]{x} = 9$ (Squaring both sides)

$\sqrt[3]{x} = 9 - 5$

$\sqrt[3]{x} = 4$

$((x)^{1/3})^3 = (4)^3$

$x^{1/3 \times 3} = 4 \times 4 \times 4$

$\boxed{x = 64}$

Hence, the correct answer is choice A.

Q18.(B) $x^{x\sqrt{x}} = (x\sqrt{x})^x$

$x^{x\sqrt{x}} = (x \cdot x^{1/2})^x$

$\Rightarrow x^{x\sqrt{x}} = (x^{3/2})^x$

$\Rightarrow x^{x\sqrt{x}} = (x^{3x/2})$

$\Rightarrow x\sqrt{x} = \frac{3x}{2}$

$\Rightarrow \sqrt{x} = \frac{3}{2}$

$\Rightarrow \boxed{x = \frac{9}{4}}$

Hence, the correct answer is choice B.

Q19.(B) $(16)^{7/4}$

$= (2^4)^{7/4}$

$= 2^{4 \times 7/4}$

$= 2^7$

$= 128$

Hence, the correct answer is choice B.

Q20.(B) $\frac{4}{5}x - \frac{2}{3}x = 8$

$\Rightarrow 12x - 10x = 120$

$\Rightarrow 2x = 120$

$\Rightarrow x = 60$

Hence, the correct answer is choice B.

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Chapter 2

MULTIPLICATION AND DIVISION

MULTIPLICATION

Multiplication is a short method of adding the same number repeatedly.

PROPERTIES OF MULTIPLICATION

1. Multiplication is commutative for rational numbers.
   Example:
$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}=\frac{c}{d}\times\frac{a}{b}$$
$$\frac{2}{3}\times\frac{5}{7}=\frac{10}{21}=\frac{5}{7}\times\frac{2}{3}$$
2. Multiplication is associative for rational numbers.
   Example:
$$\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)=\frac{ace}{bdf}=\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}$$
$$\frac{2}{3}\times\left(\frac{5}{7}\times\frac{11}{13}\right)=\frac{110}{273}=\left(\frac{2}{3}\times\frac{5}{7}\right)\times\frac{11}{13}$$
3. Multiplication is distributive over addition and subtraction for rational numbers.
   Example:
$$\frac{a}{b}\times\left(\frac{c}{d}\pm\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}\pm\frac{a}{b}\times\frac{e}{f}$$
$$\frac{2}{3}\times\left(\frac{5}{7}\pm\frac{11}{13}\right)=\frac{2}{3}\times\frac{5}{7}\pm\frac{2}{3}\times\frac{11}{13}$$
4. For any rational number $\frac{x}{y}$, $\frac{x}{y}\times 1=\frac{x}{y}=1\times\frac{x}{y}$, one is called multiplicative identity.
5. Two rational numbers $\frac{a}{b}$ and $\frac{b}{a}$ are the multiplicative inverses of each other.
$$\frac{a}{b}\times\frac{b}{a}=1=\frac{b}{a}\times\frac{a}{b}$$

Note: The sign of the product is +ive, if there are an even number of negative factors or there are no negative factors. The sign of the product is –ive, if there are an odd number of negative factors

DIVISION

The process of subtraction of the same number form a given number for a few times is called division (÷), *i.e.*,

$6 \div 2 = 3$

(2 can be subtracted 3 times from 6)

IMPORTANT POINTS

1. Division is the inverse operation of multiplication. For example $6 \div 2 = 3$ means to find the number by which 2 should be multiplied so as to obtain 6.
   Because $3 \times 2 = 6$
   Therefore, $6 \div 2 = 3$
2. When a number is divided by another number, the first number *i.e.* the number which is being divided is called the ***dividend***, the second number which divides is called the ***divisor*** and the number obtained as a result of division is called the ***quotient***. In the above example, 6 is the dividend, 2 is the divisor and 3 is the quotient.
3. The operation of division starts from the left whereas the operations of addition, subtraction and multiplication start from the right.

Divisibility:

The following table gives the rules to test the divisibility from 2 to 19.

Divisibility by	If	Example
2	Any number in the unit's place which is either even or zero.	12, 10, 26, 32, 38, 567992, 11110234
3	The sum of digits is divisible by 3.	321 : 3 + 2 + 1 = 6 is divisible by 3.
4	The last two digits of a number is divisible by 4.	725324 : 24 is divisible by 4.
5	The number ends with 5 or zero.	4112370, 5321095, 3331210, etc.
6	A number is divisible by 2 and the sum of the digits of the number is multiple of 3.	342, 63924, 154, 261 etc.
8	The last three digits of a number is divisible by 8. or The last three digits of a number are zero.	2125000, 135923120, 7792320, 1256, etc.
9	The sum of all the digits of a number is divisible by 9.	33456735 : 3 + 3 + 4 + 5 + 6 + 7 + 3 + 5 = 36 divisible by 9.
10	Any number which ends with zero.	70, 789790, 7111130, 5773313570, 112300100 etc.
11	The sum of digits at odd and even places are respectively equal or differ by a number divisible by 11.	4235682 : Sum 1 = 4 + 3 + 6 + 2 = 15 Sum 2 = 2 + 5 + 8 = 15 Sum 1 = Sum 2, the number is divisible by 11. or 283712 : Sum 1 = 2 + 3 + 1 = 6 and Sum 2 = 8 + 7 + 2 = 17, their differ 17 – 6 = 11 is divisible by 1.
12	The number which is divisible by both 4 and 3.	135792 etc.
14	The number which is divisible by both 2 and 7.	98, 504 etc.
15	The number which is divisible by 3 and 5.	360, 733352215 etc.
16	The number whose last 4 digit number is divisible by 16.	253421020, 27954204 etc.
18	Any number which is divisible by 9 and has its last digit even (or zero).	2709360, 252630 etc.
25	The number formed by the last two digits of the number is divisible by 25.	257275, 25277750 etc.

Model Examples:

Q1. Multiply 63987 by 91763 is not more than 3 lines.

Solution:

$$\begin{array}{rl} 63986 & \\ (\times)\ 91763 & \\ \hline 4031181 & \text{Multiplication by } 63 \\ 4479099\times\times & \text{Multiplication by } 700 \\ 582817\times\times\times & \text{Multiplication by } 91000 \\ \hline 5871639081 & \end{array}$$

Q2. Find the number, one-sixth of which exceeds its one-ninth by 654.

Solution: Let the number be x

$$\therefore \quad \frac{x}{6} - \frac{x}{9} = 654$$

$$\frac{x}{18} = 654$$

$$\Rightarrow \quad x = 654 \times 18 = 11772 \qquad \text{Ans.}$$

Q3. Find the quotient and remainder when $x^2 + bx - 5$ is divided by $x + 1$. For what value of 'b' will the remainder be zero?

Solution:

$$x+1\overline{)\,x^2+bx-5\,}\,(x+(b-1)$$

$$\begin{array}{l} x^2+x \\ \hline (b-1)x-5 \\ (b-1)x-1+b \\ \hline -4-b \end{array}$$

So Quotient $= x+b-1$ Ans.

Remainder $= -(b+4)$

For remainder $= 0$

$$-b-4=0$$

$$\Rightarrow \boxed{b=-4}$$

Q4. The speed of mail train is 1,370 meters per minute. Express it in miles per hour correct to three significant figures, given that 1 metre = 39.37 inches.

Solution: Speed of mail train = 1,370 metres per minute

$$= 1370 \times 60 \text{ metres per hour}$$

$$= \frac{1370\times60\times39.37}{12\times3\times1760} \text{ miles per hour}$$

$$= 51.077 \text{ miles per hour} \quad \textbf{Ans.}$$

Q5. A boy when asked to multiply a number by 7/8, divided this instead, by 7/8 and found the answer $1\frac{1}{14}$ too great. Find the number and the correct answer.

Solution: Let the number be 'x'

$$\therefore \quad \left(x\div\frac{7}{8}\right)-\left(x\times\frac{7}{8}\right)=\frac{15}{14}$$

$$\frac{8x}{7}-\frac{7x}{8}=\frac{15}{14}$$

$$\frac{64x-49x}{56}=\frac{15}{14}$$

or $$\frac{15x}{56}=\frac{15}{14}$$

$$\therefore \quad x=\frac{56\times15}{14\times15}=4 \quad \textbf{Ans.}$$

Correct answer $= 4\times\frac{7}{8}=3\frac{1}{2}$ **Ans.**

Q6. The sum of the squares of two consecutive integers is 1105. Find the integers and check your answer.

Solution: Let the two consecutive positive numbers be:

$$x, x+1$$

Then sum of the squares of these consecutive numbers = 1105

$$\therefore \quad x^2+(x+1)^2 = 1105$$

$$x^2+x^2+2x+1 = 1105$$

$$2x^2+2x-1104 = 0$$

$$x^2+x-552 = 0$$

or $$x^2+24x-23x-552 = 0$$

$$x(x+24)-23(x+24) = 0$$

$$(x-23)(x+24) = 0$$

$$\Rightarrow \quad x=23 \quad \text{or} \quad x=-24$$

As the two consecutive numbers are +ve integers, therefore, we neglect the –ve number.

Thus the two consecutive numbers are 23 and 24. **Ans.**

Chapter 3

HIGHEST COMMON FACTOR & LEAST COMMON MULTIPLE

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

Methods of finding H.C.F.

(i) ***By Prime Factors.***

Resolve the given number into their prime factors. The product of all prime common factors is known as H.C.F.

Model Example

Find the H.C.F. of 630, 1050 and 1260.

Solution: $630 = 2.3.3.5.7$

$1050 = 2.3.5.5.7$

$1260 = 2.2.3.3.5.7$

$\therefore$ H.C.F. is $2.3.5.7 = 210$. **Ans.**

(ii) ***By Division*:**

Find the H.C.F. of 5133 and 3953

$$
\begin{array}{r}
3953\overline{)\,5133\,}(1 \\
\underline{3953} \\
1180\overline{)\,3953}(3 \\
\underline{3540} \\
413\overline{)\,1180}(2 \\
\underline{826} \\
354\overline{)\,413}(1 \\
\underline{354} \\
59\overline{)\,354}(6 \\
\underline{354} \\
\times
\end{array}
$$

Various Steps:

Step I. Dividing the greatest number by the lesser, we get the remainder 1180.

Step II. Dividing the previous divisor 3953 by 1180, we get the remainder 413.

Step III. Dividing the previous divisor 1180 by 413 we get the remainder 354.

Step IV. Dividing the previous divisor 413 by 354 we get the remainder 59.

Step V. Dividing the previous divisor 354 by 59 we get no remainder.

$\therefore$ The last divisor 59 is the H.C.F.

H.C.F. is also known as Greatest Common Measure (G.C.M.)

LEAST COMMON MULTIPLE (L.C.M)

The Least Common Multiple of two or more given numbers is the least number which is exactly divisible by each of them.

Methods of Finding L.C.M.

(i) ***By Factors.*** Resolve the given numbers into prime factors, and find the product of the highest powers of all the factors that occur in the given number. The product will be the required L.C.M.

Model Example

Q1. Find the L.C.M. of 70, 80, 90.

Solution: $70 = 2 \times 5 \times 7$

$80 = 2^4 \times 5$

$90 = 2 \times 3^2 \times 5$

L.C.M. $= 2^4.3^2.5.7 = 5040$ Ans.

(ii) ***With the help of H.C.F.*** The product of two numbers is equal to the product of their L.C.M. and H.C.F.

$\therefore$ L.C.M. of two numbers

$$= \frac{\text{Product of numbers}}{\text{H.C.F.}}$$

L.C.M. and H.C.F. of Factions.

L.C.M. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

Model Examples

Q1. The H.C.F. of two numbers is 34 and their L.C.M. is 4284. If one of the numbers is 204, find the other.

Solution: As product of 2 numbers = their H.C.F. × L.C.M.

The other number is $= \frac{34 \times 4284}{204}$

$= 714$ **Ans.**

Q2. What is the highest number of four digits which will leave a remainder of 1 when divided by any of numbers 6, 9, 12, 15, or 18?

Solution: L.C.M. of 6, 9, 12, 15, 18 = 180

Greatest no. of 4 digits = 9999

Greatest no. of 4 digits divisible by

180 = 9999 − 99 = 9900

$$\begin{array}{r} 55 \\ 180 \overline{)\,9999} \\ \underline{900} \\ 999 \\ \underline{900} \\ 99 \end{array}$$

∴ Reqd. No. = 9900 + 1 = 9901 **Ans.**

Q3. Three men A, B and C go walking round a circle one mile in circumference at the rates of 160, 120 and 105 yards per minute, respectively. If they all start together and walk in the same direction, when will they first be together again?

Solution: Circumference of the circle = 1 mile or 1760 yds.

A will complete the circle in

$= \frac{1760}{160} = 11$ min.

B will complete the circle in

$= \frac{1760}{120} = \frac{44}{3}$ min.

C will complete the circle in

$= \frac{1760}{105} = \frac{352}{21}$ min.

L.C.M. of 11, $\frac{44}{3}, \frac{352}{21} = 352$ minutes.

i.e., they will be together again first after 352 min. or 5 hrs. 52 min. **Ans.**

Multiple Choice Questions (MCQs)

Q1. **A neon sign flashes every 3 seconds, another sign flashes every 5 seconds, and a third flashes every 7 seconds. If they all flash together, how many seconds will pass before they all flash simultaneously again?**

(A) 15 seconds **(B)** 35 seconds

(C) 105 seconds **(D)** 21 seconds

Q2. **The greatest number which exactly divides 1155 and 735 is:**

(A) 25 **(B)** 5

(C) 15 (D) 105

Q3. The least number which when divided by 35, leaves remainder of 25; when divided by 45 leaves a remainder of 35 and when divided by 55 leaves 45 as remainder, is:

(A) 3455 (B) 3465
(C) 3475 (D) 10

Q4. The L.C.M of 12,20,24,32 is:

(A) 240 (B) 360
(C) 480 (D) 600

Q5. How many whole bricks 6 × 12 × 24 cm will be sufficient to construct a solid cube of minimum size?

(A) 4 (B) 6
(C) 8 (D) 12

Q6. If the L.C.M and H.C.F of two numbers are 150 and 30 respectively, and one of the numbers is 18, find the other number?

(A) 250 (B) 180
(C) 150 (D) 170

Q7. The product of two numbers is 2500. If their L.C.M is 125, then their H.C.F is:

(A) 20 (B) 250
(C) 125 (D) None of these

Q8. It takes Riaz 30 minutes to mark a paper. Razi only need 25 minutes to mark a paper. If they both start marking papers at 11 : 00 AM, what is the first time they will finish marking a paper at the same time?

(A) 12 : 30 (B) 12 : 45
(C) 1 : 30 (D) 12 : 25

Q9. Sonia buys two off-cuts of ribbon in a sale. One is 153 cm long. The other is 204 cm long. She cuts them so that she ends up with a number of pieces all the same length. What is the greatest length each piece can be?

(A) 39 (B) 6
(C) 17 (D) 51

Q10. A farmer wants to fence a triangular field. He plans to put a fencing post in each corner and place other posts at equal distance along its sides. He wants the posts to be as far apart as possible. The sides of the field are 477 feet 2412 feet and 639 feet long. How far apart will the posts be?

(A) 18 feet (B) 9 feet
(C) 27 feet (D) 159 feet

Q11. Find the greatest number of 4 digits which when divided by 18, 24, 30 and 36 leaves a remainder 17 in each case.

(A) 360 (B) 9360
(C) 3600 (D) 9377

Q12. The least number which when divided by 12, 15 and 18 leaves 5 as remainder in each case is:

(A) 180 (B) 175
(C) 185 (D) 125

Q13. The greatest number which divides 2400 and 3600 leaving 48 and 60 respectively, as remainder is:

(A) 9 (B) 7
(C) 17 (D) 10

Q14. Ahmed has a rectangular garden measuring 4.32m by 3.36m. He wants to divide it into square plots of equal size. What is largest sized square he can use?

(A) 0.24 (B) $\sqrt{3}$
(C) 0.48m (D) 0.16

Q15. The chairs in the school hall can be set out in 35 equal rows or in 45 equal rows or in 105 equal rows are:

(A) 600 (B) 400

(C) 40 (D) 80

Q16. **Three bells toll after intervals of 6, 9 and 15 minutes, respectively. If they toll together at 5 p.m., when will they toll together next?**

(A) 6 : 30 (B) 5 : 30

(C) 6 : 45 (D) 5 : 45

Explanatory Answers

Q1. (C) The L.C.M of 3,5 and 7 will give the answer

3	3 – 5 – 7
5	1 – 5 – 7
7	1 – 1 – 7
	1 – 1 – 1

$= 3 \times 5 \times 7 = 105$

Q2. (D) The required number is the H.C.F of 1155 and 735

$$735 \overline{)1155}\ (1$$
$$420 \overline{)735}\ (1$$
$$315 \overline{)420}\ (1$$
$$105 \overline{)315}\ (3$$
$$315$$
$$\times$$

The greatest number required is 105.

Q3. (A) The least number which is completely divided by 35, 45 and 55, is their L.C.M. which is 3465. We want to find the least number which on dividing by 35, 45 and 55 leave remainders 25, 35 and 45 respectively *i.e.,* 10 less than the quotient in each case. Hence such a number is 3465 – 10 = 3455

Q4. (C)

2	12–20 – 24–32
2	6 – 10 – 12 – 16
2	3 – 5 – 6 – 8
2	3 – 5– 3 – 4
2	3 – 5– 3 – 2
3	3 – 5– 3 – 1
5	1 – 5 – 1 – 1
	1 – 1 – 1 – 1

The L.C.M. of 12, 20, 24 and 32 is

$2^5 \times 3 \times 5 = 32 \times 3 \times 5 = 480$

Q5. (C) One edge of the minimum cube must be 24 cms, the least common multiple of 6, 12 and 24. Thus, it will have a volume of 24 × 24 × 24 cubic centimeters which is equal to 8 bricks

i.e., $\frac{24 \times 24 \times 24}{6 \times 12 \times 24} = 8$

Q6. (A) Product of two numbers = L.C.M × H.C.F

$18 \times$ 2nd number $= 150 \times 30$

$$\text{2nd number} = \frac{150 \times 30}{18}$$

$= 250$

Q7. (A) Product of two numbers = L.C.M × H.C.F

$2500 = 125 \times$ H.C.F

$$\Rightarrow \quad \text{H.C.F} = \frac{2500}{125}$$

$= 20$

Q8. (C) The question asks for the first time they will finish at the same time. So, we must find least common multiple

5	25 – 30
5	5 – 6
6	1 – 6
	1 – 1

$6 \times 5 \times 5 = 150$ minutes

$= 2 : 30$ hours

So they will finish marking at 1 : 30 PM.

Q9. (D) The HCF of 153 and 204 gives the wanted length

3	153
3	51
17	17
	1

2	204
2	102
3	51
17	17
	1

$153 = 3 \times ③ \times \boxed{17}$

$204 = 2 \times 2 \times ③ \times \boxed{17}$

$HCF = 17 \times 3 = 51$

Greatest length $= 51$cm

Q10. (B) The HCF of 477, 2412 and 639 gives the wanted length.

3	477
3	159
	53

2	2412
2	1206
3	603
3	201
	67

3	639
3	213
	71

$477 = ③ \times \boxed{3} \times 53$

$2412 = 2 \times 2 \times ③ \times \boxed{3} \times 67$

$639 = ③ \times \boxed{3} \times 71$

$H.C.F = 3 \times 3 = 9$

Q11. (D) The number which is divisible by 18, 24, 30 and 36 is divisible by their L.C.M

2	18 – 24 – 30 – 36
2	9 – 12 – 15 – 18
3	9 – 6 – 15 – 9
3	3 – 2 – 5 – 3
2	1 – 2 – 5 – 1
5	1 – 1 – 5 – 1
	1 – 1 – 1 – 1

$\therefore$ L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

The greatest number of 4–digits is 9999. Now we find the greatest multiple of 360 less than 9999.

$$360 \overline{)\ 9999\ (\ 26}$$
$$720$$
$$2799$$
$$2160$$
$$639$$

Thus $9999 - 639 = 9360$ is exactly divisible by 360. But the required number leaves a remainder of 17 in each case. Hence, the number is

$9360 + 17 = 9377$

Q12. (C) Required number = L.C.M of 12, 15, 18

2	12, 15, 18
3	6 – 15 – 9
2	2 – 5 – 3
3	1 – 5 – 3
5	1 – 5 – 1
	1 – 1 – 1

∴ L.C.M $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

The required least number $= 180 + 5 = 185$

Q13. **(D)** As 48 and 60 remainders when 2400 and 3600 are divided by the numbers $2400 - 48 = 2352$ and

$3600 - 60 = 3540$ must be exactly divisible by the number.

The H.C.F of 2352 and 3540 is the required number.

$$\begin{array}{r|l} & 1 \\ 2352 & 3540 \\ & 2350 \quad 1 \\ \hline 1190 & 2350 \\ & 1190 \quad 1 \\ \hline 1160 & 1190 \\ & 1160 \quad 38 \\ \hline 30 & 1160 \\ & 90 \\ \hline & 260 \\ & 240 \quad 1 \\ \hline 20 & 30 \\ & 20 \quad 2 \\ \hline 10 & 20 \\ & 20 \\ \hline & \times \end{array}$$

a. The H.C.F of 2350 and 3540 is 10.

b. The required greatest number is 10.

Q14. **(C)**

$$\begin{array}{r|l} & 1 \\ 3.36 & 4.32 \\ & 3.36 \quad 3.5 \\ \hline .96 & 3.36 \\ & 3.36 \\ \hline & \times \end{array}$$

$0.96 \div 2 = 0.48\text{m}$

Q15. **(A)**

5	25 – 40 – 120
5	5 – 8 – 24
8	1 – 8 – 24
3	1 – 1 – 3
	1 – 1 – 1

$= 5 \times 5 \times 8 \times 3 = 600$ chairs

Q16. **(A)**

3	6 – 9 – 15
2	2 – 3 – 5
3	1 – 3 – 5
5	1 – 1 – 5
	1 – 1 – 1

L.C.M of 6, 9 and 15 $= 3 \times 2 \times 3 \times 5$

$= 90$

∴ The bells will toll after 90 minutes, it mean at 6 : 30.

Chapter 4

SQUARE ROOT

Methods of Finding Square Root:

(*i*) *By Factors.* Resolve the number into its prime factors. The square root is the product of the prime factors taken half as many times as they occur in the number.

(*ii*) *By Division.*

Model Example

Find the square root of 2480625.

Solution:

1	2480625	(1575
	1	
25	148	
	125	
307	2306	
	2149	
3145	15725	
	15725	
	×	

∴ **Ans.** 1575

Q2. Find the square root of 43.45 to four decimal places.

Solution:

6	43.45	(6.5916
	36	
125	7.45	
	6.25	
1309	12000	
	11781	
13181	21900	
	13181	
131826	871900	
	790956	
	80944	

As remainder is more than half 6.5917 **Ans.**

Q3. Find the value of $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$ correct to three decimal places.

Solution: $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2-\sqrt{3})}}$ (Rationalization)

$$= \sqrt{\frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}}$$

$$= \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2+\sqrt{3} = 2+1.732$$

$= 3.732$ **Ans.**

Unitary Method and Chain Rule

IMPORTANCE:

The unitary method and chain rule have quite an importance in our daily life. It is explained by the following model examples.

Model Examples:

Q1. In a kilometer race A can beat B by 40 metres and B can beat C by 50 metres. How many metres can A beat C in a 500 metres race?

Solution: When A covers 1000 m.

B covers $1000 - 40 = 960$ m.

and When B covers 1000 m.

C covers $1000 - 50 = 950$ m.

$\therefore$ When B covers 960 m.

C covers $\frac{950}{1000} \times 960\,\text{m} = 912$ m.

i.e., when A covers 1000 m. C covers $\frac{912}{2}$

$= 456$ m.

or when A covers 500 m. race, A will beat C by

$= 500 - 456 = 44$ m.

Q2. Divide Rs. 510 between A, B and C so that A gets 2/3 of what B gets and B gets ¼ of what C gets. Find the share of each.

Solution: Let C's share be Rs. x

$\therefore$ B's // // $= \frac{x}{4}$

A's // // $= \frac{2}{3} \times \frac{x}{4} = \text{Rs. } \frac{x}{6}$

Total amount $= \text{Rs. } 510$

$\therefore$ $x + \frac{x}{4} + \frac{x}{6} = 510$

$\frac{12x + 3x + 2x}{12} = 510$

$\therefore$ $x = \frac{12 \times 510}{17} = 360$

$\therefore$ A's share $= \frac{360}{6} = \text{Rs. } 60$

B's // $= \frac{360}{4} = \text{Rs. } 90$

C's // $= \text{Rs. } 360$

} **Ans.**

Q3. Divide Rs. 600 among A, B, and C so that Rs. 40 more than 2/5 of A's share, Rs. 20 more than 2/7 of B's share, Rs. 10 more than $\frac{9}{17}$ of C's share may be equal.

Solution: Let $\frac{2}{5}$ of A's share + Rs. 40 = $\frac{2}{7}$ of B's share + Rs. 20 = $\frac{9}{17}$ of C's share + Rs. 10 be = x.

$\therefore$ $\frac{2}{5}$ of A's share $= x - 40$

or A's share $= \frac{5}{2}(x - 40)$

Similarly B's share $= \frac{7}{2}(x - 20)$

$$\text{C's share} = \frac{17}{9}(x-10)$$

As total amount = Rs. 600

$$\therefore \quad \frac{5x-200}{2}+\frac{7x-140}{2}+\frac{17x-170}{9} = 600$$

$$\frac{45x-1800+63x-1260+34x+340}{18} = 600$$

or $142x - 3400 = 600 \times 18 = 10800$

$142x = 10800 + 3400 = 14200$

$$x = \frac{14200}{142} = 100$$

$$\therefore \quad \text{A's share} = \frac{5}{2}(100-40) = \text{Rs. } 150$$

$$\text{B's share} = \frac{7}{2}(100-20) = \text{Rs. } 280 \quad \text{Ans.}$$

$$\text{C's share} = \frac{17}{9}(100-10) = \text{Rs. } 170$$

Q4. A garrison has enough provision for 52 days. After 20 days, a reinforcement of 400 men arrives and the food would then last for 24 days only. How many men were there in the garrison originally?

Solution: Let there be x men in the garrison originally. After 20 days no. of men = $x + 400$.

If these men had not joined, the provision would have lasted for $50 - 20 = 32$ days more.

$\therefore$ For x men the provision can last for 32 days

// 1 // // // $32x$ //

For $(x + 400)$ men of provision can last for $\frac{32x}{x+400}$ days

But provision lasted for 24 days

$$\therefore \quad \frac{32x}{x+400} = 24$$

or $32x = 24x + 9600$

$8x = 9600$

or $x = 1200$ men. **Ans.**

Multiple Choice Questions (MCQs)

Q1. **What is the least positive integer which is to be added to 57592910 so that the sum may be a perfect square?**

(A) 7588 (B) 7

(C) 11 (D) 15166

Q2. **A rectangular field which is twice as long as it is broad, has an area of 14450 m^2, what is its perimeter?**

(A) 85 m (B) 510 m

(C) 165 m (D) 170 m

Q3. **The cost of the planting sugarcane at the rate of 6 paisa per square meter is Rs. 5840.64. What is the length of side of this square field:**

(A) 312 m (B) 622 m

(C) 97344 m (D) 459 m

Q4. **What is the smallest number which when subtracted from 1.00060219 gives a perfect square number?**

(A) 0.00210 (B) 210

(C) 0.00000210 (D) 0.210

Q5. The product of 313 with itself is:

(A) 97969 (B) 17.69

(C) 5536.97 (D) 195938

Q6. The size of the square which can be made using 256 square shapes with a side length of 6 cm is:

(A) 16 cm (B) 36 cm

(C) 2.67 (approx) cm (D) 96 cm

Q7. An instructor having 9224 students under him, arranges them into a square and finds 8 students to be excess. What is the number of students in the front row?

(A) 97 (B) 88

(C) 104 (D) 96

Q8. The difference between the first two perfect squares that end with 9 is:

(A) 11 (B) 40

(C) 30 (D) 120

Q9. A rectangular field which is 10 times as long as its breadth has an area of 75690 *sq m*. What is its perimeter?

(A) 275 m (B) 2750 m

(C) 1914 m (D) 191 m

Q10. A square lawn having area 0.25 *sq km* has to be enclosed with iron railings at the rate of Rs. 101-00 per metre. What will be its cost?

(A) Rs. 20200 (B) Rs. 2000

(C) Rs. 202000 (D) Rs. 100100

Q11. How many 1cm square pieces of paper can Rachael cut from a square sheet of paper with a side length of 2.1m?

(A) 22100 cm^2 (B) 44100 cm^2

(C) 27100 cm^2 (D) 36100 cm^2

Q12. If length of a rectangular field is twice that of its width. What is the perimeter of the field if its area is 1152 m^2?

(A) 256 m (B) 144 m

(C) 24 m (D) 96 m

Q13. $\sqrt{x^2+y^2}$ is equal to:

(A) $(x+y)(x-y)$ (B) $\sqrt{x^2}+\sqrt{y^2}$

(C) $x+y$ (D) None of these

Q14. $\sqrt{\frac{a^2}{16}+\frac{a^2}{25}} =$

(A) $\frac{a}{10}$ (B) $\frac{2a}{9}$

(C) $\frac{a\sqrt{41}}{20}$ (D) $\frac{41a}{20}$

Q15. The product of $\sqrt{24a}$ and $\sqrt{6a}$ is:

(A) 12a (B) $6a^2$

(C) $12a^2$ (D) $3a\sqrt{8}$

Explanatory Answers

Q1. (C) 57592910 is greater than the square of 7588 (using calculator). The next squared is the square of 7589. $(7589)^2 = 57592921$.

$\therefore$ Now $57592921 - 57592910 = 11$, which is the required integer to be added.

Q2. (B) As the length is twice as long as width and so its rectangle can be divided into 2 square regions

$$\text{The area of each square region} = \frac{14450}{2} = 7225\text{m}^2$$

$$\text{Now length of each region} = \text{Width of the field} \times 2$$
$$= \sqrt{7225}$$
$$= 85\text{m}$$
$$= 85 \times 2 = 170\text{m}$$
$$\text{Perimeter} = 2(170 + 85)$$
$$= 2(255)$$
$$= 510\text{m}$$

Q3. (A)

$$\text{Cost} = \text{Rs. } 5840.64$$
$$= 584064 \text{ paisas}$$
$$\text{Area} = \frac{584064}{6} = 97344$$
$$\text{Side} = \sqrt{97344}$$
$$= 312 \text{ m}$$

Q4. (C)

$$\begin{array}{r|l} 1 & \overline{1.00\,06\,02\,19}\ (1.0003 \\ & 1 \\ \hline 2.0003 & 00\,06\,02\,19 \\ & 6\,00\,09 \\ \hline & 210 \end{array}$$

There are eight places after the decimal in the given number. so after subtracting .0000021 from the given number the remainder would be zero. So .0000021 is the required number.

Q5. (A) $313 \times 313 = 97969$

Q6. (D) Side length $= 6\text{cm}$

Sides of 256 square shapes $= \sqrt{256} = 16$

Size of the square is $= 16 \times 6 = 96$

Q7. (D)

$$\text{Number of students in front row} = \sqrt{9224 - 8}$$
$$= \sqrt{9216}$$
$$= 96$$

Q8. (B) The first two integers those square end with 9 are 3 and 7.

The difference between there is

$$49 - 9 = 40$$

Q9. (C) Suppose breadth $= x$

Length $= 10x$

Area $= (10x) \times (x) = 75690$

$\Rightarrow$ $10x^2 = 75690$

$\Rightarrow$ $x^2 = 7569$

$\Rightarrow$ $x = 87$ = breadth

$\Rightarrow$ $10x = 870$ = length

Perimeter $= 2$(Length + breadth)

$= 2(87 + 870)$

$= 1914$ m

Q10. (C) Side of each square $= \sqrt{0.25} = 0.5$km

Length of the railings $= (.5 + .5)2 = 2$km

$= 2 \times 1000 = 2000$ m

Cost of railing $= 2000 \times 101 =$ Rs. 202000

Q11. (B) Size of the square sheet $= 2.1\text{m} = 2.1 \times 100 = 210\text{cm}$

Area of square root $= 210 \times 210$

$= 44100\ \text{cm}^2$

So, No. of 1cm square pieces will be $= 1 \times 44100 = 44100$

Q12. (C) Let the width $= b$, then length $= 2b$

Area $= b \times 2b = 2b^2$

Now $2b^2 = 1152$ (given) $\Rightarrow b^2 = 576$

$\Rightarrow$ $b = \sqrt{576} = 24$

length $= 2b \Rightarrow$ length $= 2 \times 24 = 48$

Perimeter of the field $= 2(24 + 48)$

$= 2(72) = 144$m

Q13. (D) There is no way to simplify $\sqrt{x^2 + y^2}$.

Q14. (C) $\sqrt{\frac{a^2}{16} + \frac{a^2}{25}} = \sqrt{\frac{25a^2 + 16a^2}{400}} = \sqrt{\frac{41a^2}{400}} = \frac{a\sqrt{41}}{20}$

Q15. (A) $\sqrt{24a} \times \sqrt{6a}$

$= \sqrt{144a^2}$

$= 12a$

****

Chapter 5

FRACTIONS AND DECIMALS

FRACTIONS:

If any unit be divided into any number of equal parts, one or more of these parts is called a fraction of the unit.

Example: The fractions one-fourth, two-third and three-fourth are respectively written as $\frac{1}{4}, \frac{2}{3}$ and $\frac{3}{4}$

NUMERATOR AND DENOMINATOR:

The upper number, which shows the number of parts taken to form the fraction, is called numerator.

The lower number, which indicates the number of equal parts in which the unit is divided, is called denominator.

Terms of The Fraction:

The numerator and the denominator of a fractions are called its terms.

Note: A fraction is also called a rational number.

Lowest Terms of a Fraction:

When the numerator and the denominators of a fraction have no common factor, the fraction is said to be is its lowest terms:

Example: $=\frac{6}{10}=\frac{3\times2}{5\times2}$

In the above example denominator and the numberator have a common factor, thus $\frac{6}{10}$ is not is its lowest terms. If we cancel out 2 by dividing numerator and denominator by 2 we find $\frac{3}{5}$, which has no common factor. hence $\frac{3}{5}$ is in its lowest terms.

Proper Fraction:

A proper fraction is one whose numerator is less than the denominator.

Example: $\frac{2}{3}, \frac{5}{7}, \frac{23}{46}$ are proper fractions.

Note: The value of proper fractions is always less than 1

IMPROPER FRACTION:

A fraction whose numerator is equal to or greater than the denominator is called improper fraction.

Example: $\frac{15}{13}, \frac{13}{5}$, and $\frac{21}{14}$ are improper fractions.

Note: The value of an improper fractions is always more than as equal to 1.

Mixed Fraction:

When an improper fraction is changed to consist of a whole number and a fraction, it is called a mixed fraction.

Example: The improper fraction $\frac{15}{13}$ can be written as $\frac{2}{13}$, which is a mixed fraction.

$$13\overline{)\,15}\ (1$$
$$\underline{13}$$
$$2$$

Compound Fraction:

A fractions of a fraction is called a compound fraction.

Example: $\frac{1}{3}$ of $\frac{3}{5}$ is a compound fraction.

Thus $\frac{1}{3}$ of $\frac{3}{5}=\frac{1}{\cancel{3}}\times\frac{\cancel{3}}{5}=\frac{1}{5}$

Complex Fractions:

A complex fraction is one in which the numerator or denominator or both are fractions.

Example: $\frac{3/2}{5}$, $\frac{3}{2/5}$, $\frac{2/5}{3/7}$ and $\frac{1/3+1/2}{2/3-1/5}$ are complex fractions.

Example 1: One third of one-seventh of a plot is sold Rs. 45000. What is the value of six-twnety fifth of the plot.

Solution: One third of one seventh $= \frac{1}{3} \times \frac{1}{7} = \frac{1}{21}$

Now, $\frac{1}{21}$ of a plot costs = Rs. 45000.

$\frac{6}{25}$ of the plot will cost $= \frac{45000}{1/2} \times \frac{6}{25}$

$$= \frac{45000 \times 21 \times 6}{25}$$

Example 2: A sum of money increased by its sixth part amount to Rs. 56. Find the sum.

Solution: Let x be the amount of money, thus

$$x + \frac{x}{6} = 56 \Rightarrow \frac{6x+x}{6} = 56$$

$$\Rightarrow \frac{7x}{6} = 56$$

$$\Rightarrow 7x = 56 \times 6$$

$$\Rightarrow x = \frac{56 \times 6}{7} = 48$$

VULGAR FRACTIONS

In questions of fractions signs +, –, ×, ÷, "*of*" ('of' signifies *multiplication*) and brackets are often involved. In simplifying these questions the following order must be followed:

IMPORTANT POINTS

(i) Remove the brackets.
(ii) Then quantities which are connected by *'of'* should be simplified.
(iii) Then division and multiplication are carried out.
(iv) Operation of addition and subtraction are performed at last.

Note: The above rules can be easily remembered by the word 'BODISA' of which 'B' stands for brackets, O for 'of', D for division, I for into, S for subtraction and A for addition.

Model Examples:

Example 3: $7\frac{1}{2} - \frac{1}{9}\left[3\frac{3}{4} \div \left\{\frac{5}{6} of \frac{2}{3}\left(\frac{1}{3} - \frac{\overline{1}}{4} - \frac{1}{6}\right)\right\}\right]$

Solution:

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} of \frac{2}{3}\left(\frac{1}{3} - \frac{3-2}{12}\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} of \frac{2}{3}\left(\frac{1}{3} - \frac{1}{12}\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} of \frac{2}{3}\left(\frac{3}{12}\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} of \frac{1}{6}\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \frac{5}{36}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \times \frac{36}{5}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9} \times 3 \times 9 = 7\frac{1}{2} - 3 = 4\frac{1}{2} \text{ Ans.}$$

Example 4: Simplify

$$\frac{\frac{1}{6} + \frac{5}{12} \times \left(\frac{4}{5} - \frac{5}{7}\right)}{\frac{3}{4} \text{ if } 1\frac{2}{3} - \frac{3}{5} \text{ of } 1\frac{4}{7}} \div \frac{\frac{1}{3} + \frac{1}{7} - \frac{2}{5}}{\frac{1}{5} + \frac{1}{9} - \frac{2}{7}}$$

Solution:

$$= \frac{\frac{1}{6} + \frac{5}{12} \times \left(\frac{4}{5} - \frac{5}{7}\right)}{\frac{3}{4} \times \frac{5}{3} - \frac{3}{5} \times \frac{11}{7}} \div \frac{\frac{1}{3} + \frac{1}{7} - \frac{2}{5}}{\frac{1}{5} + \frac{1}{9} - \frac{2}{7}}$$

$$= \frac{\frac{1}{6} + \frac{5}{12} \times \left(\frac{28 - 25}{35}\right)}{\frac{5}{4} - \frac{33}{35}} \div \frac{\frac{35 + 15 - 42}{105}}{\frac{63 + 35 - 90}{315}}$$

$$= \frac{\frac{1}{6} + \frac{1}{28}}{\frac{5}{4} - \frac{33}{35}} \div \frac{8}{105} \times \frac{315}{8}$$

$$= \frac{\frac{14 + 3}{84}}{\frac{175 - 132}{140}} \div 3 = \frac{17}{84} \times \frac{140}{43} \times \frac{1}{3}$$

$$= \frac{85}{387} \text{ Ans.}$$

Continued Fraction:

The fractions of the form $a + \cfrac{b}{c + \cfrac{d}{e + \cfrac{f}{g}}}$

etc. are known as continued fractions where *a, b, c,* ..... etc., are any numbers.

Note: In order to simplify such fractions, we begin with the lowest part and proceed step by step, upwards.

Model Examples:

Example 5: Simplify:

$$\left\{1 + \frac{1}{2 + \frac{2}{3 + \frac{3}{4}}}\right\} \div \left\{\frac{4}{4 + \frac{4}{3 + \frac{3}{2}}}\right\}$$

Solution:

$$\left\{1+\frac{1}{2+\frac{2}{\frac{15}{4}}}\right\}\div\left\{\frac{4}{4+\frac{4}{\frac{9}{2}}}\right\}$$

$$=\left\{1+\frac{1}{2+\frac{8}{15}}\right\}\div\left\{\frac{4}{4+\frac{8}{9}}\right\}$$

$$=1+\left\{\frac{1}{\frac{38}{15}}\right\}\div\left\{\frac{4}{\frac{44}{9}}\right\}$$

$$=\left\{1+\frac{15}{38}\right\}\div\left\{\frac{36}{44}\right\}$$

$$=\frac{53}{38}\times\frac{44}{36}=\frac{583}{342}$$ **Ans.**

DECIMAL FRACTION

A fraction involving decimal point is called decimal fraction.

Conversion of a decimal fraction into vulgar fraction:

Rule. Write down the given number in the numerator omitting the decimal point and for the denominator write 1 followed by as many zeroes as there are figures on the right of the decimal point.

As $46.76=\frac{4676}{100}$

and $199.0083=\frac{1990083}{10000}$

Model Example

Q1. Simplify

$$\frac{0.1\times0.1\times0.1+0.01+0.01\times0.01}{0.2\times0.2\times0.2+0.08+0.04\times0.02}$$

Solution:

$$=\frac{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}+\frac{1}{100}+\frac{1}{100}\times\frac{1}{100}}{\frac{2}{10}\times\frac{2}{10}\times\frac{2}{10}+\frac{8}{100}+\frac{4}{100}\times\frac{2}{100}}$$

$$=\frac{\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}}{\frac{8}{1000}+\frac{8}{100}+\frac{8}{10000}}$$

$$= \frac{\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}\right)}{8\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}\right)}$$

$$= \frac{1}{8}$$ **Ans.**

Multiple Choice Questions (MCQs)

Q1. If $\frac{5}{x}, \frac{8}{x}$, and $\frac{13}{x}$ are all in lowest terms. Then how many integers, x, between 30 and 40?

(A) 5 (B) 1
(C) 2 (D) 3
(D) None of these

Q2. $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$ equals:

(A) $\frac{1}{120}$ (B) $\frac{1}{2}$
(C) $\frac{1}{30}$ (D) 1
(D) None of these

Q3. If $\frac{4}{13}$ of a number is 39, what is $\frac{8}{13}$ of that number?

(A) $\frac{39}{4}$ (B) 78
(C) 16 (D) $\frac{39}{8}$

Q4. $\frac{3}{4}$ of 28 is equal to $\frac{30}{7}$ of what number?

(A) 90 (B) 45
(C) 30 (D) 56
(D) None of these

Q5. Which of the following is less than $\frac{5}{11}$?

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$
(C) $\frac{1}{2}$ (D) $\frac{2}{5}$
(D) None of these

Q6. There are 20 boys in a class. Five of them are left-handed. What fraction of the class is left handed?

(A) $\frac{1}{5}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{2}{11}$

Q7. A chemical solution contains 8% of acid. If there is 15ml of acid, what is the volume of the solution?

(A) 125.5 mL (B) 187.5 mL
(C) 225.5 mL (D) 171.5 mL

Q8. What fractional part of a week is 98 hours?

(A) $\frac{7}{98}$ (B) $\frac{7}{12}$
(C) $\frac{1}{20}$ (D) $\frac{1}{7}$

Q9. A village has 5860 voters, of whom 7% usually forget to vote. In order to win an election, a candidate must gain at least 50% of the remaining votes. How many votes does he need in order to win?

(A) 2725 (B) 410
(C) 5450 (D) None of these

Q10. What fraction is exactly midway between $\frac{1}{3}$ and $\frac{1}{4}$?

(A) $\frac{7}{12}$ (B) $\frac{7}{24}$
(C) $\frac{29}{11}$ (D) $\frac{1}{2}$

Q11. $\frac{4}{9}$ of a number is 12. What is the number?

(A) 27 (B) 36
(C) 18 (D) 16

Q12. Ali purchased some goldfish. During the first week, $\frac{1}{5}$ of them died, and during the second week, $\frac{3}{8}$ of those still alive at the end of the first week died. What is the fraction of the original goldfish still alive after two weeks?

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$
(C) $\frac{5}{2}$ (D) $\frac{4}{3}$

Q13. $\frac{3}{8}$ of a number is 10. What is the number?

(A) 91 (B) 81
(C) 23 (D) 27

Q14. $\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what number?

(A) 15 (B) 105
(C) 35 (D) 7

Q15. A German class has 12 boys and 18 girls. What is the fraction of the class boys?

(A) $\frac{1}{6}$ (B) $\frac{3}{5}$
(C) $\frac{2}{3}$ (D) $\frac{4}{15}$

Explanatory Answers

Q1. (D) If x is even, then $\frac{8}{x}$ will not be in lowest term. This is because, both x and 8 are divisible by 2.

Now we take the odd number between 30 and 40, these are; 31, 33, 35, 37, 39. In these numbers, we see that 35 and 39 are divisible by 5 and 13, respectively. Thus only 31, 33 and 37 are required numbers.

Q2. (A) Simplifying $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$

$$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$

Q3. (B) As $\frac{4}{13}$ of a number is 39. Therefore the $\frac{8}{13}$ of that number will be 78

Because $\frac{8}{13} = \frac{4}{13} \times 2$, and $\frac{4}{13}$ of a number is 39, therefore double of $\frac{4}{13}\left(\frac{4}{13} \times 2 = \frac{8}{13}\right)$ should be equal to $39 \times 2 = 78$.

Q4. (A) Let x be the required number, then by given condition

$$28 \div \frac{4}{3} = x \div \frac{30}{7}$$

$$28 \times \frac{3}{4} = x \times \frac{7}{30}$$

$$21 = x \times \frac{7}{30}$$

$$\frac{21 \times 30}{7} = x$$

$$\Rightarrow \boxed{x = 90}$$

Q6. (C) Left handed $= 5$

Total $= 20$

So, fraction $= \frac{5}{20} = \frac{1}{4}$

Q7. (B) 8mL acid in solution $= 100$mL

1mL acid in solution $= \frac{100}{8} = 12.5$ mL

15mL acid in solution $= 12.5 \times 15$

$= 187.5$ mL

Q8. (B) There are 7 days in a week, and each day has 24 hours. Therefore, Hours in a week $= 24 \times 7 = 168$

The required fraction is: $\frac{98}{168} = \frac{7}{12}$

Q9. (A) People does not give vote $= \frac{7}{100} \times 5860$

$= 7 \times 58.6$

$= 410.2$

People does not give vote $\cong 410$ people

Remaining people $= 5860 - 410$

$= 5450$ people

Candidate must gain vote $= 5450 \times \frac{50}{100}$

$= 2725$ vote

Q10. (B) The midway fraction of the fractions $\frac{1}{3}$ and $\frac{1}{4} = \frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right) = \frac{1}{2}\left(\frac{7}{12}\right) = \frac{7}{24}$

Q11. (A) Let the required number be "x", then according to given condition $\frac{4}{9} \times x = 12 \Rightarrow x = \frac{12}{\frac{4}{9}}$

$$= 12 \times \frac{9}{4} = 27$$

Q12. (A) Let the number of fish purchased $= x$

During first week (died fish) $= \frac{1}{5} \times x = \frac{x}{5}$

Still alive $= x - \frac{1}{5}x = \frac{4}{5}x$

During second week (died fish) $= \frac{4}{5}x \times \frac{3}{8} = \frac{3}{10}x$

Fish at the end of two weeks $= \frac{4x}{5} - \frac{3x}{10} = \frac{8x-3x}{10} = \frac{5x}{10} = \frac{1}{2}x$

So fraction $= \frac{\frac{1}{2}x}{x} = \frac{1}{2}$

Q13. (D) Let the number $= x$

Then $\frac{3}{8} \times x = 10$

$\Rightarrow x = \frac{80}{3}$

$\Rightarrow x = 26.67 = \boxed{27}$

Q14. (D) Let the number $= x$

Then $\frac{15}{7} \times x = \frac{5}{8} \times 24$

$\Rightarrow \frac{15 \times x}{7} = 15$

$\Rightarrow x = \frac{7 \times 15}{15} = \boxed{7}$

Q15. (D) No. of boys $= 12$

No. of girls $= 18$

Total $= 12 + 18 = 30$

Required fraction $= \frac{12}{30} = \boxed{\frac{4}{15}}$

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Chapter 6

PERCENTAGE

Percentage:

The term 'percent' is a short form of the Latin word 'per centum' meaning 'out of hundred'. It can best be defined as:

> "A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called the rate percent."

A rate percent is reduced to an equivalent fraction dividing it by 100.

Change of percentage into Fraction or Decimal:

To convert a percentage to a fraction, mixed number or decimal, divide it by 100, and reduce, if possible. If necessary, the relating fraction may then be changed to a decimal.

Example:

(i) Express $2\frac{1}{7}\%$ to a fraction

(ii) Change $\frac{3}{4}\%$ to a decimal.

Solution:

(i) $2\frac{1}{7}\% = \frac{15}{7}\%$

$$= \frac{15}{7} \times \frac{1}{100}\left(\text{Replace } \% \text{ by } \frac{1}{100}\right)$$

$$= \frac{3}{140}$$

$$= \frac{3}{140}$$

(ii) $\frac{3}{4}\% = \frac{3}{4} \times \frac{1}{100}\left(\text{Replace } \% \text{ by } \frac{1}{100}\right)$

$$= \frac{3}{400} = .0075$$

Change of Fraction into Percentage:

To change a fraction or a mixed numbers to a percent.

a. Multiply the fraction or mixed number by 100%.

b. Reduce, if possible

c. Affix a % sign.

Example 2:

(i) Change $\frac{1}{80}$ to a percent.

(ii) Change 0.05 to a percent

Solution:

(i) $\frac{1}{80} = \frac{1}{80} \times 100\%$

$= 1.25\%$

(ii) $0.05 = 0.05 \times 100\%$

$$= \frac{5}{100} \times 100\%$$

$$= 5\%$$

Expressing One Quantity as a Percentage of Another:

To express one quantity "p" as a percentage of another quantity "q".

a. Write p as a fraction of q.

b. Multiply the fraction $\frac{p}{q}$ by 100% to convert it to a percentage.

Example 3:

There are 56 boys in a class of 140 students. What is the percentage of the boys?

Solution:

Total students $= q = 140$

Boys $= p = 56$

Fraction $= \frac{p}{q}$

$$= \frac{56}{140}$$

Percentage $= \frac{56}{140} \times 100\%$

$$= 40\%$$

Important Tip:

If a salary of a man is first increased by x% and then it has decreased x%, the change in its initial salary is less by x% of x or $\frac{x^2}{100}$.

Note:

If two values are respectively a% and b% more than a third value, then the first is $\frac{100+a}{100+b} \times 100$'s of the second.

Example 4:

Two numbers are respectively 20% and 50% more than a third, what percentage is the first to the second?

Solution:

Following the above, we have the value

$$= \frac{100+20}{100+50} \times 100\%$$

$$= \frac{120}{150} \times 100\%$$

$$= 80\%$$

Important Tip:

If the first value is r% more than the second value, then the second is $\left[\frac{r}{100+r} \times 100\right]$% less than the first value.

Example 5:

If Hamza's salary is 35% more than that of Osama, then how much percent is Osama's salary less than that of Hamza?

Solution:

Following the above theorem, we have the value

$$= \left[\frac{35}{100+35}\times 100\right]\%$$

$$= \left[\frac{35}{135}\times 100\right]$$

Important Tip:

If the first value is r% less than the second value, then the second is $\left[\frac{r}{100-r}\times 100\right]\%$ more than the first value.

Example 6:

If Maryam's salary is 25% less than that of Fatima, then how much percent is Fatima's salary more than that of Maria?

Solution:

Following the above theorem, we have

$$\left[\frac{25}{100-25}\times 100\right]\%$$

$$= 33\frac{1}{3}\%$$

Important Tip:

a% of a quantity is taken by the first, b% of the remaining is taken by the second and c% of the remaining is taken by the third person. Now if X is left then there was

$$\frac{X\times 100\times 100\times 100}{(100-x)(100-y)(100-z)}$$

in the beginning.

Example 7:

After deduction 20% from a certain sum, and then 30% from the remainder, there is 3500 left. Find the original sum.

Solution:

Following the above theorem, we have

$$= \frac{3500\times 100\times 100}{(100-20)(100-30)}$$

$$= \frac{3500\times 100\times 100}{80\times 70}$$

$$= 6250$$

Model Examples

Example 8:

In an examination paper of 5 questions, 5 percent of the candidates answered all of them and 5 percent none of the rest, 25 percent answered only one question, and 20 percent answered only 4. If 24½ percent of the entire candidates answered only 2 questions and 200 candidates answered only 3, how many candidates appeared at the examination?

Solution: Let the total no. of candidates be x

$\frac{5x}{100}$ answered all the questions and $\frac{5x}{100}$ answered none.

$$\text{The remaining candidates} = x - \left(\frac{5x}{100} + \frac{5x}{100}\right)$$

$$= \frac{9x}{10}$$

$$\text{No. of candidates answering only one question} = \frac{25}{100} \times \frac{9x}{10} = \frac{9x}{40}$$

$$\text{No. of candidates answering four questions} = \frac{20}{100} \times \frac{9x}{10} = \frac{9x}{50}$$

$$\text{No. of candidates answering two questions} = \frac{49}{200} \times x$$

No. of candidates who answered three questions

$$= x - \left(\frac{5x}{100} + \frac{5x}{100} + \frac{9x}{40} + \frac{9x}{50} + \frac{49x}{200}\right) = 200$$

$$\Rightarrow x - \left(\frac{10x + 10x + 45x + 36x + 49x}{200}\right) = 200$$

$$\Rightarrow \frac{(200 - 50)x}{200} = 200$$

$$\Rightarrow \quad 50x = 40000$$

$$x = 800 \quad \textbf{Ans.}$$

Example 9:

The following table gives the number of the candidates (boys and girls) who appeared an examination. To complete the missing figure, find the number of candidates and their passing percentage.

Candidates	Appeared	Passed	Passing percentage *(correct to one decimal place)*
Boys	6720	3528	---
Girls	4750	---	62.4
Total	11470	---	---

Solution: Passing percentage of boys $= \frac{3528}{6720} \times 100$

$= 52.5\%$ **Ans.**

$$\text{No. of girls passed} = \frac{62.4}{100} \times 4750$$

$= 2964$ **Ans.**

Total no. of candidates passed $= 3528 + 2964$

$= 6492$ **Ans.**

$$\text{Total pass percentage} = \frac{6492}{11470} \times 100$$

$= 56.6\%$ **Ans.**

Example 10:

In 1990, the population of a town is given below:

Men	7640
Women	6675
Boys	5628

Girls	4872
Total	24815

In 1992, men increased by 5 percent, women by 8 percent and total population by 20 percent, and for every 7 boys there are 6 girls. Find the number of boy in 1992.

Solution: Increase in the no. of men in 1992. $= \frac{5}{100} \times 7640 = 382$

Total no. of men in 1992 $= 7640 + 382 = 8022$

Increase in the no. of women in 1992 $= \frac{8}{100} \times 6675 = 534$

Total no. of women in 1992 $= 6675 + 534 = 7209$

Increase in the total population in 1992 $= \frac{20}{100} \times 24815 = 4963$

Total population in 1992 $= 24815 + 4963 = 29778$

Total no. of boys and girls in 1992 $= 29778 - (8022 + 7209)$

$= 29778 - 15231$

$= 14547$

No. of boys $= \frac{7}{13} \times 14547$

$= 7833$ **Ans.**

Example 11:

Complete the following table which gives the enrolment in professional courses of one country in (1990-91) and (1995-96) and calculate the percentage increase in the first three totals of 1995-96 over those of 1990-91.

Solution:

Categories	1990-91		
	Boy	Girls	Total
Higher Secondary stage	57,676	9,511	67,187
Undergraduate stage	169,259	15,715	184,974
Postgraduate & Research	12,052	898	12,950
Total of all stages	238,987	26,124	265,111

Categories	1995-96		
	Boy	Girls	Total
Higher Secondary stage	93,760	14,585	108,345
Undergraduate stage	256,040	33,415	289,455
Postgraduate & Research	20,200	2,000	22,200
Total of all stages	370,000	50,000	420,000

Solution: Increase in no. of boys

$= 370,000 - 238987 = 131013$

% increase $= \frac{131013}{238987} \times 100$

$= 54.8\%$ **Ans.**

Increase in no. of Girls

$= 50,000 - 26124 = 23876$

% increase $= \frac{23876}{26124} \times 100$

$= 91.39\ \%$ **Ans.**

Total increase $= 420{,}000 - 265{,}111$

$= 154889$

% increase $= \dfrac{154{,}889}{265{,}111} \times 100$

$= 58.42\ \%$ **Ans.**

Example 12:

The civilian industrial products of a country are given below in million dollars. Complete the columns of 'increase over the preceding year' and 'percentage increase'.

Solution:

		Increase over the preceding year	*Percentage increase*
1951	27,003		
1952	29,314	2,311	8.60
1953	32,439	3,125	10.68
1954	36,335	3,896	12.00
1955	40,033	3,698	10.18

Example 13:

In an examination, 75% of candidates passed in English and 65% in mathematics while 15% failed both in English as well as mathematics. If 495 candidates passed in both the subjects, find the total number of candidates who took the examination.

Solution: Let the total number of students be 100

25 failed in English

35 failed in Mathematics

15 failed in both

No. of failed students $= 25 + 35 - 15 = 45$

No. of passed in both subjects $= 100 - 45 = 55$

If no. of passed students is 55, then total no. of students $= 100$

If no. of passed students is 495, total no. of students $= \dfrac{100}{55} \times 495$

$= 900$ **Ans.**

Example 14:

A candidate who gets 30 percent marks in an examination fails by 30 marks, but another candidate who gets 42 percent marks gets 42 marks more than that necessary for passing. Find the maximum number of marks and the percentage necessary for passing.

Solution: Let the passing marks $= x$

Now the candidate gets 30% marks and by this he gets 30 marks less than passing marks.

It means, he gets $(x - 30)$ marks

If he gets 30 marks, it means, maximum marks $= 100$

If he gets $(x - 30)$ marks, maximum marks $= \dfrac{100}{30}(x - 30)$ ...(i)

In the second case the candidate gets 42% marks such that he gets 42 marks above the passing marks.

It means, he gets $(x + 42)$ marks.

If he gets 42 marks, then maximum marks $= 100$

If he gets $(x + 42)$ marks, maximum marks $= \dfrac{100}{42}(x + 42)$ ...(ii)

As (i) and (ii) are equal because maximum marks are the same in both cases.

$$\frac{100}{30}(x-30) = \frac{100}{42}(x+42)$$

or $$\frac{x-30}{5} = \frac{x+42}{7}$$

or $$7x - 210 = 5x + 210$$

$$\Rightarrow \quad x = 210$$

Putting the value of x in (i) we get

Maximum marks $= \frac{100}{30}(210-30) = 600$ **Ans.**

If maximum marks are 600 then passing marks $= 210$

If maximum marks are 100 then passing marks $= \frac{210}{600} \times 100 = 35\%$

$\therefore$ Passing percentage $= 35\ \%$ **Ans.**

Multiple Choice Questions (MCQs)

Q1. **If the base of a rectangle is increased by 40% and its altitude is decreased by 20%, then its area is:**

(A) decreased by 20% **(B)** increased by 12%

(C) decreased by 12% **(D)** increased by 16%

Q2. **If x% of y is 20, then $y =$**

(A) $2000\,x$ **(B)** $\frac{100}{x}$

(C) $\frac{2000}{x}$ **(D)** $\frac{x}{200}$

Q3. **12 is $\frac{1}{3}$% of what number?**

(A) 4 **(B)** 400

(C) 36 **(D)** 3600

Q4. **If p is a positive number, 400% of p is what percent of 400 p?**

(A) 4 **(B)** 25

(C) 40 **(D)** 1

Q5. **What is 10% of 30% of 40%?**

(A) 0.12% **(B)** 0.012%

(C) 12% **(D)** 1.2%

Q6. **What percent of 75 is x?**

(A) $\frac{3}{4} - x$ **(B)** $\frac{4}{3}x$

(C) $4x$ **(D)** $3x$

Q7. **If 35 students took an exam and 13 of them failed, what percent of them passed?**

(A) 20% approx **(B)** 63% approx

(C) 25% approx **(D)** 22% approx

Q8. **There are twice as many boys as girls in an economics class. If 20% of the boys and 35% of the girls have already handed over their result cards, what percent of the students have not yet handed over their cards?**

(A) 75 (B) 65
(C) 55 (D) 15

Q9. A dealer bought an ornamental jar for Rs. 7,000 and after some days sold it for Rs. 21,000. By what percent did the value of jar increase?

(A) 300 (B) 200
(C) 150 (D) 20

Q10. On a test consisting of 60 problems, Sonia solved 75% of first 40 problems correctly. What percent of the other 20 questions does she need to solve correctly for her grade on the entire exam to be 90%?

(A) 95% (B) 65%
(C) 85% (D) cannot achieve 90%

Q11. If 60% of A is 30% of B, then B is what percent of A?

(A) 300% (B) 30%
(C) 200% (D) 3%

Q12. What percent of p is q?

(A) $\frac{q}{p}$ (B) $\frac{q}{p}$
(C) $\frac{100q}{p}$ (D) $\frac{100p}{q}$

Q13. What percent of $\frac{1}{2}$ is $\frac{5}{4}$?

(A) 2.5 (B) 1.5
(C) 250 (D) 150

Q14. In a school of 820 students, 55% are boys. The number of girls and the number of boys are:

(A) 369 boys, 451 girls (B) 281 boys, 539 girls
(C) 539 boys, 281 girls (D) 451 boys, 369 girls

Q15. Jafer drew a square. He then erased it and drew a second square whose sides were 3 times the sides of the first square. By what percent was the area of the square increased?

(A) 300% (B) 800%
(C) 400% (D) 200%

Q16. A team has won 60 percent of the 20 games for all this season. If the team plays a total 50 games all season and wins 80 percent of the remaining games, how many games will the team win for the entire season?

(A) 36 (B) 25
(C) 42 (D) 39

Q17. Local telephone calls increased in price from 25Pa to 30Pa. What percentage increase was this?

(A) 15% (B) 25%
(C) 5% (D) 20%

Q18. A worker pays Rs. 350 tax per month, which is 15% of his income. What is his income?

(A) 3500 (B) 5250
(C) 2333.30 (D) 2523.30

Q19. If "x" is a positive number, 400% of x is what percent of $400x$?

(A) 1 (B) 0.1
(C) 0.01 (D) 100

Q20. Babar gave 15% of his baseball cards to Laeeq and 20% to Sarfraz. If he still had 520 cards, how many did he have originally?

(A) 800 (B) 720
(C) 820 (D) 600

Q21. A certain country has an infant mortality rate of 6.8% of 20000 babies born in a certain year, how many survived?
(A) 1360 (B) 18640
(C) 18000 (D) 17640

Q22. 20% of 50% of 80 is:
(A) 40 (B) 16
(C) 8 (D) 60

Q23. The price of a can of acid was increased by 20%. How many cans can be purchased for the amount of money that used to buy 300 cans?
(A) 250 (B) 320
(C) 150 (D) 240

Q24. In a basket containing 180 pears, 9 Pears are spoiled. What percent of the pears in the basket are not spoiled?
(A) 85% (B) 5%
(C) 95% (D) 9%

Q25. A silo (container for storing grain) is filled to capacity with p kilograms of wheat. Rats eats q kilograms a day. After 21 days, what percentage of the silo's capacity have the rats eaten?
(A) $\frac{21q}{30p} \times 100$ (B) $\frac{q}{p} \times 100$
(C) $\frac{21q - p}{p} \times 100$ (D) $2100\left(\frac{q}{p}\right)$

Q26. A factory normally employs 100 people. During a slow spell, it fired 20% of its employees. By what percentage must it now increase its staff to return to full capacity?
(A) 25% (B) 20%
(C) 80% (D) 40%

Q27. Six students in a class failed in geometry. This represents $16\frac{2}{3}\%$ of the class. How many students passed the course?
(A) 36 (B) 30
(C) 42 (D) 24

Q28. If 30% of all women are voters and 42% of the population are women, what percent of the population are women voters?
(A) 17.4% (B) 25.20%
(C) 12.60% (D) None of these

Q29. If the length of the rectangle is increased by 16% and the width is decreased by 25%, then the area:
(A) increases by 9% (B) decreases by 41%
(C) decreases by 13% (D) increases by 59%

Q30. If the base of a rectangle is increased by 40% and the altitude is decreased by 30%, the area is
(A) increased by 10% (B) increased by 12%
(C) decreased by 10% (D) decreased by 2%

Explanatory Answers

Q1. (B) If the value firstly increased by $x\%$ and then decreased by $y\%$ then there is $\left[x - y - \frac{xy}{100}\right]\%$ increase or decrease according as the sign +ve or –ve, respectively. In this problem, $x = 40$ and $y = 20$. Therefore

$$\left[40 - 20 - \frac{(40)(20)}{100}\right]\%$$

$$\left[20 - \frac{800}{100}\right]\%$$

$[20 - 8]\% = 12\%$

Because sign is +ve therefore its area is increased by 12%.

Q2. (C) $y \times \frac{x}{100} = 20$

$\Rightarrow \quad xy = 20 \times 100 \quad \Rightarrow xy = 2000$

$\Rightarrow \quad y = \frac{2000}{x}$

Q3. (D) Using, $\frac{\text{Part}}{\text{Whole}} = Y$ percent, here $P = 12$, W = ? and Y percent $= \frac{1}{300}$

$$\frac{P}{W} = \frac{Y}{100} \quad \Rightarrow \quad \frac{P}{W} = Y \times \frac{1}{100}$$

$$\frac{12}{W} = \frac{1}{3} \times \frac{1}{100} \quad \Rightarrow \quad W = 3 \times 1200 = 3600$$

Q4. (D) 400% of $p = \frac{400}{100} \times p = 4p$, which is 1% of 400 p.

Q5. (D) 30% of 40% $= \frac{30}{100} \times \frac{40}{100} = \frac{12}{100} = 0.12$

Now 10% of 30% of 40% $= \frac{10}{100} \times 0.12 = 0.012 = 1.2$

Q6. (B) $\frac{P}{W} = \frac{y}{100} \quad \Rightarrow \quad x = \frac{y}{100} \times 75$

$$\Rightarrow \quad x = \frac{3y}{4} \quad \Rightarrow \quad y = \frac{4x}{3}$$

Q7. (B) If 13 students failed, then the number of passed students = 35 – 13 = 22

Thus, $\frac{22}{35} \times 100 = 63\%$ approx

Q8. (A) Let the number of girls = 100, then

Number of boys = 200

Then 35 girls (35% of 100) and 40 boys (20% of 200), have handed in their cards. Hence 75 of the 300 (100 + 200) students have handed them in. It means that 300 – 75 = 225 have not handed them in. Thus

$\frac{225}{300} \times 100 = 75\%$

Q9. (B) The increment in the value of the jar = Rs. 21000 – Rs. 7000 = Rs. 14000

The %age increase in the value of the jar

$$= \frac{\text{Increment}}{\text{Actual}} \times 100$$

$$= \frac{14000}{7000} \times 100 = 200\%$$

Q10.(D) To achieve 90% grade on the entire examination, Sonia needs 54 (as calculated below) problems:

$$\frac{P}{W} = y\% \Rightarrow \frac{P}{60} = \frac{90}{100} \Rightarrow P = \frac{90}{100} \times 60$$
$$\Rightarrow P = 54$$

to solve correctly. So far she has solved 30 $\left(\frac{P}{40} = \frac{75}{100} \Rightarrow P = \frac{75}{100} \times 40 = 30\right)$ problems correctly. Therefore, on the last 20 problems she needed 54 – 30 = 24 correct answers, which is impossible to get from 20 problems.

Q11.(C) 60% of A is 30% of B, *i.e.*, $\frac{60}{100}A = \frac{30}{100}B$.

$\Rightarrow .60A = .30B, \Rightarrow B = \frac{.60}{.30}A \Rightarrow B = 2A$

Now we find B is what percent of A. *i.e.*,

$$B = \frac{x}{100}A \text{ or } B = (x\%)(A)$$

$\Rightarrow$ B = (200%)A

Q12.(A) Using the relation $\frac{\text{Part}}{\text{Whole}} = y\%$

$$\frac{q}{p} = y\%$$

Second Method: What % p is q

x % p = q

$\Rightarrow x\% = \frac{q}{p}$

Q13.(C) Using $\frac{\text{Part}}{\text{Whole}} = y\%$

$$\frac{5}{4} \div \frac{1}{2} = y\%$$

$$\frac{5}{4} \times 2 = y\% \Rightarrow y\% = \frac{5}{2} = 2.5$$

$$\Rightarrow y\% = 250\%$$

Q14. (D) Total no of students = 820

No. of boys $= 820 \times \frac{55}{100} = 451$ boys

No. of girls $= 820 - 451 = 369$ girls

Q15. (B)

Let the length of first square = 1 inch

Then Area of first square = 1 square inch

Then sides of the second square = 3 inch

Area of the second square = 9 square inch

∴ Increase in the area of the 2nd square = 8 square inches

%age increase in the second square = $\boxed{800\%}$

Q16. **(A)** Total No. of games that

the team has won so far $= \frac{60}{100} \times 20 = 12$ games

The total number of games left $= 50 - 20 = 30$

80% of 30 games will the team win

$\therefore \quad \frac{80}{100} \times 30 = 24$ games

The total number of wins $= 12 + 24 = \boxed{36}$

Q17. **(D)** Increase in local call $= 30 - 25 = 5$Pa

$$\% \text{ increase} = \frac{5}{25} \times 100 = 20\%$$

Q18. **(C)** Let "x" be his income then

$$15\% \text{ of } x = 350$$

$$x = 350 \div 15\% = 350 \times \frac{100}{15}$$

$$x = 2333.33$$

Q19. **(A)** 400% of $x = 4x$. Which is 1% of $400x$.

Q20. **(A)** Actually, Babar had 100% of the cards. After distributing 35% (20% + 15%) of them, he had 100% – 35% = 65% of them left. So

$$520 = \frac{65}{100}x \Rightarrow x = \frac{520 \times 100}{65} = \boxed{800}$$

Q21. **(B)** Infant mortality $= 20000 \times \frac{6.8}{100} = 1360$

survived bodies $= 20000 - 1360 = \boxed{18640}$

Q22. **(C)** 50% of 80 $= 80 \times \frac{50}{100} = 40$

20% of 40 $= 40 \times \frac{20}{100} = \boxed{8}$

Q23. **(A)** Let the can of acid used to cost Rs. 1

After increasing 20% cost it became $= 1 + \frac{1}{20} = 1.20$

Then 300 cans of acid used to cost = Rs. 300

Each can be bought for Rs 300

$\therefore 300 \div 1.20 = \boxed{250}$

Q24. **(C)** The pears that are not spoiled $= 180 - 9 = 171$

$$\text{Percentage} = \frac{171}{180} \times 100$$

$$= 19 \times 5$$

$$= 95\%$$

Q25. **(D)** After 21 days the rats have eaten wheat = 21q kilograms.

So, the required fraction in percentage $= \frac{21q}{p} \times 100$

$$= 2100\left(\frac{q}{p}\right)$$

Q26. **(A)** 20% of 100 = 20 employee

employees left = 100 – 20 = 80 employees

If it again increases by 20, the percentage of increase

$$= \frac{20}{80} \times 100 = 25\%$$

Q27. **(B)** Let x be the number of students, then

$$16\frac{2}{3}\% = \frac{1}{6} \Rightarrow \frac{1}{6}x = 6$$

$$\Rightarrow x = 36$$

36 students in class, 6 failed, 30 passed

Q28. **(C)** 30% of the 42% of the population who are women are voters so

(.30)(.42) = 0.126 = 12.60% of the population are women voters.

Q29. **(C)** Let L be the original length and W be the original width.

The new length = 100% + 16% = 116% of L $\Rightarrow$ $\boxed{1.16L}$

Since the width decreases by 25% so the new width is 75% of W $\Rightarrow$ .75W

Area = LW

$\Rightarrow$ New Area = (1.16)(.75)LW

= 0.87 LW = 87% of Area

Since the area is 87% of the original area. Thus the area has decreased by (100 – 87) = 13%

Q30. **(D)** Let "b" be the base and "a" altitude. Then the new base will be (b + 0.4b). The new altitude after decreasing 30% is (a – 0.3a).

So the area is

((a – .3a)(b + 0.4b) = (0.7)(1.4)ab

= 0.98ab

The new area is 42% of the old. So the new area (98% – 100%)

= –2% is decreased by 2%

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Chapter 7

RATIO AND PROPORTION

RATIO:

The number of times one quantity contains another quantity of the same kind is called the ratio of the two quantities.

Note: The ratio of two quantities is equivalent to the fraction that one quantity is to the other.

Example: There can be ratio between Rs. 30 and Rs. 40, but there can be no ratio between Rs. 30 and 40 apples.

Remember: The ratio 3:5 is written as 3:5 or $\frac{3}{5}$, 3 and 5 are called the terms of the ratio. 3 is the first and 5 is the second term.

Note: The first term of a ratio is called the antecedent and the second the consequent.

If a set of objects is divided into two groups in the ratio $a : b$, then the first group contains $\frac{a}{a+b}$ of the total objects. The second group contain $\frac{b}{a+b}$ of the total number of objects.

Important Example:

If a bag containing twelve mirrors is dropped, which of the following cannot be the ratio of the broken mirrors to unbroken mirrors?

(i) 2: 1 ii) 3:1 iii) 3:2 iv) 1:1 v) 7:5

Solution:

Since there are 12 mirrors is the bag. So 12 must be divisible by the sum of terms in the ratio exactly. We see that 2+1=3 divides 12 exactly 3+1=4 also divides exactly. Only the ratio 3+2=5 doesn't divide 12 exactly. Thus the correct answer is (iii)

PROPORTION:

The equality of ratios is called proportios.

Example:

Consider the two ratios

Ist ratio	2nd ratio
5:15	7:21

Since 5 is one-third of 15, and 7 is one-third of 21, the two ratios are equal.

Note: The first and fourth terms are called extremes, and the second and third terms, are called the means. In above example 5 and 21 are extremes, while 15 and 7 are means.

Important Points:

1. If four quantities be in proportion, the product of the extremes is equal to the product of the means.
2. Three quantities of the same kind are said to be in continued proportion when the ratio of the first to the second is equal to the ratio of the second to the third.

Aid to Memory:

The mean proportional between two numbers is equal to the square root of their product.

Example: Find

i). Fourth proportions to 5, 10, 5

ii) Third proportion of 5 and 10.

iii) Mean proportions between .04 and 0.09.

Solution:

(i) Let $5 : 10 :: 5 : x$

Then $5x = 10 \times 5 \Rightarrow 5x = 50 \Rightarrow \boxed{x = 10}$

(ii) Let $5 : 10 :: 10 : x$

Then $5x = 10 \times 10 \Rightarrow 5x = 100 \Rightarrow \boxed{x = 20}$

(iii) Mean proportion between .04 and .09

$$= \sqrt{.04 \times .09} \quad = \sqrt{.0036}$$

$$= \sqrt{\frac{36}{1000}} \quad = \frac{6}{100} = 0.06$$

Direct Proportion:

If the given two quantities are so related to each other that if one of them is multiplied (or divided) by any number, the other is also multiplied (or divided) by the same number.

Inverse Proportion:

If two quantities are so related that if one of them is multiplied by any number, the other is divided by the same number.

Example:

1. If 5 balls cost Rs. 7, what do 15 balls cost?

Solution: This example is an illustration of direct proportion. Therefore, setting a proportion.

$\therefore \quad 5 : 7 :: 15 : x$

$$\Rightarrow 5x = 15 \times 7 \Rightarrow x = \frac{15 \times 7}{5} = 21$$

2. If 5 men can build a house in 28 days, in how many days will 10 men build it?

Solution:

This example is an illustration of inverse proportion. Here, if we increase number of men.

2, 3, 4 ..... times, the number of days will be decreased.

2, 3, 4..... times, Thus the inverse ratio of the number of men is equal to the ratio of the corresponding number of days.

$$\therefore \quad \frac{1}{15} : \frac{1}{10} :: 28 : x \text{ days}$$

$$\Rightarrow \quad x = \frac{1}{5} = \frac{1}{10} \times 28$$

$$\Rightarrow \quad x = \frac{28 \times 15}{10} = 42 \text{ days.}$$

DOUBLE RULE OF THREE:

Example: If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days.

Solution: We resolve this problem in two parts.

Ist Part: If 8 men can reap 80 hectors, how many hectares can 36 men reap.

Setting a proportion

8 men : 36 men : : 80 hectares : x hecter

$$x = \frac{36 \times 80}{8} = 360 \text{ hectres}$$

2nd Part: If 360 hectares can be reaped in 24 days, how many hectares can be reaped in 30 days?

24 days : 30 days = 360 hectares : x hector

$$x = \frac{360 \times 30}{24} = 450$$

SINGLE STEP:

8 men : 36 men 24 days : 30 days	: : 80 hectare : x hector

Required No. of hectares : $= \frac{\text{Multiplication of means}}{\text{Multiplication of Ist terms}}$

$= \frac{80\times36\times30}{8\times24} = 450$

Model Examples:

Example 1: Three liquids contain petrol and spirit mixed in the ratio 2 : 3, 3 : 4 and 4 : 5, respectively. A motor owner mixes 20 litres of the first, 21 litres of the second and a few litres of third. If the ratio of petrol to spirit in the mixture is 29 : 39, find the number of litres of the third liquid taken for the mixture.

Solution: 20 litres of the first liquid has $2/5 \times 20$

= 8 litres of petrol and 12 litres of spirit

21 litres of the second liquid have $3/7 \times 21$

= 9 litres of petrol and 12 litres of spirit.

Suppose x litres of the third liquid are taken, it will have $\frac{4x}{9}$ litres of petrol and $\frac{5x}{9}$ litres of spirit.

$\therefore$ Total petrol in the mixture $= 8 + 9 + \frac{4x}{9}$ litres

and total spirit $= 12 + 12 + \frac{5x}{9}$ litres.

Ratio of these $= \frac{17+\frac{4x}{9}}{24+\frac{5x}{9}} = \frac{29}{39}$

$\therefore$ $663 + \frac{156}{9}x = 696 + \frac{145}{9}x$

or $\left(\frac{156}{9} - \frac{145}{9}\right)x = 696 - 663$

or $\frac{11}{9}x = 33$

or $x = \frac{9}{11}\times 33 = 27$ litres.

Example 2: In a regiment the number of officers to men was 3:31 before the battle. In the battle, 6 officers and 22 men were killed and the ratio become 1:13. Find the number of officers and men in the regiments.

Solution: In the beginning *i.e.*, before the battle

Let no. of officers $= x$

// // men $= y$

then $x : y :: 3 : 31 \Rightarrow \frac{x}{y} = \frac{3}{31}$ .....(*i*)

After the battle

No. of officers $= x - 6$

// // men $= y - 22$

Then $\frac{x-6}{y-22} = \frac{1}{13}$ .....(*ii*)

From (*i*) we get $x = \frac{3}{31}y$

Substituting this value in (*ii*), we get

$$\frac{\frac{3}{31}y-6}{y-22}=\frac{1}{13} \quad \ldots\ldots\ldots(iii)$$

Solving for y in (iii), we get

$$y=217$$

$$x=\frac{3}{31}\times 217=21$$

$\therefore$ No. of officers $= 21$
No. of men $= 217$ } **Ans.**

Example 3: Of two kinds of alloy, silver and copper are contained in one in the ratio of 5 : 1 and in the other in the ratio of 7 : 2. What weights of the two alloys should be melted and mixed together so as to make up a 5 lb mass with 80% of silver?

Solution: Let the alloys taken be in the ratio of $6 : 9x$

So in 1st alloy wt. of silver $= 5$
// // // copper $= 1$
in 2nd alloy wt. of silver $= 7x$
// // // copper $= 2x$
$\therefore$ wt. of silver $= 5 + 7x$
and Total wt. $= 9x + 6$

But $$(5+7x)\frac{100}{80}=9x+6$$

$$25+35x=36x+24$$

$\therefore$ $$x = 1$$

So alloys are taken in 6 : 9 ratio.
Total wt. of 2nd alloys = 5 lb

// // 1st alloy $= \frac{6}{15}\times 5 = 2$ lb
// // 2nd // $= \frac{9}{15}\times 5 = 3$ lb } **Ans.**

Example 4: An alloy contains copper and zinc in the ratio of 5 : 3 and another alloy contains copper and tin in the ratio 8 : 5. If equal weights of both the alloys are melted together find the weight of tin in the resulting alloy per kg.

Solution: Let weight of both alloys be taken to be 13 kg.

$\therefore$ wt. of resulting alloy $= 26$ kg.
In 2nd alloy in 13 kg. wt. of tin $= 5$ kg.
$\therefore$ in resulting alloy of
26 kg. wt. of tin $= 5$ kg.
$\therefore$ wt. of tin per kg. of resulting alloy $= \frac{5}{26}$ kg.

Multiple Choice Questions (MCQs)

Q1. In a city 90% of the population own a car, 15% own a motorcycle, and everybody owns one or the other or both. What is the percentage of motorcycle owners to who own cars?

(A) 15% (B) 5%
(C) 75% (D) $33\frac{1}{3}\%$

Q2. Concrete consists of cement, sand and screenings in the ratio of 1 : 5 : 4, what is the percentage of the sand mixed?

(A) 10% (B) 40%

(C) 50% (D) 60%

Q3. Three business partners shares have profit of Rs. 24000 in the ratio 5 : 4 : 3. What is the amount of the least share?

(A) 6000 (B) 8000
(C) 10,000 (D) 1200

Q4. A machine produces 1280 parts in 16 hours. How many parts would it make in a working week of 44 hours?

(A) 2530 (B) 3520
(C) 2122 (D) 3960

Q5. If the ratio of x and y is $\frac{11}{3}$, what is the value of $2x$ to y?

(A) $\frac{11}{6}$ (B) $\frac{22}{6}$
(C) $\frac{22}{3}$ (D) $\frac{11}{5}$

Q6. If 80% application to a program were rejected, what is the ratio of the number accepted to the number rejected?

(A) 1 : 4 (B) 4 : 1
(C) 1 : 8 (D) 3 : 8

Q7. What is the ratio of the circumference of a circle to its radius?

(A) π (B) $\frac{\pi}{2}$
(C) $2\pi r$ (D) 2π

Q8. Win/Loss ratio for two teams are A, 5 : 2 and B, 7 : 3 which team has the better record?

(A) A (B) B
(C) both A and B (D) wrong question

Q9. If 15 workers can paint a certain number of houses in 24 days, how many days will 40 workers take, working at the same rate, to do the same job?

(A) 12 days (B) 18 days
(C) 15 days (D) 9 days

Q10. If a jet travels 1280 km in 2 hours, how far will it travel in $5\frac{1}{2}$ hours, at the same speed?

(A) 2100 (B) 3300
(C) 2700 (D) 3520

Q11. If the ratio of $a : b$ is 9 : 7 then $a + b$ is:

(A) 14 (B) 16
(C) 63 (D) not possible

Q12. If you can buy A apples for n nickels (five cent coin), how many apples can you buy for d dimes and q quarters?

(A) $\frac{A(d+q)}{n}$ (B) $\frac{A}{n}(10d + 25q)$
(C) $\frac{A}{n}(2d + 5q)$ (D) $\frac{d+q}{An}$

Q13. If the ratio of boys and girls in a class is 3 : 5 and the class contains 24 students, how many additional boys would have to enroll to make the ratio of boys to girls 1:1?

(A) 9 (B) 15
(C) 6 (D) 12

Q14. A recipe requires 13 gram of sugar and 18 gram of flour. If only 100gram of sugar is used, how much flour, to the nearest gram, should be used?

(A) 16 (B) 13
(C) 14 (D) 17

Q15. Green paint is obtained from blue and yellow paint in the ratio 3 : 5. How much of each colour is

needed to make 40 litres of this green paint?

(A) Blue paint 15 litres, yellow paint: 25 litres
(B) Blue paint: 25 litres, yellow paint: 15 litres
(C) Blue paint: 10 litres, yellow paint: 30 litres
(D) Blue paint: 13 litres, yellow paint: 27 litres

Explanatory Answers

Q1. (D) Let x stand for the percentage who own both a car and a motorcycle. Then
(The %age who own a motorcycle) + (The %age who own a car) – (The %age who own one or the other or both) = 100% own one or other or both.

$\therefore \quad 15\% + 90\% - A = 100\%$

$\Rightarrow \quad 105\% - A = 100\% \Rightarrow \boxed{A = 5\%}$

The %age of motorcycle owners to who own car is

$$= \frac{5\%}{15\%} = \frac{1}{3} = \boxed{33\frac{1}{3}\%}$$

Q2. (C) Ratio $= 1 : 5 : 4$

Sum of ratio $= 1 + 5 + 4 = 10$

$$\text{Sand} = \frac{5}{10} \times 100 = 50\%$$

Q3. (A) Ratio $= 5 : 4 : 3$

Sum of ratio $= 5 + 4 + 3 = 12$

$$\text{least share} = \frac{3}{12} \times 24000$$

$= \text{Rs. } 6000$

Q4. (B) Let "x" be the number of parts in 44 hours

Then $16 : 1280 :: 44 : x$

$$\Rightarrow \quad \frac{16}{1280} = \frac{44}{x} \Rightarrow x = \frac{44 \times 1280}{16}$$

$$x = \boxed{3520}$$

Q5. (C) The ratio of x to y can be written as $\frac{x}{y}$. The ratio of x to y is $\frac{11}{3}$, which can be written as

$$\frac{x}{y} = \frac{11}{3}$$

If $\frac{x}{y} = \frac{11}{3}$, then $2\left(\frac{x}{y}\right) = 2\left(\frac{11}{3}\right)$

$$\frac{2x}{y} = \boxed{\frac{22}{3}}$$

Q6. (A) Since 80% of the application were rejected. Therefore, 20% = (100% – 80%) were accepted, the ratio of accepted to rejected is

$20\% : 80\% = 1 : 4$

Q7. (D) The ratio of the circumference to the diameter of the circle is π. Therefore

$$\pi = \frac{C}{d} \Rightarrow \frac{C}{2r} \Rightarrow 2\pi = \frac{C}{r}$$

Q8. (A)

A	B
5 : 2	7 : 3
$= \frac{5}{2} : 1$	$= \frac{7}{3} : 1$
2.5 : 1	= 2.3 : 1

Team A has the better record.

Q9. **(D)** Clearly, the more workers are there, the less time will be required, therefore, $15 : 40 :: \frac{1}{24} : \frac{1}{x}$

$$\Rightarrow \frac{15}{40} = \frac{x}{24} \quad \Rightarrow x = \frac{15 \times 24}{40} = 9 \text{ days}$$

Q10. **(D)** It's a direct variation question

$$1280 : 2 :: x : \frac{11}{2}$$

$$\frac{1280}{2} = \frac{x}{11/2} \quad \Rightarrow 2x = \frac{1280 \times 11}{2}$$

$$\Rightarrow x = 3520 \text{ km}$$

Q11. **(D)** In this question if a is 18 and b is 14, then the ratio a : b is 9 : 7 but a + b = 32. The point in this question that a and b can take on many possible values. It is not possible here to establish one definite value for the sum of a and b.

Q12. **(C)** $\frac{\text{A apples}}{\text{n nickels}} = \frac{\text{A apples}}{\text{5n cents}} = \frac{\text{x apples}}{(10d + 25q)\text{cents}}$

$$\Rightarrow \frac{A}{5n} = \frac{x}{10d + 25q} \quad \Rightarrow 5nx = A(10d + 25q)$$

$$\Rightarrow x = \frac{A5(2d + 5q)}{5n}$$

$$\Rightarrow x = \frac{A}{n}(2d + 5q)$$

Q13. **(C)** Given ratio 3 : 5 of boys and girls. Total number of students in the class is 24.

$$\text{Number of boys} = \frac{3}{8} \times 24 = 9 \text{ boys}$$

$$\text{Number of girls} = \frac{5}{8} \times 24 = 15 \text{ girls}$$

In order of have same number of boys and girls, 6 additional boys would have to enroll.

Q14. **(B)** This is a direct proportion, because the more sugar, the more flour

$$\frac{13}{18} = \frac{10}{x}$$

$$13x = 180$$

$$\Rightarrow x = 13\frac{11}{13}$$

Q15. **(A)**

The ratio 3 : 5 gives (3 + 5) = 8 parts

$$\text{Blue paints} = \frac{3}{8} \times 40 = 15 \text{ litres}$$

$$\text{Yellow paints} = \frac{5}{8} \times 40 = 25 \text{ litres}$$

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Chapter 8

AVERAGE

In Mathematics, average is a representative of a number of given quantities. Average is of several kinds.

METHOD OF FINDING AVERAGE

To find average of any number of quantities of the same kind is to add all the items together and then divide the sum by the number of items.

$$\therefore \quad \text{Average} = \frac{\text{Sum of all the items}}{\text{No. of items}}$$

Model Examples

Example 1: The average daily temperature from 9th January to 16th January (both inclusive) was 38.6° and that from the 10th to 17th January (inclusive) was 39.2°. What was the temperature on 17th January?

Solution: Total temp. from 9th Jan. to 16th Jan.

$= 38.6 \times 8°C$

$= 308.8°C$

Since the temp. on 9th $= 34.6°C$

$\therefore$ Total temp. from 10th Jan. to 16 Jan.

$= 308.8 - 34.6$

$= 274.2°C$

Total temp. from 10 to 17th Jan.

$= 39.2 \times 8°C$

$= 313.6°C$

$\therefore$ Temp on 17th Jan. $= 313.6 - 274.2$

$= 39.4°C$

Example 2: A goods train in five successive minutes from its start runs 68 metres, 127 metres, 208 metres, 312 metres and 535 metres and for next five minutes in maintains average speed of 33 km/hr. Find the whole distance covered and the average speed of train in km/hour.

Solution: Distance covered in first five minutes.

$$= \frac{68+127+208+312+535}{1000} \text{ kms.}$$

$$= \frac{5}{4} \text{ kms.}$$

Now average speed for next five minutes

$= 33$ km/hr.

$\therefore$ Distance covered in next five minutes $= \frac{33 \times 5}{60} = \frac{11}{4}$ km

Total distance covered in 10 minutes $= \frac{5}{4} + \frac{11}{4} = \frac{16}{4}$

$= 4$ kms. **Ans.**

Average speed $= \frac{4}{10}$ km/min.

$$= \frac{4}{10} \times 60$$

$= 24$ km/hr. **Ans.**

Example 3: The average salary per head of all the workers of an institution is Rs. 60. The average salary per head of 12 officers is Rs. 400. The average salary per head of the rest is Rs. 56. Find the total no. of workers in the institute.

Solution: Let the total No. of workers $= x$

$\therefore$ Total salary drawn $= 60x$ .....(*i*)

Salary of 12 officers $= 12 \times 400 = 4800$ Rs.

// // // // the rest $= (x - 12) \times 56$

Hence total salary of the workers

$= (x - 12)56 + 4800$ .....(*ii*)

Equating (*i*) and (*ii*)

$60x = 4800 + 56x - 672 \Rightarrow 4x = 4128$

$\Rightarrow x = 1032$ **Ans.**

Example 4: On a journey across Karachi the overage speed of a taxi 20 m.p.h. for 70% of the distance, 25 m.p.h. for 10% of it and 8 m.p.h. for the remainder. Find the average speed for the whole journey.

Solution: Let the distance be 100 miles.

Time taken for 70% journey at 20 m.p.h. $= \frac{70}{20} = 3.5$ hrs.

Time taken for 10% journey at 25 m.p.h. $= \frac{10}{25} = 2/5$ hrs.

Total taken for 20% journey at 8 m.p.h. $= \frac{20}{8} = 2.5$ hrs.

Total time taken $= 3.5 + \frac{2}{5} + 2.5 = 6\frac{2}{5}$ hrs.

$\therefore$ Average speed $= \frac{100}{\frac{32}{5}} = \frac{125}{8}$

$= 15.625$ m.p.h. **Ans.**

Example 5: A batsman has a certain average of runs for 16 innings. In the 17th innings, he makes a score of 85 runs there by increasing his average by 3. What is the average of the 17th inning.

Solution: To increase the average by 3 runs he has to make $17 \times 3 = 51$ runs more than the average of previous innings.

$\therefore$ Average of 16 innings $= 85 - 51 = 34$

// // 17 // $= 34 + 3 = 37$ **Ans.**

Example 6: A motorist set out at 10 a.m. to travel from Lahore to Gujrat, suppose a distance of 80 miles. He estimated that he could maintain an average speed of 25 m.p.h. For the first 44 miles from Lahore to Gujranwala his speed, averaged 30 m.p.h. but afterwards he was delayed by traffic and reached Gujrat 24 minutes later than the estimated time. Calculate:

(i) His time of arrival in Gujrat

(ii) His average speed from Gujranwala to Gujrat

Solution: Total distance from Lahore to Gujrat = 80 miles.

Average speed estimate from Lahore to Gujrat = 25 m.p.h.

Estimated time taken from Lahore to Gujrat $= \frac{80}{25} = \frac{16}{5}$ hours

But he was late by $\frac{24}{60}$ hours.

$\therefore$ Total actual time taken $= \frac{16}{5} + \frac{24}{60}$

$= \frac{18}{5}$ hours

= 3 hours 36 minutes.

He starts at 10:00 A.M. and will reach there at Gujrat at 1:36 P.M.

Now distance from Lahore to Gujranwala = 44 miles

Average speed from Lahore to Gujranwala = 30 m.p.h.

Time taken from Lahore to Gujranwala $= \frac{44}{30}$ hours

$= \frac{22}{15}$ hours

Now distance from Gujranwala to Gujrat = 80 – 44

= 36 miles.

Time taken from Gujranwala to Gujrat $= \frac{18}{5} - \frac{22}{15}$

$= \frac{32}{15}$ hours

$\therefore$ Average speed from Gujranwala to Gujrat $= \frac{36}{\frac{32}{15}}$ miles/hr.

=16.9 miles per hour (app.) **Ans.**

Multiple Choice Questions (MCQs)

Q1. The average of even integers from 2 to 100 inclusive is:

(A) 49 **(B)** 52

(C) 51 **(D)** 50

Q2. What is the average of first hundred natural numbers?

(A) 50 **(B)** 50.5

(C) 49.5 **(D)** 100

Q3. What is the average of x, y and z? If $x + y = 5$, $y + z = 8$ and $x + z = 11$.

(A) $\frac{11}{3}$ **(B)** $\frac{1}{2}$

(C) $\frac{13}{5}$ **(D)** 4

Q4. The average of five numbers is 54. If three of the numbers are 26, 28 and 30, what is the average of the other two?

(A) 91 **(B)** 93

(C) 54 (D) 186

Q5. Which of the following is the average of $x^2 - 16$, $39 - x^2$ and $3x + 10$?

(A) $x + 3$ (B) $2x + 13$

(C) $x + 11$ (D) $\frac{x + 11}{3}$

Q6. 8 students in a class obtained 60%, 3 obtained 75%, 2 obtained 80% and 7 obtained 45% in a class test. What is the average marks?

(A) 49% (B) 59%

(C) 29% (D) 51%

Q7. The average number of goals a team has scored in 7 matches is 8. They averaged 10 goals for the first 3 matches and they scored 5 goals in each of the next two matches. What is the average score of the last two matches?

(A) 5 goals (B) 4 goals

(C) 6 goals (D) 8 goals

Q8. If the mean (average) of 6 numbers is 4.5. What is the sum of the numbers?

(A) 0.75 (B) 10.5

(C) 12 (D) 27

Q9. A worker is paid R rupees per hour for the first 8 hours daily. For every hour after the first 8 hours, she is paid S rupees per hour. If she works 12 hours in one day, what is her average hourly for the day?

(A) 8R + S (B) $\frac{8R + 4S}{4}$

(C) $\frac{12R - 8S}{4}$ (D) $\frac{2R + S}{3}$

Q10. Asim had an average of 60 on his first four math tests. After taking the next test, his average dropped to 58. Find his recent test grade.

(A) 40 (B) 50

(C) 48 (D) 32

Q11. If a + b = 8, b + c = 9, and c + a = 11, what is the average of a, b and c?

(A) $\frac{14}{3}$ (B) $\frac{28}{3}$

(C) $\frac{14}{6}$ (D) $\frac{7}{3}$

Q12. If the average of 3,5, 10 and S is 6, what is the value of S?

(A) 4 (B) 6

(C) 12 (D) 0

Q13. What is the average of 3^{10}, 3^{20} and 3^{30}?

(A) 3^{59} (B) $3^9 + 3^{19} + 3^{29}$

(C) 3^{57} (D) $3^{11} + 3^{21} + 3^{31}$

Q14. If $20x + 20y = 70$, what is the average of x and y?

(A) $\frac{7}{2}$ (B) 7

(C) $\frac{7}{4}$ (D) $\frac{4}{7}$

Q15. Which of the following is the average of $x^4 - 20$, $40 - x^4$, and $3x + 4$?

(A) $x^4 - 24$ (B) $x + 8$

(C) $x^4 + 3x + 24$ (D) $x + 24$

Explanatory Answers

Q1. (C) As sum of the first n even numbers $= n(n+1)$
Now, the sum of even numbers from 2 to 100 is
$2 + 4 + 6 + 8 + \ldots\ldots + 100$ (or 50 even number)

$$= 50(50+1) = 2550$$

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

$$= \frac{2550}{50} = 51$$

Q2. (B) The first 100 natural numbers are {1,2,3,.....,100}

Now, sum of all the first n numbers $= \frac{n(n+1)}{2}$

$$\text{Sum of first 100 natural numbers} = \frac{100(100+1)}{2}$$

$$= 5050$$

$$\text{Now, average} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

$$= \frac{5050}{100} = 50.5$$

> ***Shortcut:*** The average of first "n" natural number is $\frac{n+1}{2}$
>
> Thus, average $= \frac{100+1}{2} = \frac{101}{2} = 50.5$

Q3. (D) Adding the given three equations:

$$(x+y) + (y+z) + (z+x) = 5 + 8 + 11$$
$$2x + 2y + 2z = 24$$
$$2(x+y+z) = 24$$

Dividing both sides by 2

$$x + y + z = 12$$

Now average of x, y and z is

$$\frac{x+y+z}{3} = \frac{12}{3} = 4$$

Q4. (B) Let the missing numbers be a and b, then by given condition,

$$\frac{a+b+26+28+30}{5} = 54$$

$$a + b + 84 = 270 \text{ (Multiplying both sides by 5)}$$
$$a + b = 186$$

Hence average of a and b is

$$\frac{a+b}{2} = \frac{186}{2} = 93$$

Q5. (C)

$$\text{Average} = \frac{\text{Sum of the terms}}{\text{No. of terms}}$$

$$= \frac{x^2 - 16 + 39 - x^2 + 3x + 10}{3}$$

$$= x + 11$$

Q6. (B)
8 students with 60%, total = 480 marks
3 students with 75%, total = 225 marks
2 students with 80%, total = 160 marks
7 students with 45%, total = 315 marks
$\therefore$ 20 students obtain a total = 1180 marks

$\therefore$ Average $= \frac{1180}{20} = 59\%$

Q7. (D) Total goals for 7 matches $= 7 \times 8 = 56$
Total goals for 3 matches with average score of 10 $= 30$
Total goals for 2 matches with average score of 5 $= 10$
$\therefore$ Total goals for remaining 2 matches $= 56 - 30 - 10$
$= 16$

Average goals in last two matches $= \frac{16}{2}$
$= 8$ goals

Q8. (D) Average of 6 numbers $= \frac{\text{Sum of numbers}}{6}$

$\Rightarrow$ Sum of the numbers $=$ (Average of 6 numbers) $\times 6$
$= 4.5 \times 6 = 27$

Q9. (D)
For first 8 hours, she is paid $= 8R$
Next 4 hours, she is paid $= (12 - 8) = 4S$
Total pay $= 8R + 4S$

Average $= \frac{8R + 4S}{12} = \boxed{\frac{2R + S}{3}}$

Q10. (B) Let "x" be the required grade, then

$\frac{4(60) + x}{5} = 58$

$\Rightarrow \quad 240 + x = 290 \quad \Rightarrow x = 290 - 240 = 50$

Q11. (A)

$\frac{(a+b)+(b+c)+(c+a)}{3} = \frac{8+9+11}{3}$

$\Rightarrow \quad \frac{2(a+b+c)}{3} = \frac{28}{3}$

$\Rightarrow \quad a + b + c = 14 \quad \ldots\ldots\ldots(i)$

Put $a + b = 8 \Rightarrow 8 + c = 14 \Rightarrow \boxed{c = 6}$
Now put $b + c = 9 \Rightarrow a + 9 = 14 \Rightarrow \boxed{a = 5}$
again put $c + a = 11 \Rightarrow 11 + b = 14 \Rightarrow \boxed{b = 3}$

Average of a, b and c $= \frac{6+5+3}{3} = \boxed{\frac{14}{3}}$

Q12. (B) $\frac{3+5+10+S}{4} = 6 \Rightarrow 18 + S = 24 \Rightarrow \boxed{S = 6}$

Q13. (B) $\frac{3^{10} + 3^{20} + 3^{30}}{3} = (3^{10} + 3^{20} + 3^{30})3^{-1}$

$= 3^{10-1} + 3^{20-1} + 3^{30-1}$
$= 3^9 + 3^{19} + 3^{29}$

Q14. (C) $20x + 20y = 70 \Rightarrow 20(x + y) = 70 \Rightarrow x + y = \frac{7}{2}$

$\Rightarrow$ Average of x and y $= \frac{x+y}{2} = \frac{7}{2 \times 2} = \boxed{\frac{7}{4}}$

Q15. (B) $\frac{(x^4 - 20) + (40 - x^4) + (3x + 4)}{3} = \frac{3x + 24}{3} = \frac{3(x+8)}{3}$

$= x + 8$