FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2014

Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE:(i) Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper.**
(ii) Attempt FIVE questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B. ALL** questions carry **EQUAL** marks.
(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(iv) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** Prove that if n is a positive integer which is not a perfect square, then $\sqrt{n}$ is an irrational number. **(10)**

(b) Show that every non-empty set of real numbers which has a lower bound has the infimum. **(10)**

Q. No. 2. **(a)** For what value of a, m, and b does the function **(10)**

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

Satisfy the hypotheses of the Mean Value Theorem on the interval $[0,2]$? **(10)**

(b) For what value of a is

$$f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \ge 3 \end{cases}$$

Continuous at every x?

Q. No. 3. **(a)** Find the area of the surface generated by revolving $r = 2aSin^{-}$ about the polar axis. **(6)**

(b) Find the area enclosed by the graph of the cardioid $r = a(1 - Sin^{-})$. **(7)**

(c) Evaluate the integral $\int_1^{10} \frac{dx}{(x-2)^{2/3}}$ **(7)**

Q. No. 4. **(a)** Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. **(6)**

(b) For what value of x does the series converges absolutely, converges conditionally and diverges? **(7)**

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+3}}$$

(c) Let $f(x,y)=\begin{cases} \dfrac{x^3}{x^3+y^3} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$ **(7)**

Show that f is not continuous at the origin.

Q. No. 5. (a) Let x be a non-empty set and define **(10)**
$d : X_x X \rightarrow R$ by

$$d(a,b)=\begin{cases} 1 & \text{if } a\neq b \\ 0 & \text{if } a=b \end{cases}$$

Show that d is a metric on X
Also describe open and closed balls in this metric space.

(b) Prove that a function f from a metric space (x, d) into a metric space (Y, d^1) is continuous if and only if $f^{-1}(A)$ is a closed subset of X for every closed subject A of Y. **(10)**

SECTION-B

Q. No. 6. (a) Using De Moivre's Theorem evaluate **(10)**

$$\left(\frac{1+i}{\sqrt{3}+i}\right)^6$$

(b) Find real constants a, b, c and d so that the given function is analytic **(10)**

$$f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$$

Q. No. 7. (a) Evaluate $\oint_c \dfrac{dz}{z^2+1}$, where c is the circle $|z| = 4$. **(10)**

(b) Expand $(z)=\dfrac{1}{z(z-1)}$. in a Laurent series valid for $1<|z-2|<2$. **(10)**

Q. No. 8. (a) Find the Fourier transform of $f(z)=e^{-|x|}$. **(10)**

(b) Evaluate $\oint_c \dfrac{1}{(z-1)^2(z-3)}\,dz$, where the contour C is the rectangle defined by $x= 0, x\ 4, y= -1, y=1$ **(10)**

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