FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2010

Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** questions in all by selecting at least **THREE** questions from **SECTION–A** and **TWO** questions from **SECTION–B**. All questions carry **EQUAL** marks. (ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) If f is continuous on [a,b] and if $\propto$ is of bounded variation on [a,b], then $f \in R(\propto)$ on [a, b] i.e. f is Riemann – integrable with respect to $\propto$ on [a,b] **(10)**

(b) Let $\sum a_n$ be an absolutely convergent series having sum S. then every rearrangement of $\sum a_n$ also converges absolutely & has sum S. **(10)**

Q.2. (a) For what +ve value of P, $\int_0^1 \frac{dn}{(1-x)^p}$ is convergent? **(10)**

(b) Evaluate $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$ **(10)**

Q.3. (a) Find the vertical and horizontal asymptotes of the graph of function:
$f(x) = (2x+3)\sqrt{x^2-2x+3}$ **(10)**

(b) Let (i) $y = f(x) = \frac{(x+2)(x-1)}{(x-3)^2}$

(ii) $y = f(x) = \frac{(x-1)}{(x+3)(x-2)}$ **(10)**

Examine what happens to y when $x \to -\infty$ & $x \to +\infty$

Q.4. (a) Find a power series about 0 that represent $\frac{x}{1-x^3}$ **(6)**

(b) Let $\sum s_n$ be any series, Justify. **(5+5+4)**

(i) if $\lim_{n\to\infty} \left|\frac{Sn+1}{Sn}\right| = r < 1$, then $\sum s_n$ is absolutely convergent.

(ii) if $\lim_{n\to\infty} \left|\frac{Sn+1}{Sn}\right| = r$ and ($r > 1$ or $r=\infty$), then S_n diverges.

(iii) if $\lim_{n\to\infty} \left|\frac{Sn+1}{Sn}\right| = 1$, then we can draw no conclusion about the convergence or divergence.

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Q.5. (a) Show that $\int_{0}^{\Pi 12} Sin^{2m-1}\theta \cos^{2n-1} \theta d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}; m, n > 0$ **(10)**

(b) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}; m, n, > 0$ **(10)**

Q.6. (a) Let A be a sequentially compact subset of a matrix space X. Prove that A is totally bounded. **(10)**

(b) Let A be compact subset of a metric space (X,d) and let B be a closed subset of X such that $A \cap B = \Phi$ show that d(A,B)>0 **(10)**

SECTION – B

Q.7. (a) Show that if tanZ is expanded into Laurent series about $Z = \frac{\Pi}{2}$, then **(10)**

(i) Principal is $\frac{-1}{z - \Pi/2}$

(ii) Series converges for $0 < |Z - \frac{\Pi}{2}| < \frac{\Pi}{2}$

(b) Evaluate $\frac{1}{2\Pi i} \oint_C \frac{e^{zt}}{z^2(z^2 + 2z + 2)} dz$ around the circle with equation |z|=3. **(10)**

Q.8. (a) Expand $f(x) = x^2$; $0<x<2\Pi$ in a Fourier series if period is 2Π. **(10)**

(b) Show that $\int_{0}^{\infty} \frac{Cosxdx}{x^2 + 1} = \frac{\Pi}{a} e^{-x}; x \geq 0$ **(10)**

Q.9. (a) Let f(z) be analytic inside and on the simple close curve except at a pole of order m inside C. Prove that the residue of f(Z) at a is given

by $a_{-1} = \lim_{Z \to a} \frac{1}{(m-1)!} \frac{{}^{m-1}d}{dz^{m-1}} \{(z-a)^m f(z)\}$ **(10)**

(b) If f(z) s analytic inside a circle C with center at a, then for all Z inside C.

$f(z) = f(a) + f^{/}(a)(z-a) + f^{//}\frac{(a)}{2!}(z-a)^2 + f^{///}\frac{(a)}{3!}(z-a)^3 + ...$ **(10)**

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