FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

PURE MATHEMATICS, PAPER-I

Time Allowed: 3 Hours | Maximum Marks: 80

Note: (i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q.** Paper.

(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** question from **SECTION-B. ALL** questions carry **EQUAL** marks.

(iii) Extra attempt of any question or any part of the attempted question will not be considered.

(iv) **Use of Calculator is allowed.**

SECTION-A

Q.1. (a) For any integer n let $a_n : Z \to Z$ by such that $a_n(m) = m + n$, $m \in Z$.

Let A = $\{a_n; n \in Z\}$. Show that A is the group under the composition of mappings.

(b) Show that the group of all inner automorphisms of a group G is isomorphic to the factor group of G by its center.

Q.2. (a) Let A and B be cyclic groups of order n. Show that the set Hom(A.B) of all homomorphisms of A to B is a cyclic group.

(b) Prove that group G is abelian iff G/Z(G) is cyclic, where Z(G) is Centre of the group.

Q.3. (a) Define the dimension of a vector space V, prove that all basses of a finite dimension vector space contain same number of elements.

(b) Show that the vectors (3, 0, -3), (-1, 1, 2), (4, 2, -2) and (2, 1, 1) are linearly dependent.

Q.4. (a) The set $\{v_1, v_2, \ldots\ldots, v_n\}$ of vectors is a vector space V is linearly dependent if and only if some v_i is the linear combination of the other vectors.

(b) Let A, B be two ideals of the ring R. Then show that

$$\frac{A+B}{A} \cong \frac{B}{A \cap B}.$$

Q.5. (a) If A is $n \times n$ matrix then

(i) Determinant of $(A-\lambda I)$ where λ is a scalar in a polynomial $P(\lambda)$.

(ii) The eigenvalues of A are the solutions of $P(\lambda) = 0$.

(b) If A is an ideal of the ring R with unity such that $1 \in A$, then $A = R$

SECTION-B

Q.6. (a) Find an equation of the straight line joining two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are given. Hence find equations of the tangent and normal at any point 'θ' on the ellipse.

(b) Prove that an equation of the normal to the asteroid

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

is $x \sin t - y \cos t + a \cos 2t = 0$, t being parameter.

Q.7. (a) Show that the pedal equation of the curve

$x = 2a \cos\theta - 2 \cos 2\theta$, $y = 2a \sin\theta - a \sin 2\theta$ is $9(r^2 - a^2) = 8p^2$

(b) Find the length of the arc of the curve $x = e^{\theta} \sin\theta$, $y = e^{\theta} \cos\theta$

from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

Q.8. (a) Find the shortest distance between the straight line joining the points A(3, 2, -4) and B(1, 6, -6) and the straight line joining the points C(-1, 1, -2) and D(-3, 1, -6). Also find equation of the line of shortest distance and coordinates of the feet of the common perpendicular.

(b) Find an equation of the sphere for which the circle

$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y - 4z - 8 = 0$ is a great circle.