FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2012

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE:
(i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B**. **ALL** questions carry **EQUAL** marks.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) **Use of Scientific Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let H be a normal subgroup and K a subgroup of a group G. Prove that HK is a subgroup of G and $H \cap K$ is normal in K and $\frac{HK}{H} \cong \frac{K}{H \cap K}$. **(12)**

(b) Show that number of elements in a Conjugacy class Ca of an element 'a' in a group G is equal to the index of its normaliser. **(8)**

Q. 2. **(a)** Prove that if G is an Abelian group, then for all $a, b \in G$ and integers n, $(ab)^n = a^n b^n$. **(6)**

(b) Show that subgroup of Index 2 in a group G is normal. **(7)**

(c) If H is a subgroup of a group G, let $N(H) = \{a \in G \mid aHa^{-1} = H\}$ Prove that $N(H)$ is a subgroup of G and contains H. **(7)**

Q. 3. **(a)** Show that set C of complex numbers is a field. **(6)**

(b) Prove that a finite integral domain is a field. **(6)**

(c) Show that $Z_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$ is a ring under addition mod 6 and multiplication mod 6 but not a field. Find the divisors of Zero in Z_6. **(8)**

Q. 4. **(a)** Let F be a field of real numbers, show that the set V of real valued continuous functions on the closed interval $[0,1]$ is a vector space over F and the subset Y of V containing all functions whose nth derivatives exist, forms a subspace of V. **(10)**

(b) Prove that any finite dimensional vector space is isomorphic to F^n. **(10)**

Q. 5. **(a)** State and prove Cayley-Hamilton theorem. **(10)**

(b) Use Cramer's rule to solve the following system of linear equations: **(10)**

$$x + y + z + w = 1$$
$$x + 2y + 3z + 4w = 0$$
$$x + y + 4z + 5w = 1$$
$$x + y + 5z + 6w = 0$$

SECTION-B

Q. 6. **(a)** Prove that an equation of normal to the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ can be written in the form: **(10)**

$$y\,Cos\theta - x\,Sin\theta = a\,Cos2\theta$$

Hence show that the evolute of the curve is

$$(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$$

(b) If and are radii of curvature at the extremities of any chord of the Cardioid $r = a(1 + Cos\theta)$ which passes through the pole, then prove that $= \frac{16a^2}{9}$. **(10)**

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Q. 7. **(a)** Find an equation of the normal at any point of the curve with parametric equations: **(10)**
$x = a(Cos\,t + t\,Sin\,t), \quad y = a(Sin\,t - t\,Cos\,t)$.
Hence deduce that an equation of its evolute is $x^2 + y^2 = a^2$.

(b) Find equations of the planes bisecting the angle between the planes $3x + 2y - 6z + 1 = 0$ and $2x + y + 2z - 5 = 0$. **(10)**

Q. 8. **(a)** Define a surface of revolution. Write equation of a right elliptic-cone with vertex at origin. **(6)**

(b) Identify and sketch the surface defined by **(6)**
$x^2 + y^2 = 2z - z^2$.

(c) If *y=f(x)* has continuous derivative on $[a,b]$ and S denotes the length of the arc of *y=f(x)* between the lines *x=a* and *x=b*, prove that **(8)**

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,.$$

Find the length of the parabolas $y^2 4ax$

(i) From vertex to an extremity of the latus rectum.

(ii) Cut off by the latus rectum.

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