FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2010

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** questions in all by selecting at least **THREE** questions from **SECTION–A** and **TWO** questions from **SECTION–B**. All questions carry **EQUAL** marks. (ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) Let W be a subspace of a finite dimensional vector space V, then W is finite dimensional and dim (w) $\leq$ dim (v). Also if dim (w) = dim (V), then V = W. **(10)**

(b) Let V & W be vector space and let T : V $\rightarrow$ w be a linear if V is finite dimensional, then nullity (T) + rank (T) = dim v **(10)**

Q.2. (a) Show that there exist a homomorphism from S_n onto the multiplication group {-1,1} of 2 elements ($n \geq 1$). **(7)**

(b) If H is the only subgroup of a given finite order in a group G. Prove that H is normal in G. **(7)**

(c) Show that a field K has only two ideals (namely K & (o)). **(6)**

Q.3. (a) Find all possible jordan canonical forms for 3x3 matrix whose eiganvalues are -2,3,3 **(10)**

(b) Show that matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ **(10)**

is diagonalizable with minimum calculation

Q.4. (a) Every group is isomorphic to permutation group **(7)**

(b) Show that for n $\geq$ 3 Z (s_n) = I **(6)**

(c) Let A, B be two ideal of a ring, then $\frac{A+B}{A} = \frac{B}{A \cap B}$. **(7)**

Q.5. (a) Verify Cayley – Hamilton theorem for the matrix **(7)**

$$A=\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) Prove that ring A = $\underset{1}{Z}$, the set of all integers is a principal ideal ring. **(7)**

(c) Under what condition on the scalar, do the vectors (1,1,1), $(1, \xi, \xi^2)$, $(1, -\xi, \xi^2)$ form basis of c^3? **(6)**

SECTION – B

Q.6. (a) Show that T.N. = 0 for the helix **(10)**

R(t) = (acos wt) $\hat{z}$ + (a sinwt) $\hat{j}$ + (bt) $\hat{k}$

(b) The vector equation of ellipse :r(t) = (2 cos t) $\hat{i}$ + (3 Sint) $\hat{j}$; $(0 \leq t \leq 2\Pi)$

Find the eurvature of ellipse at the end points of major & minor axes. **(10)**

Q.7. (a) Discuss & sketch the surface **(12)**

$x^2+4y^2=4x-4z^2$

(b) Show that an equation to the right circular cone with vertex at 0, axis oz & semi – vertical angle $\propto$ is $x^2+y^2=z^2 \tan^2 \propto$ **(8)**

Q.8. (a) Show that hyperboloids of one sheet and hyperbolic parabolas are ruled surface. **(6+6)**

(b) Find an equation of the plane which passes through the point (3,4,5) has an x – intercept equal to -5 and is perpendicular to the plane 2x+3y-z = 8. **(8)**

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