# Cambridge International Examinations 

## Cambridge

Cambridge Pre-U Certificate

## MATHEMATICS (PRINCIPAL)

Paper 3 Applications of Mathematics

## SPECIMEN PAPER

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on the work you hand in.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10 \mathrm{~ms}^{-1}$.
The use of a electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80 .
You are advised to spend no more than 1 hour on Section A and 1 hour on Section B.

## Section A: Probability (40 marks)

## You are advised to spend no more than 1 hour on this section.

1 The times for a motorist to travel from home to work are normally distributed with a mean of 24 minutes and a standard deviation of 4 minutes. Find the probability that a particular trip from home to work takes
(i) more than 27 minutes,
(ii) between 20 and 25 minutes.

2 (a) A music club has 200 members. 75 members play the piano, 130 members like Elgar, and 30 members do not play the piano, nor do they like Elgar.
(i) Calculate the probability that a member chosen at random plays the piano but does not like Elgar.
(ii) Calculate the probability that a member chosen at random plays the piano given that this member likes Elgar.
(b) The music club is organising a concert. The programme is to consist of 7 pieces of music which are to be selected from 9 classical pieces and 6 modern pieces. Find the number of different concert programmes than can be produced if
(i) there are no restrictions,
(ii) the programme must consist of 5 classical pieces and 2 modern pieces,
(iii) there are to be more modern pieces than classical pieces.

3 The table shows fuel economy figures in miles per gallon (mpg) for some new cars.

| Car | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mpg | 57 | 40 | 34 | 33 | 11 | 17 | 30 | 27 | 31 | 20 | 35 | 24 | 26 | 23 | 32 |

(i) Find the median and quartiles for the mpg of these fifteen cars.
(ii) Use the values in part (i) to identify any cars for which the mpg is an outlier.

4 A survey into left-handedness found that $13 \%$ of the population of the world are left-handed.
(i) State the assumptions necessary for it to be appropriate to model the number of left-handed children in a class of 20 children using the binomial distribution $\mathrm{B}(20,0.13)$.
(ii) Assuming that this binomial model is appropriate, calculate the probability that fewer than $13 \%$ of the 20 children are left-handed.

5 James plays an arcade game. Each time he plays, he puts a $£ 1$ coin in the slot to start the game. The possible outcomes of each game are as follows:

James loses the game with a probability of 0.7 and the machine pays out nothing,
James draws the game with a probability of 0.25 and the machine pays out a $£ 1$ coin,
James wins the game with a probability of 0.05 and the machine pays out ten $£ 1$ coins.
The outcomes can be modelled by a random variable $X$ representing the number of $£ 1$ coins gained at the end of a game.
(i) Construct a probability distribution table for $X$.
(ii) Show that $\mathrm{E}(X)=-0.25$ and find $\operatorname{Var}(X)$.

James starts off with $10 £ 1$ coins and decides to play exactly 10 games.
(iii) Find the expected number of $£ 1$ coins that James will have at the end of his 10 games.
(iv) Find the probability that after his 10 games James will have at least $10 £ 1$ coins left.

## Section B: Mechanics (40 marks)

## You are advised to spend no more than 1 hour on this section.



The diagram shows two horizontal forces $\mathbf{P}$ and $\mathbf{Q}$ acting at the origin $O$ of rectangular coordinates $O x y$. The components of $\mathbf{P}$ in the $x$ - and $y$-directions are 12 N and 17 N respectively. The components of $\mathbf{Q}$ in the $x$ - and $y$-directions are -5 N and 7 N respectively.
(i) Write down the components, in the $x$ - and $y$-directions, of the resultant of $\mathbf{P}$ and $\mathbf{Q}$.
(ii) Hence, or otherwise, calculate the magnitude of this resultant and the angle the resultant makes with the positive $x$-axis.

7 A particle travels along a straight line. Its velocity $v \mathrm{~ms}^{-1}$ after $t$ seconds is given by

$$
v=t^{3}-9 t^{2}+20 t
$$

When $t=0$, the particle is at rest at $P$.
(i) Find the times, other than $t=0$, at which the particle is at rest.
(ii) Find the displacement of the particle from $P$ when $t=2$.

8 Two trucks, $S$ and $T$, of masses 8000 kg and 10000 kg respectively, are pulled along a straight, horizontal track by a constant, horizontal force of $P \mathrm{~N}$. A resistive force of 600 N acts on $S$ and a resistive force of 450 N acts on $T$. The coupling between the trucks is light and horizontal (see diagram).


The acceleration of the system is $0.3 \mathrm{~ms}^{-2}$ in the direction of the pulling force of magnitude $P$.
(i) Calculate the value of $P$.

Truck $S$ is now subjected to an extra resistive force of 1800 N . The pulling force, $P$, does not change.
(ii) Calculate the new acceleration of the trucks.
(iii) Calculate the force in the coupling between the trucks.


Three particles $A, B$ and $C$, having masses of $1 \mathrm{~kg}, 2 \mathrm{~kg}$ and 5 kg respectively, are placed 1 metre apart in a straight line on a smooth horizontal plane (see diagram). The particles $B$ and $C$ are initially at rest and $A$ is moving towards $B$ with speed $14 \mathrm{~ms}^{-1}$. The coefficient of restitution between each pair of particles is 0.5 .
(i) Find the velocity of $B$ immediately after the first impact and show that $A$ comes to rest.
(ii) Show that $B$ reversed direction after the impact with $C$.
(iii) Find the distances between $B$ and $C$ at the instant that $B$ collides with $A$ for the second time.


Particles $A$ and $B$ of masses $2 m$ and $m$, respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley $P$. The particle $A$ rests in equilibrium on a rough plane inclined at an angle $\alpha$ to the horizontal, where $\alpha \leqslant 45^{\circ}$ and $B$ is above the plane. The vertical plane defined by $A P B$ contains a line of greatest slope of the plane, and $P A$ is inclined at angle $2 \alpha$ to the horizontal (see diagram).
(i) Show that the normal reaction $R$ between $A$ and the plane is $m g(2 \cos \alpha-\sin \alpha)$.
(ii) Show that $R \geqslant \frac{1}{2} m g \sqrt{2}$.

The coefficient of friction between $A$ and the plane is $\mu$. The particle is about to slip down the plane.
(iii) Show that $0.5<\tan \alpha \leqslant 1$.
(iv) Express $\mu$ as a function of $\tan \alpha$ and deduce its maximum value as $\alpha$ varies.

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