CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2013 series

	9794 MATHEMATICS
9794/03	Paper 3 (Applications of Mathematics), maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Where appropriate, accept answers to 3 sf or better, then, except in **Q4 (iii)**, ISW if rounded to 2sf or fewer. Answers given to 2 sf or fewer without an "unrounded" answer score A0.

1	$\bar{x} = \frac{192}{100} = 1.92$	M1 A1		Use of correct formula for mean; may be implied. c.a.o.
	$s = \sqrt{\frac{488}{100} - 1.92^2} = \sqrt{1.1936} = 1.09(25...)$	M1 A1 **[4]**	**[4]**	Use of correct formula for standard deviation; may be implied. c.a.o. Accept unbiased estimate 1.09(80…) If no working shown, answer must be correct to 3 sf (or better) to score.
2 (i)	$P(A \cap B) = P(A) \times P(B \mid A)$ $= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$	M1 A1 **[2]**		Conditional probability rule applied, s.o.i. c.a.o. Accept solutions based on Venn diagrams.
(ii)	$P(B) = P(A \cup B) - P(A) + P(A \cap B)$ $= \frac{5}{6} - \frac{1}{2} + \frac{1}{8} = \frac{11}{24}$	M1 A1 **[2]**	**[4]**	Probability rule applied, s.o.i. Ft (i) provided both $P(A \cap B)$ and $P(B)$ lie between 0 and 1.
3 (i)	$S_{xy} = 77532 - \frac{1002 \times 1865}{25} = 2782.8$	M1		Use of formula for numerator.
	$S_{xx} = 43508 - \frac{1002^2}{25} = 3347.84$	M1		Use of formula for either term in denominator.
	$S_{yy} = 142749 - \frac{1865^2}{25} = 3620$			
	$r = \frac{2782.8}{\sqrt{3347.84 \times 3620}} = 0.799(36...)$	M1 A1 **[4]**		Use of formula for r. c.a.o.
(ii)	Form $y = ax + b$			
	$a = \frac{S_{xy}}{S_{xx}} = \frac{2782.8}{3347.84} = 0.83(122...)$	M1 A1		Use of formula for a. S_{xy} and S_{xx} from above. **AG.**
	$b = \bar{y} - a\bar{x}$			
	$\therefore b = \frac{1865}{25} - 0.83122... \times \frac{1002}{25}$ $= 74.6 - 0.83122... \times 40.08 = 41.28(46...)$	M1 A1 **[4]**		Use of formula for b. **AG.** Must be convincing. Allow M1 for use of $a = 0.83$ to find b (= 41.33...), or $b = 41.28$ to find a (= 0.83133…), but not both, but do not award the corresponding A mark.

(iii)	When $x = 50$, $y = 82.78\ldots \approx 82.8$	B1		Accept a.w.r.t. 82.8
	This is ok; it is within the range of the data.	B1		At least one of the comments must refer to within/beyond the range of the data. (o.e.)
	When $x = 65$, $y = 95.23\ldots \approx 95.2$	B1		Accept a.w.r.t. 95.2
	This is not ok; it is beyond the range of the data.	B1 **[4]**	**[12]**	
4 (i)	$X \sim N(85.1, 3.4^2)$			
	$P\left(Z < \frac{80 - 85.1}{3.4}\right)$	M1		Standardising.
	$= \Phi(-1.5) = 1 - \Phi(1.5) = 1 - 0.9332$	M1		1 – … to deal with negative z value.
	= 0.0668	A1 **[3]**		
(ii)	P(B(6, 0.0668) ‹1)	M1		Recognise need for B(6, p).Possibly implied by partially correct terms in the next line.
	$= 0.9332^6 + 6 \times 0.9332^5 \times 0.0668$	M1 M1		Either term correct. Sum of two correct terms.
	= 0.66046 … + 0.28366 …			
	= 0.944(12 …)	A1 **[4]**		Ft *their p* from (i).
(iii)	250 × (1 – 0.9441)	M1		250 × …
		M1		(1 – (ii)).
	=13.975 ≈ 14.0	A1 **[3]**	**[10]**	Must be at least 1 dp. Do not allow answer rounded to the nearest integer, even following an answer to 3sf or better.

5	(i)	$\frac{7!}{2!} = \frac{5040}{2} = 2520$	M1 M1 A1 **[3]**		7! ÷ 2! c.a.o.
	(ii)	$^6C_5$	M1		Consider selections when all digits are different.
		$^6C_5 \times {}^5P_5$ or $^6P_5 = 720$	A1		Arrangements when all digits different.
		$^5C_3$	M1		Consider selections of the form 11xxx.
		$(10) \times \frac{5!}{2!} = 600$	M1 A1		Arrangements of 11xxx
		720 + 600	M1		Adding two (or more) relevant cases.
		= 1320	A1 **[7]**		Fully correct.
		OR: (e.g.) Using no 1's + one 1 + two 1's $= {}^5P_5 + 5 \times {}^5P_4 + 10 \times {}^5P_3$ = 120 + 600 + 600 = 1320		**[10]**	
6	(i)	$v = t(t-2)(t-4)$	M1		Set $v = 0$ and attempt to solve.
		$t \neq 0$ so $t = 2$ and 4.	A1		Fully correct. SC: B1 for both $t = 2$ and $t = 4$ found by substitution or stated without working, and B1 if shows/explains there are no other values.
		Cubic graph crossing the t axis at 0 & 2 other places.	B1		
		Fully correct <u>curve</u>, axes and intercepts labelled and curve only between $t = 0$ and 4.	B1 **[4]**		
	(ii)	$a = 3t^2 - 12t + 8$	M1 A1		Differentiate v. All terms correct. Allow if found in (i) and used here.
		= 12 – 24 + 8 = –4 (ms^{-2})	A1 **[3]**		Substitute $t = 2$. c.a.o

(iii)	$x = \frac{t^4}{4} - 2t^3 + 4t^2 + c$	M1 A1		Integrate v. All terms correct; condone omission of "+ c". Allow definite integral as alternative.
	$x = 0$ when $t = 0$ therefore $c = 0$	A1		Deal with c correctly or consider lower limit of definite integral.
	When $t = 2$, $x = 4 - 16 + 16 = 4$	A1		Indep of previous A1.
	So average speed = 4 / 2	M1		Use formula for average speed.
	= 2 (ms^{-1})	A1 **[6]**	**[13]**	Ft *their* x when $t = 2$.
7 (i)	Let the velocities of A and B after the collision be v and w.			
	$4mu = 4mv + 2mw$ $\therefore 2u = 2v + w$	M1		Use of conservation of momentum: a correct equation, consistent with a diagram, if present.
	$eu = w - v$	M1		Use of N.E.L.: a correct equation, consistent with a diagram, if present.
	$\therefore v = \frac{1}{3}(2-e)u$ and $w = \frac{2}{3}(1+e)u$	M1 A1 **[4]**		Solve simultaneous equations. Both correct. Accept "w" unsimplified.
(ii)	If $e = \frac{1}{2}$ then $v = \frac{1}{2}u$ and $w = u$	B1 **[1]**		Ft *their* v and w in (i).
(iii)	After A collides with B velocities are: $u/2$, u (and 0) respectively.	M1		Apply the result from (i) at least once. Or a complete correct method for the BC collision.
	After B collides with C velocities are: $u/2$, $u/2$ and u respectively.	A1 **[2]**		All correct, including A.
(iv)	A and B have the same velocity and C is moving away from them so there can be no further collisions.	B1 **[1]**	**[8]**	Ft (iii). Must consider all 3 particles.
8 (i)	$x = Ut\cos\theta$	B1		
	$y = Ut\sin\theta - \frac{1}{2}gt^2$	B1		Allow $g = 9.8$.
	$t = \frac{x}{U\cos\theta}$	M1		Make t the subject of x equation and substitute.
	$\therefore y = U\left(\frac{x}{U\cos\theta}\right)\sin\theta - \frac{1}{2}g\left(\frac{x}{U\cos\theta}\right)^2$ $= x\tan\theta - \frac{gx^2}{2U^2\cos^2\theta}$	A1 **[4]**		Accept any correct form/unsimplified.

(ii)	$y = 0$ and $x \neq 0$ gives $x = \frac{U^2}{g}\sin 2\theta$	M1		Set $y = 0$ and attempt to make x or $\sin 2\theta$ the subject. Allow other equivalent methods e.g by solving a quadratic ($t^2 - 4t + 1 = 0$) in $\tan\theta$ ($= 2 \pm \sqrt{3}$).
	$\therefore \sin 2\theta = \frac{gx}{U^2} = \frac{10 \times 45}{30^2} = 0.5$	A1		Substitute and obtain 0.5 (or $\tan\theta$) correctly.
	This has 2 solutions so there are 2 trajectories.	B1		Require an explicit statement to this effect.
	$\therefore \theta = 15°$ or $75°$	A1 **[4]**		Both correct.
(iii)	$\theta = 15°$ is fast (and low).	B1		"Advantage" of one. (ft (ii))
	$\theta = 75°$ is high (more likely to clear any obstacles).	B1 **[2]**		"Advantage" of the other. (ft (ii))
			[10]	SC B1 only for just "high" and "low". Allow other reasonable "advantages".
9 (i)	Diagram with weight, normal contact and friction forces added.	B1 **[1]**		Do not accept both T and the components of T shown.
(ii)	$F = T\cos\theta$	B1		Resolve horizontally.
	$mg = R + T\sin\theta$	B1		Resolve vertically.
	$F = \mu R$	M1		Limiting friction
	$T\cos\theta = \mu(mg - T\sin\theta)$ $\therefore T = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$	M1 **[4]**		Eliminate F and R and rearrange to given answer. Must be convincing – require at least one intermediate line.
(iii)	With $\mu = 0.75$, min T occurs at max ($\cos\theta + 0.75\sin\theta$).	M1		Allow substitution for μ at any stage.
	EITHER $-\sin\theta + 0.75\cos\theta = 0$	M1 A1		Differentiate and set = 0.
	$\tan\theta = 0.75$ $\therefore \theta = \text{invtan}(0.75) = 36.9°$	A1 **[4]**		
	OR Use of $R\cos(\theta - \alpha)$ or $R\sin(\theta + \alpha)$.	M1		And set cos(…) or sin(…) = 1.
	$\alpha = 36.9°$ or $53.1°$	A1		As appropriate.
	$\theta = 36.9°$	A1	**[9]**	