CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2013 series

9794 MATHEMATICS

9794/02 Paper 2 (Pure Mathematics 2), maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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CAMBRIDGE
International Examinations

1	**(i)**	$\mathbf{u}+\mathbf{v}=\begin{pmatrix}1\\8\end{pmatrix}$, $\mathbf{u}-\mathbf{v}=\begin{pmatrix}7\\4\end{pmatrix}$	B1 B1 **[2]**	
	(ii)	$\lvert\mathbf{u}+\mathbf{v}\rvert=\sqrt{1+64}=\sqrt{65}$ $\lvert\mathbf{u}-\mathbf{v}\rvert=\sqrt{49+16}=\sqrt{65}$	M1 A1 **[2]**	**[4]**
2	**(i)**	Any correct complete method 43	M1 A1 **[2]**	
	(ii)	$r=\frac{1}{3}$ $S_\infty=\frac{a}{1-r}$ $=\frac{162}{1-\frac{1}{3}}=243$	B1 M1 A1 **[3]**	
	(iii)	All four of –1, 3, –1, 3 It is periodic o.e.	B1 B1 **[2]**	**[7]**
3	**(i)**	$x^2+2x-3=(x+1)^2-4$ $(a=1, b=-4)$	B1 B1 **[2]**	
	(ii)	u-shaped parabola Vertex at (–1, –4) Let $x=0$ and solve Intersecting: x-axis at –3 and 1, y-axis at –3	B1 B1 ft M1 A1 B1 **[5]**	**[7]**
4	**(i)**	Substitute $z=-1$ and convincingly obtain 0	B1 **[1]**	
	(ii)	3 term quadratic $z^3+5z^2+9z+5=(z+1)(z^2+4z+5)=0$ Solve $z^2+4z+5=0$ Obtain $-2+\mathrm{i}$ and $-2-\mathrm{i}$	M1 A1 M1 A1 **[4]**	
	(iii)	Argand diagram showing their three roots	B1 ft **[5]**	**[10]**

<table>
<tr><td>5</td><td>(i)</td><td>Differentiate implicitly, using product rule</td><td>M1</td><td></td></tr>
<tr><td></td><td></td><td>$y + x\frac{\mathrm{d}y}{\mathrm{d}x}$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>final term $2y\frac{\mathrm{d}y}{\mathrm{d}x}$</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td>complete $2x + y + x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, and manipulate to given answer</td><td>A1 [4]</td><td></td></tr>
<tr><td></td><td>(ii)</td><td>Substitute $x = 2, y = 3$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{7}{8}$</td><td>M1</td><td></td></tr>
<tr><td></td><td></td><td>Gradient of normal is $\frac{8}{7}$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>Line through (2, 3) with <i>their m</i>.</td><td>M1</td><td></td></tr>
<tr><td></td><td></td><td>Obtain $8x - 7y + 5 = 0$</td><td>A1 [4]</td><td>[8]</td></tr>
<tr><td>6</td><td>(i)</td><td>Obtain $\log N = \log a + t\log b$ o.e. w.w.w.</td><td>M1</td><td></td></tr>
<tr><td></td><td></td><td>Compare with $y = mx + c$</td><td>A1 [2]</td><td></td></tr>
<tr><td></td><td>(ii)</td><td><table><tr><td>t</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>log <i>N</i></td><td>0.9</td><td>1</td><td>1.2</td><td>1.38</td><td>1.52</td><td>1.6</td><td>1.67</td><td>1.84</td></tr></table></td><td>M1
A1</td><td></td></tr>
<tr><td></td><td></td><td>Plot points (condone 1 error)</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td>Line of best fit</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td>Obtain <i>a</i> between 5.5 and 6.5</td><td>B1</td><td></td></tr>
<tr><td></td><td></td><td><i>b</i> between 1.32 and 1,42</td><td>B1 [6]</td><td></td></tr>
<tr><td></td><td></td><td>SC M1A1 for <i>a</i> and <i>b</i> from data in the table only if no graph drawn</td><td></td><td></td></tr>
<tr><td></td><td>(iii)</td><td>Follow through <i>their a</i> and <i>b</i> given answers in these ranges</td><td></td><td></td></tr>
<tr><td></td><td></td><td><table><tr><td></td><td>Model</td></tr><tr><td>2008</td><td>50–95</td></tr><tr><td>2020</td><td>1400–5500</td></tr></table></td><td>B1 ft
B1 ft [2]</td><td></td></tr>
<tr><td></td><td>(iv)</td><td>Use logs (or <i>their</i> expression from part (i)), or evaluate enough terms to get $N > 500$</td><td>M1</td><td></td></tr>
<tr><td></td><td></td><td>Solve for <i>t</i> and interpret as a year</td><td>M1</td><td></td></tr>
<tr><td></td><td></td><td>2013–2017</td><td>A1 ft [3]</td><td></td></tr>
<tr><td></td><td>(v)</td><td>Any reasonable observation about the <i>model</i>, e.g:
• It predicts unrestricted growth which is unrealistic.
• It predicts that the growth rate is not constant, but increases with population size, which is realistic.
• Extrapolation is not valid when breeding conditions may change, so not suitable.</td><td>B1 [1]</td><td>[14]</td></tr>
</table>

7	**(i)**	Attempt product rule	M1		
		Obtain $2xe^{-x}$	A1		
		Obtain $\pm x^2e^{-x}$	M1		
		Obtain $xe^{-x}(2-x)$ **AG**	A1	**[4]**	
	(ii)	Set equal to zero and solve	M1		
		At least two correct x or y values	A1		
		(0, 0) and $(2, 4e^{-2})$	A1	**[3]**	**[7]**
8	**(i)**	Since most terms cancel, $(1 + 30^{-1})$	M1		
		$= 1\frac{1}{30}$	A1	**[2]**	
	(ii)	$S = -1 + 2 - 3 + 4 - \ldots -99 + 100$	M1		
		$= 50 \times 1 = 50$	A1	**[2]**	**[4]**
9	**(i)**	$\text{cosec}2x = \frac{1}{\sin 2x}$, $\cot 2x = \frac{\cos 2x}{\sin 2x}$	B1		
		OR $\frac{1}{\tan 2x}$ seen			
		$\text{cosec}\,2x - \cot 2x = \frac{1-\cos 2x}{\sin 2x}$			
		$= \frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$	M1 M1		
		$= \frac{2\sin^2 x}{2\sin x\cos x}$	A1		
		$= \frac{\sin x}{\cos x} = \tan x$	A1		
		$\tan\frac{3}{8}\pi = \text{cosec}\ \frac{3}{4}\pi - \cot\ \frac{3}{4}\pi = 1 + \sqrt{2}$	B1	**[6]**	

(ii)	$\int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}(\operatorname{cosec}2x-\cot 2x)^2\,\mathrm{d}x = \int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}\tan^2 x\mathrm{d}x$	M1		
	$= \int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}\sec^2 x \pm 1\mathrm{d}x$	A1		
	$= \left[\tan x - x\right]_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}$	M1 A1		
	$= \sqrt{2}-\frac{1}{8}\pi$	M1 A1	**[6]**	
	Alternate solution:			
	$\int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}(\operatorname{cosec}2x-\cot 2x)^2\,\mathrm{d}x$			
	$= \int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}\operatorname{cosec}^2 2x - 2\operatorname{cosec}2x\cot 2x + \cot^2 2x\mathrm{d}x$	M1		
	$= \int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}2\operatorname{cosec}^2 2x - 2\operatorname{cosec}2x\cot 2x - 1\mathrm{d}x$	A1		
	$= \left[-\cot 2x + \operatorname{cosec}2x - x\right]_{\frac{1}{4}\pi}^{\frac{3}{8}\pi}$	M1 A1		
	$= \sqrt{2}-\frac{1}{8}\pi$	M1 A1	**[6]**	**[12]**
10 (i)	$\frac{\mathrm{d}V}{\mathrm{d}t} \propto \sqrt{h}$	M1		
	Since the tank is a prism $V \propto h$ so			
	$\frac{\mathrm{d}V}{\mathrm{d}t} = a\sqrt{V}$ where a is a constant	A1	**[2]**	
(ii)	Separating variables			
	$\int \frac{1}{\sqrt{V}}\mathrm{d}v = \int a\mathrm{d}t$	M1		
	$2\sqrt{V} = at\ (+c)$	M1 A1		
	Use $t = 0$, $V = V_0$ to obtain $c = 2\sqrt{V_0}$	B1		
	and $t = 1$, $V = \frac{1}{2}V_0$ in an equation involving a and c (or using definite integrals) to find a in terms of V_0 only	M1		
	$a = 2\sqrt{V_0}\left(\frac{1}{\sqrt{2}}-1\right)$	A1		
	convincingly substitute and rearrange to get			
	$V = V_0\left(\left(\frac{1}{\sqrt{2}}-1\right)t+1\right)^2$	A1	[7]	

(iii)	$V = 0$ implies $t = \dfrac{-1}{\dfrac{1}{\sqrt{2}} - 1} = 2 + \sqrt{2} = 3.41...$ 3.41 hours is 3 hours 24 mins and 51 seconds Condone verification only if $5.16 \times 10^{-6}\ V_0$ seen	M1 A1 **[2]**	**[11]**