UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9794 MATHEMATICS

9794/02 Paper 2 (Pure Mathematics 2), maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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International Examinations

1 **(i)** Using the quadratic formula or equivalent, $x=\dfrac{8\pm\sqrt{64-16}}{2}=4\pm2\sqrt{3}$	M1 A1 [2]		up to 1 error
(ii) $\left(6+2\sqrt{3}\right)\left(2-\sqrt{3}\right)=12-6\sqrt{3}+4\sqrt{3}-2\sqrt{3}\sqrt{3}$ $=12-6\sqrt{3}+4\sqrt{3}-6$ $=6-2\sqrt{3}$	M1 B1 A1 [3]	**[5]**	Multiply out $\sqrt{3}\sqrt{3}=3$
2 **(i)** **(a)** $AB=\sqrt{6^2+8^2}=10$	M1 A1 [2]		Use Pythagoras
(b) Midpoint of AB is (6, –3)	B1 [1]		
(c) $(x-6)^2+(y+3)^2=25$ AEF	√B1 √B1 √B1 [3]		ft *their* values from **(a)** + **(b)**
(ii) Gradient of AC is –0.5 Use of $y = mx + c$ or equivalent Required equation: $y=-\frac{1}{2}x+\frac{11}{2}$	M1 M1 A1 [3]	**[9]**	$\Delta y/\Delta x$
3 $\int_0^1\left(e^x-x\right)dx=\left[e^x-\frac{1}{2}x^2\right]_0^1$ $=e-\frac{3}{2}$	M1 A1 M1 A1 [4]	**[4]**	ke^x+mx^2 Use of limits Without + c
4 Take logarithms and apply log rule $(2x-1)\log2=\log5$ Rearrange to make x the subject Obtain $x=\dfrac{1}{\log4}=1.66096\ldots$ AEF	M1 A1 M1 A1 [4]	**[4]**	Up to 1 error
5 **(i)** Sine wave through the origin, showing intersections with the x-axis at (0), π and 2π.	B1 [1]		
(ii) Sine wave translated in the negative x-direction x-intercepts $\frac{5}{6}\pi$, $\frac{11}{6}\pi$, y-intercept 0.5 and symmetrical about the x-axis.	M1 A1 [2]	**[3]**	
6 **(i)** **(a)** $u_1=5$, $u_5=37$ implies $4d=32$ $d=8$ $u_n=8n-3$ AEF ft *their d*	M1 A1 M1 √A1 [4]		seen in either part
(b) $S_n=\dfrac{n}{2}(2+8n)$ AEF ft *their d*	M1 √A1 [2]		
(ii) $S_{25}-S_4=2525-68$ $=2457$	M1 A1 [2]	**[8]**	Or equivalent

7 **(i)** Attempt product rule $\frac{dy}{dx} = 2e^{-2x} - 2(2x-3)e^{-2x}$ $= (8-4x)e^{-2x}$	M1 A1 A1 [3]		
(ii) $\frac{dy}{dx} \geq 0$ seen y is increasing when $x \leq 2$.	B1 B1 [2]	**[5]**	
8 Separate variables, prior to integration: $\int \frac{-1}{y^2}dy = \int x^3 dx$ $\frac{1}{y} = \frac{1}{4}x^4$ $(+c)$ Subs into expression including c and solve $c = \frac{1}{4}$ so $y = \frac{4}{x^4+1}$ AEF	M1 A1 A1 A1 M1 A1 [6]	**[6]**	
9 **(i)** $P = 2r + 2rx$ $A = r^2x$	B1 B1 [2]		
(ii) $P = 20$ implies $r = \frac{10}{1+x}$ so $A = \left(\frac{10}{1+x}\right)^2 x = \frac{100x}{(1+x)^2}$ AG	M1 A1 [2]		$r = f(x)$
(iii) Use quotient rule $\frac{dA}{dx} = \frac{100(1+x)^2 - 200x(1+x)}{(1+x)^4}\left[= \frac{100(1-x)}{(1+x)^3}\right]$ Set equal to zero and find $x = 1$ Attempt to show with first differential test that it is max. Completely correct solution	M1 A1 A1 M1 A1 [5]	**[9]**	Allow ±1 Or equivalent

10 **(i)** **(a)** $\ln(1 + e^x)$	B1		
$+ c$	B1 [2]		
(b) $\ln(1 + e^{\ln 3}) - \ln(1 + e^0)$	M1		Use of limits
$= \ln 4 - \ln 2$			
Use log rule correctly	M1		
$= \ln 2$ CAO	A1 [3]		
(ii) **(a)** Make substitution, including attempt at changing dx to du.	M1		
Attempt to simplify to obtain… $\int \frac{u-1}{u^2}\,du$	A1		
$= \int \frac{1}{u} - \frac{1}{u^2}\,du$ Deal with integrand,	M1		
$= \ln(u) + \frac{1}{u} + c$	√A1		
$= \ln(1+e^x) + \frac{1}{1+e^x} + c$ CAO	A1 [5]		
(b) $V = \pi \int_{-\ln 3}^{\ln 3} \left(\frac{e^x}{1+e^x}\right)^2 dx$ and attempt to integrate	M1		
$= \pi\left[\ln(1+e^x) + \frac{1}{1+e^x}\right]_{-\ln 3}^{\ln 3} = \pi(\ln 3 - \tfrac{1}{2})$	A1 [2]	**[12]**	

11 **(i)** $\frac{\mathrm{d}f}{\mathrm{d}t} = 2\cos t - 2\sin 2t$	M1		$k\cos t + m\sin 2t$
$= 2\cos t - 4\sin t\cos t$ $= 2\cos t(1 - 2\sin t)$	A1 [2]		AG
(ii) Find local maxima/minima	M1		
f′(x) = 0 implies $x = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{1}{6}\pi, \frac{5}{6}\pi$	A1		Any four
Values of f 1, –3, 1.5, 1.5	A1		All eight
Values of f at endpoints 1, 1	B1		
Hence range is [–3, 1.5]	A1 [5]		
(iii) Substitute for x and y, and multiply out: $(2\cos t + \sin 2t)^2 + (2\sin t + \cos 2t)^2$ $= 4\cos^2 t + 4\cos t\sin 2t + \sin^2 2t$ $+4\sin^2 t + 4\sin t\cos 2t + \cos^2 2t$	M1		Including cross-terms
$= 4(\cos^2 t + \sin^2 t) + (\cos^2 2t + \sin^2 2t)$ $+4(\cos t\sin 2t + \sin t\cos 2t)$	DM1		Pythagorean identity OR addition formula
$= 5 + 4\sin(t + 2t) = 5 + 4\sin 3t$	A1 [3]		AG
(iv) $x^2 + y^2 = r^2$ is a circle centre the origin $5 + 4\sin 3t \in [1,9]$ so C lies between and on circles of radius 1 and 3.	B 0, 1, 2		Either statement B1 Both and conclusion B2
(v) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1		$\frac{a\cos t + b\sin 2t}{c\sin t + d\cos 2t}$
$= \frac{2\cos t - 2\sin 2t}{-2\sin t + 2\cos 2t}$	A1		
at $t = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2-0}{-0+2} = 1$	A1 [3]	**[15]**	