Cambridge Pre-U

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics and Mechanics **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
You are advised to spend no more than 2 hours on Section A and 1 hour on Section B.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

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UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

Section A: Pure Mathematics (79 marks)

You are advised to spend no more than 2 hours on this section.

1 **(i)** Show that $x = 4$ is a root of $x^3 - 12x - 16 = 0$. [2]

(ii) Hence completely factorise the expression $x^3 - 12x - 16$. [3]

2 **(i)** Expand and simplify $(7 - 2\sqrt{3})^2$. [2]

(ii) Show that

$$\frac{\sqrt{125}}{2 + \sqrt{5}} = 25 - 10\sqrt{5}.$$ [4]

3 Use integration by parts to find $\int x \sin 3x \, dx$. [5]

4 **(i)** On the same diagram, sketch the graphs of $y = 2 \sec x$ and $y = 1 + 3 \cos x$, for $0 \leqslant x \leqslant \pi$. [4]

(ii) Solve the equation $2 \sec x = 1 + 3 \cos x$, where $0 \leqslant x \leqslant \pi$. [5]

5 Diane is given an injection that combines two drugs, Antiflu and Coldcure. At time t hours after the injection, the concentration of Antiflu in Diane's bloodstream is $3e^{-0.02t}$ units and the concentration of Coldcure is $5e^{-0.07t}$ units. Each drug becomes ineffective when its concentration falls below 1 unit.

(i) Show that Coldcure becomes ineffective before Antiflu. [3]

(ii) Sketch, on the same diagram, the graphs of concentration against time for each drug. [2]

(iii) 20 hours after the first injection, Diane is given a second injection. Determine the concentration of Coldcure 10 hours later. [2]

6 **(i)** Using the substitution $u = x^2$, or otherwise, find the numerical value of

$$\int_0^{\sqrt{\ln 4}} x e^{-\frac{1}{2}x^2} \, dx.$$ [4]

(ii) Determine the exact coordinates of the stationary points of the curve $y = xe^{-\frac{1}{2}x^2}$. [4]

7 Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto x^2 - 2x,$$
$$\mathrm{g} : x \mapsto x^2,$$
$$\mathrm{h} : x \mapsto \sin x.$$

(i) **(a)** State whether or not f has an inverse, giving a reason. [2]

(b) Determine the range of the function f. [2]

(ii) **(a)** Show that gh(x) can be expressed as $\frac{1}{2}(1 - \cos 2x)$. [2]

(b) Sketch the curve C defined by $y = \mathrm{gh}(x)$ for $0 \leqslant x \leqslant 2\pi$. [3]

8 **(i)** A curve C_1 is defined by the parametric equations

$$x = \theta - \sin\theta, \quad y = 1 - \cos\theta,$$

where the parameter θ is measured in radians.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\frac{1}{2}\theta$, except for certain values of θ, which should be identified. [5]

(b) Show that the points of intersection of the curve C_1 and the line $y = x$ are determined by an equation of the form $\theta = 1 + A\sin(\theta - \alpha)$, where A and α are constants to be found, such that $A > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [4]

(c) Show that the equation found in part **(b)** has a root between $\frac{1}{2}\pi$ and π. [2]

(ii) A curve C_2 is defined by the parametric equations

$$x = \theta - \tfrac{1}{2}\sin\theta, \quad y = 1 - \tfrac{1}{2}\cos\theta,$$

where the parameter θ is measured in radians. Find the y-coordinates of all points on C_2 for which $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$. [4]

9 The curve $y = x^2$ intersects the line $y = kx$, $k > 0$, at the origin and the point P. The region bounded by the curve and the line, between the origin and P, is denoted by R.

(i) Show that the area of the region R is $\frac{1}{6}k^3$. [3]

The line $x = a$ cuts the region R into two parts of equal area.

(ii) Show that $k^3 - 6a^2k + 4a^3 = 0$. [3]

The gradient of the line $y = kx$ increases at a constant rate with respect to time t. Given that $\frac{\mathrm{d}k}{\mathrm{d}t} = 2$,

(iii) determine the value of $\frac{\mathrm{d}a}{\mathrm{d}t}$ when $a = 1$ and $k = 2$, [4]

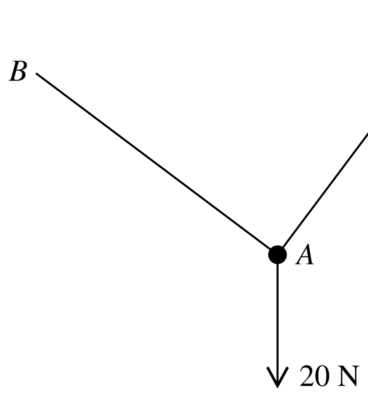
(iv) determine the value of $\frac{\mathrm{d}a}{\mathrm{d}t}$ when $a = 1$ and $k \neq 2$, expressing your answer in the form $p + q\sqrt{3}$, where p and q are integers. [5]

Section B: Mechanics (41 marks)

You are advised to spend no more than 1 hour on this section.

10 The points A, B and C lie in a vertical plane and have position vectors $4\mathbf{i}$, $3\mathbf{j}$ and $7\mathbf{i} + 4\mathbf{j}$, respectively. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertically upwards, respectively. The units of the components are metres.

(i) Show that angle BAC is a right angle. [2]



Strings AB and AC are attached to B and C, and joined at A. A particle of weight 20 N is attached at A (see diagram). The particle is in equilibrium.

(ii) By resolving in the directions AB and AC, determine the magnitude of the tension in each string. [3]

(iii) Express the tension in the string AB as a vector, in terms of $\mathbf{i}$ and $\mathbf{j}$. [3]

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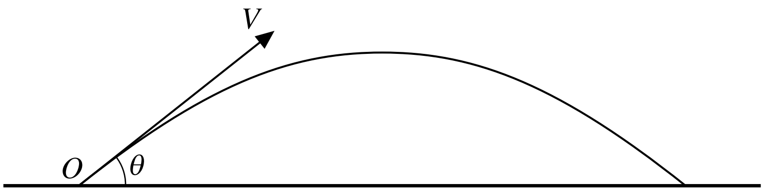

A projectile is fired from a point O in a horizontal plane, with initial speed V, at an angle θ to the horizontal (see diagram).

(i) Show that the range of the projectile on the horizontal plane is

$$\frac{2V^2 \sin\theta \cos\theta}{g}.$$ [4]

There are two vertical walls, each of height h, at distances 30 m and 70 m, respectively, from O with bases on the horizontal plane. The value of θ is 45°.

(ii) If the projectile just clears both walls, state the range of the projectile. [1]

(iii) Hence find the value of V and of h. [5]

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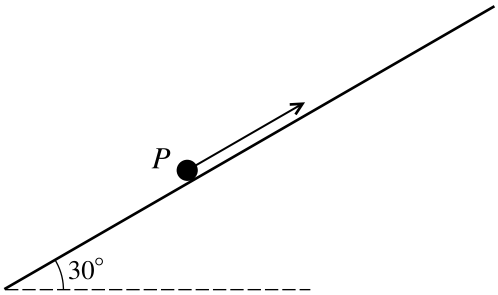

A particle P of mass 2 kg can move along a line of greatest slope on a smooth plane, inclined at 30° to the horizontal. P is initially at rest at a point on the plane, and a force of constant magnitude 20 N is applied to P parallel to and up the slope (see diagram).

(i) Copy and complete the diagram, showing all forces acting on P. [1]

(ii) Find the velocity of P in terms of time t seconds, whilst the force of 20 N is applied. [4]

After 3 seconds the force is removed at the instant that P collides with a particle of mass 1 kg moving down the slope with speed $5\,\text{m}\,\text{s}^{-1}$. The coefficient of restitution between the particles is 0.2.

(iii) Express the velocity of P as a function of time after the collision. [6]

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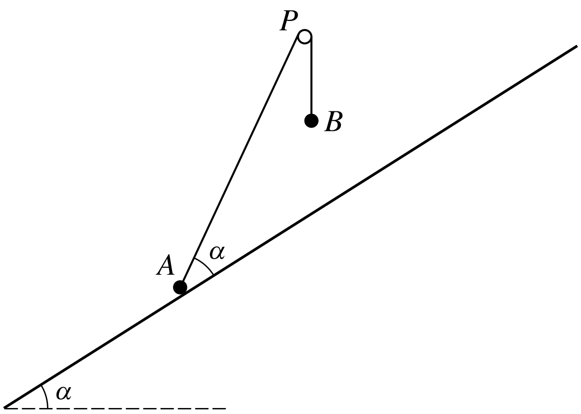

Particles A and B of masses $2m$ and m, respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley P. The particle A rests in equilibrium on a rough plane inclined at an angle α to the horizontal, where $\alpha \leqslant 45°$ and B is above the plane. The vertical plane defined by APB contains a line of greatest slope of the plane, and PA is inclined at angle 2α to the horizontal (see diagram).

(i) Show that the normal reaction R between A and the plane is $mg(2\cos\alpha - \sin\alpha)$. [3]

(ii) Show that $R \geqslant \frac{1}{2}mg\sqrt{2}$. [3]

The coefficient of friction between A and the plane is μ. The particle A is about to slip down the plane.

(iii) Show that $0.5 < \tan\alpha \leqslant 1$. [3]

(iv) Express μ as a function of $\tan\alpha$ and deduce its maximum value as α varies. [3]

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