UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9794 MATHEMATICS

9794/01 Paper 1 (Pure Mathematics and Probability), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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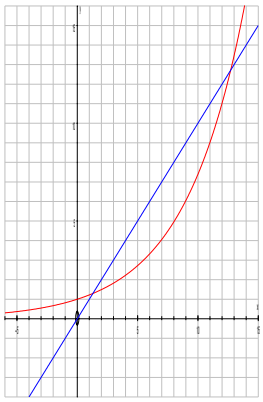
UNIVERSITY of CAMBRIDGE
International Examinations

1	**(i)**	Gradient = – 2	B1		Accept aef or method e.g sim eqns $5 = -2m + c$ and $-7 = 4m + c$ and substitution for c
		Attempt eqn of line $y = -2x + 1$	M1 A1	**[3]**	Accept any simplified form
2	**(i)**	State $2r + r\theta = 18$	B1		Accept work in degrees. Formula must be correct
		Obtain correctly $\theta = \dfrac{18-2r}{r}$ AG	A1		
	(ii)	Substitute $S = \dfrac{1}{2}r^2\left(\dfrac{18-2r}{r}\right)$	M1		Accept work in degrees. Award for substituting for θ in correct expression for S.
		Obtain $9r - r^2$	A1	**[4]**	
3		Method for modulus eqn, maybe implied.	M1		e.g graphical or if algebraic, must consider $3 + 2x = 7 - 4x$ and $3 + 2x = 4x - 7$
		State $x = 5$	B1		Ignore y co-ordinates
		State $x = \dfrac{2}{3}$	B1	**[3]**	Accept unsimplified
4	**(i)**	$\ln x^4 - \ln(3x-2) - \ln x^2$	M1 M1		Use power law at least once Use division or multiplication law at least once
		$\ln \dfrac{x^2}{3x-2}$	A1		AG so NIS
	(ii)	$\dfrac{x^2}{3x-2} = 1$	B1		Use $e^0 = 1$ or state $x^2 = 3x - 2$ from $\ln x^2 = \ln(3x - 2)$
		$x^2 - 3x + 2 = 0$	M1		Attempt soln of 3 term quadratic
		$x = 2$ or $x = 1$	A1	**[6]**	Obtain 2 and 1
5		Attempt use of $(\pi)\ \int(16 - x^2)\,dx$	B1		
		Attempt integration	M1		At least one power must rise in their single variable integral
		Obtain $(\pi)\ [\ 16x - \frac{1}{3}x^3\ ]$	A1		
		Use of correct limits in correct order	M1		
		Obtain $\dfrac{41\pi}{3}$ or 42.9 or better	A1	**[5]**	

6	**(i)**		B1 B1 B1*	A positive exponential graph Straight line with positive slope Show or state two intersections or roots
	(ii)	$x_{n+1} = x_n - \dfrac{e^{0.2x} - x}{0.2e^{0.2x} - 1}$	M1 B1 M1*	Use correct NR formula Correct derivative Starts at 0 and states at least two iterates States 1.296
		0, 1.25, 1.2958, 1.2959	A1 **[7]**	
7	**(i)**	$2 - 3\lambda = 2 - \mu$ $-3 + \lambda = 4 - 2\mu$	B1	Obtain correct eqns
		Obtain $\lambda = 1 \quad \mu = 3$	B1	
		Obtain $a = 3$ and $b = 1$	B1	
	(ii)	$\begin{pmatrix}-1\\-2\\1\end{pmatrix}$ and $\begin{pmatrix}-3\\1\\1\end{pmatrix}$	M1	Use correct vectors aef
		$\dfrac{(-3)(-1) + 1(-2) + 1(1)}{(\sqrt{11})(\sqrt{6})}$ $= \dfrac{2}{\sqrt{66}}$ (= 0.246)	M1 B1	Use correct dot product formula Find the length of any vector
		Obtain 75.7 °	A1 **[7]**	Obtain acute answer only cao

8	**(i)**	$(1-4a)^{\frac{1}{2}} = 1 + \frac{1}{2}(-4a) + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2}(-4a)^2$ $= 1 - 2a - 2a^2$	M1 A1 M1 A1		Attempt first two terms State $1 - 2a$ Attempt third term Obtain $-2a^2$
	(ii)	$x = \frac{1 \pm \sqrt{1-4a}}{2}$	B1		State $\frac{1 \pm \sqrt{1-4a}}{2}$
		$x = \frac{1 \pm (1 - 2a - 2a^2)}{2}$	M1		Substitute answer to (i) for discriminant
		$x = 1 - a - a^2$	A1		Obtain $x = 1 - a - a^2$ AG
		$x = a + a^2$	A1	**[8]**	Obtain $x = a + a^2$ AG
9	**(i)**	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	M1		Attempt $\sin(2\theta + \theta)$. Needs two terms but may be incorrect
		$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$	M1		Use any $\cos 2\theta$ or $\sin 2\theta$ identity
		$= 2\sin\theta(1 - \sin^2\theta) + (1 - 2\sin^2\theta)\sin\theta$	M1		Use $\cos^2\theta = 1 - \sin^2\theta$ anywhere
		$= 3\sin\theta - 4\sin^3\theta$	A1		Obtain the AG NIS
		$\sin\theta + \sin 3\theta$ $= 4\sin\theta - 4\sin^3\theta$ $= 4\sin\theta(1 - \sin^2\theta)$ $= 4\sin\theta\cos^2\theta$	B1		Factorise $4\sin\theta$ or $\sin\theta$ and correctly obtain the AG
	(ii)	$\frac{\cos^2\theta}{\sin^2\theta} = 4\sin\theta\cos^2\theta$	M1		Identify $\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$
		$\frac{1}{4} = \sin^3\theta$	M1		Attempt to solve for θ
		$\theta = 39.0$ or 141.0	A1		Obtain 39.0 and 141.0
			B1	**[9]**	Recognise $\cos^2\theta = 0$ and state 90°
10	**(a)**	Re $z = -1$ Im $z = -\sqrt{3}$	B1 B1		State -1 State with or without i, $-\sqrt{3}$
	(b)	$uv = (1 + ia)(b - i)$	B1		State uv
		$a + b = 7$	M1		Attempt to equate real and imaginary parts Allow aef
		$ab - 1 = 9$	A1		
		$a^2 - 7a + 10 = 0$	M1		Solve simultaneous eqns to obtain a quadratic
			A1		Obtain $a^2 - 7a + 10$
		$(a - 2)(a - 5)$	M1		Attempt soln of quadratic
		$a = 2$ $b = 5$	A1	**[9]**	Obtain $a = 2$ and $b = 5$

11	**(i)**	a, $a + 8d$ and $a + 13d$	M1	State nth term of an A.P for at least one term. Must be correct formula
		$a + 8d = ar$ $a + 13d = ar^2$	A1	Equate to ar and ar^2
		$r = \frac{a+13d}{a+8d}$ or $\frac{a+8d}{a}$	B1	State an expression for r, d or r^2
		$a(a + 13d) = (a + 8d)^2$	M1	Equate 2 expressions and make at least one step to solve
		$(a = \frac{-64d}{3})$	A1	Obtain an expression for d or a
		$d = \frac{-3a}{64}$	M1	Substitute their value for d or a to find r
		$r = \frac{5}{8}$	A1	Obtain $\frac{5}{8}$
	(ii)	$S = \frac{8a}{3}$	M1	Substitute any r into $\frac{a}{1-r}$
			A1 **[9]**	Obtain S
12		Attempt to separate variables	M1*	
		Attempt to use partial fractions of the form $\frac{A}{x} + \frac{Bx + c}{1 + x^2}$	M1*	
		$A = 1$	A1	
		$B = -1$	A1	
		$C = 0$	A1	
		Obtain $\ln y$	B1	
		Obtain $\ln x - \frac{1}{2}\ln(1 + x^2)$	B1	
		Attempt to combine logs	M1*	Attempt to find an eqn not including logs
		Attempt to deal with $+c$	M1*	Must be valid use of log or its inverse
		Obtain $y = \frac{Cx}{\sqrt{1 + x^2}}$	A1 **[10]**	

13	**(a)**	**(i)**	Boys mean = 14.8 Girls mean = 14.7	B1		Allow better 14.8013 and 14.6996 or rounding to 14.8 and 14.7
			Boys sd = 1.21 Girls sd = 2.29	B1 B1		Allow answers in range [1.21, 1.23] or 2.29
		(ii)	Almost the same mean but ages more spread for girls.	B1		Award only for correct mean and sd. Comment must be made on mean and sd.
	(b)		Permutations of DFATD = $\frac{5!}{2!} = 60$	B1		Sight of 60 or $\frac{5!}{2!}$
			E's can be inserted in 3 of 6 positions ${}^6C_3 = 20$	B1		20 seen or ${}^6C_3$
			No of permutations = 20 × 60 = 1200	B1	**[7]**	Accept 1200 or 20 × 60
14	**(a)**	**(i)**	$A(A-1) + A(A+3) + 50 + 2 = 92$ $A^2 + A - 20 = 0$ or equiv	M1 A1		Attempt xy products Obtain $A^2 + A - 20 = 0$ or equiv 3 termed expression
			$A = 4$	A1		State $A = 4$ only
		(ii)	The points exactly lie on a straight line	B1		The line is $3y - x = 5$
	(b)	**(i)**	$240 - x + x + 100 - x = 250$	M1		Valid method seen
			X or P($A \cap B$) = 90	A1		Award if 90 seen in the diagram
			$\frac{150}{300}$	A1		State $\frac{150}{300}$ aef
		(ii)	$\frac{90}{100}$	M1		Use conditional probability $\frac{theirx}{300} / \frac{100}{300}$
				B1	**[9]**	Obtain 0.9 or equiv

15	**(i)**	$z = \frac{114-120}{6} = -1$	B1	State or imply $z = \pm 1$
		$P(X > 114) = P(Z > -1)$	M1	Attempts $P(Z > \pm 1)$
		$= 0.8413$	A1	Concludes 0.8413
		Expected profit $= 15 \times 0.8413$	M1	Uses Profit = Number × Prob anywhere
		$= 12.6195$p $=$ £12.62	A1	£12.62 or equiv 84.13 (= 100* 0.8413) seen. No units seen A0
	(ii)	$20P(X > x) + 3P(X \leq x) = 19.17$	M1	State probabilities (may be wrong way round) and make one further step to reduce to a single probability
		$20(1 - P(X \leq x) + 3P(X \leq x) = 19.17$		
		$P(X \leq x) = 0.04882$	A1	Obtain 0.04882 or 0.9512
		$(1 - 0.04882) = 0.9512$		
		$\Phi^{-1}(0.9512) = 1.657$	M1	Use $\Phi^{-1}(0.9512)$
		so $z = -1.657$.	A1	Allow ± [1.655,1.660]
		$\frac{x-120}{6} = -1.657$ or equiv negative version	M1 B1	Award for sight of $\frac{x-120}{6} = \pm$ (their) z value
		$x = 110$	A1 **[12]**	Obtain 110 (= 110.058)
16	**(i)**	$P(X = 0) = p^8$ where X is the number of faulty chips	B1	State p^8 or $(1 - q)^8$
		P(accept when $X = 1$) $= 8qp^7 \times p^4$	M1	Attempt product of two binomial terms of correct form
		$= 8qp^{11}$	A1	Correct simplified form seen
		P(accept) $= p^8 + 8p^{11} - 8p^{12}$	M1	Use $q = 1 - p$ to write their expression in terms of p.
		$= p^8 (1 + 8p^3 - 8p^4)$	M1 A1	Sum their $P(X = 0)$ and $P(X = 1)$ Obtain given answer
	(ii)	+8 seen in their $E(X)$	B1	Accept P(selecting 8) = 1
		P(selecting 12) involving qp^7 or equiv	M1	Recognize that P(12) required.
		$= 8qp^7$ or $(8p^7 - 8p^8)$	A1	Obtain correct expression
		$E(X) = 8P(8) + 4P(12)$	M1	Attempt sum of their two np's
		$E(X) = 8 + 32qp^7$	A1	
		$E(X) = 9.07$	A1 **[12]**	Obtain 9.07 (= 9.06787)