MATHEMATICS

Paper 9794/01

Pure Mathematics and Probability

Key Messages

Candidates should remember to quote any formulae that they are using in order to gain method marks if an error is made in the calculations. Candidates should also be encouraged, when asked for a sketch, to give a careful and accurate sketch to show the salient features of a graph without resorting to plotting it on graph paper.

Centres are reminded that, from June 2012, this syllabus will be assessed using three two-hour papers. Papers 1 and 2 will assess the pure mathematics content of the syllabus and Paper 3 will assess the probability and mechanics content.

General comments

The paper appeared to be well received by candidates who were able to demonstrate their knowledge not only in questions which tested technique but also in those which required thought and allowed a variety of approaches. Many excellent scripts were seen with a high level of algebraic competence and the level of presentation by candidates was high. Time pressure was not felt to be an issue and where little attempt was made on the last questions in a section, this was felt to reflect the degree of challenge presented by the questions rather than candidates running out of time. As mentioned above, two general points might be made to assist candidates in improving their exam technique. One is to encourage a careful and accurate sketch to show the salient features of a graph without using graph paper. The second concerns the application of formulae and particularly the quadratic formula. It is important for candidates to state the formula they are using and show which values they are substituting into that formula. Examiners will still award a method mark for use of the formula even with a slip in substitution, but it is not possible to do this if a minimal level of working is seen and no formula quoted as it often suggests a wrong formula has been used.

Comments on specific questions

Section A: Pure Mathematics

Question 1

This question was intended to be accessible to all candidates and so it proved in the main with most candidates scoring full marks. It was, however, disturbing at this stage to find a small minority of candidates who could not form a gradient correctly and found its reciprocal or perpendicular gradient instead.

Answer: $y = -2x + 1$

Question 2

Many candidates again scored full marks in this question. However, a sizeable minority of candidates not only considerably increased the difficulty of their work but also jeopardized a successful outcome for their efforts by using degrees rather than radians. At this level, ease of use and familiarity with radian measure is essential and candidates should be encouraged to use the formulae for sector area and arc length in radian form wherever this is appropriate.

Answer: **(ii)** $S = 9r - r^2$

Question 3

This was another question which afforded many candidates the opportunity to earn full marks. Many considered two cases involving $3 + 2x = -(7 - 4x)$ and $3 + 2x = +(7 - 4x)$. It was noticeable, however, that those who chose to use the squaring method were often not successful because they squared only the side containing the modulus.

Answer: $x = 5$ or $x = \frac{2}{3}$

Question 4

Knowledge of log laws and their correct application was clearly evident in almost all scripts and most candidates obtained the correct quadratic equation without trouble. It was also pleasing to note that when an answer was given in the question paper, full working was shown to justify this.

Answer: **(ii)** $x = 2$ or $x = 1$

Question 5

Many candidates were challenged by this question because they were unsure of the formula associated with volumes of revolution. It may prove useful to candidates to be regularly tested on formulae associated with the syllabus, especially those not appearing in the formulae booklet. A revolution about the x-axis may have suggested to some candidates that a constant of 2π was required but there was also confusion as to whether y or y^2 was to be integrated, and some thought the inverse function was required.

Answer: $\frac{41\pi}{3}$

Question 6

It may be helpful to note that when questions are asked on numerical methods it will usually be the case that candidates are asked to determine whether one or more roots exist within a certain range. Algebraically, this may require that a function of the form $f(x) = 0$ is formed to use a change of sign indicator or similar procedure or candidates may be asked to establish the existence of roots graphically. In this case, candidates who do not have a firm knowledge of the general features of curves and rely on plotting curves not only penalize themselves in the time taken but also mislead themselves. Thus, many graphs either stopped short and showed only one intersection or did not intersect at all. Candidates need to be reminded also that stating that there are two intersections when their evidence contradicts this are liable to be penalized. When sketches were made, they were often very poor with supposed asymptotes to the x-axis dipping below the axis or curving upwards again as if they might be a quadratic. Again it may be helpful to candidates to emphasize the need for careful accurate sketches and firm knowledge of the graphs of the basic functions in the syllabus. It may also be helpful to emphasise to candidates that in applying the Newton-Raphson formula, the first essential is to form a function $f(x) = 0$. Many did not do this and lost all credit since they could not form a correct derivative. As a method was specified in the question, it was not possible to credit candidates who resorted to other methods like $x = g(x)$ or trial and improvement even if their efforts eventually gave the right answer.

Answer: **(ii)** 1.296

Question 7

This question was well answered on the whole, although again it may be worth pointing out to candidates that a complete grasp of the implications of the scalar product formula is essential. Thus, only the direction vectors play any part in the calculation and that it is the cosine and not the sine ratio which is involved. Candidates who did not have this knowledge were given some credit for finding the length of any vector. In the first part of the question, most candidates were able to find the required values of a and b. The more successful method was to consider each line separately along with the given point, finding first a value for the parameter and then deducing the missing value. Those who solved simultaneous equations to find values for the two parameters sometimes struggled to then find the required values as they ended up with an equation that linked a and b, but did not use the given point and so failed to make any further progress.

Answer: **(i)** $a = 3$ $b = 1$ **(ii)** 75.7°

Question 8

This question was well handled with a confident application of the binomial expansion, though some candidates did confuse signs and used $4a$ instead of $-4a$ in the first part. No doubt the given answers to the second part of the question helped candidates see their way through the problem but the general comment made about quoting any formulae used and showing the substitutions made is particularly relevant here since conclusions could not be credited from invalid argument. Most candidates attempted to use the quadratic formula, with a few opting instead for completing the square. The stronger candidates attempted to use their result from part **(i)**, and this was usually done correctly, but a few candidates started with the given roots and tried to work backwards to show the given quadratic; this was never successful.

Answer: **(i)** $1 - 2a - 2a^2$

Question 9

There were many impressive attempts to answer this question with confident use of trigonometric identities used to produce a coherent and rational argument, although a few candidates lost the final mark by not addressing the final part of the request. In the second part of the question, most candidates spotted that the easiest way of dealing with the problem was to use $\cot^2\theta$ in terms of $\sin\theta$ and $\cos\theta$. Only the best candidates, however, then factorised $\cos^2\theta$ and found the 90° root. Most divided by $\cos^2\theta$ and lost sight of the root. A few candidates used the cot identity and ended up with a quintic equation, which a pleasing number managed to then solve successfully, and this method was more likely to also give 90° as the third angle.

Answer: **(ii)** 39.0° 141.0° 90°

Question 10

Candidates spent a great deal of time attempting an algebraic approach to the first part of the question, regrettably without success in most cases. Thorough facility with the Argand diagram would have given them the correct answer with very little work and candidates should be encouraged to make use of their familiarity with this aspect of the syllabus wherever appropriate. That said, the second part of the question relied more on standard algebraic techniques and a great deal of success was seen, though some inaccuracy in achieving the quadratic spoilt a few attempts.

Answer: **(a)** Re $z = -1$ Im $z = -\sqrt{3}$ **(b)** $a = 2$ $b = 5$

Question 11

Examiners were impressed by the quality of the solutions seen to this question. It permitted strong candidates to adopt a number of approaches to the problem. For example, expressions were obtained either for r or d which were then equated and simplified to a quadratic with a very commendable level of algebraic accuracy. Knowledge of formulae associated with nth terms of arithmetic and geometric progressions was clearly evident in this question and even candidates who were nearing the limit of their algebraic capability were able to state two correct equations and then make some attempt at rearranging and eliminating. Weak candidates often could benefit from greater clarity in showing their methods and prevent themselves from becoming confused. For example, a few seemed to get confused as to which variable they were actually trying to eliminate and could not get beyond rearranging one equation, often to several different formats. All candidates could benefit also from careful reading of questions as a surprising number found the value of r first, but then neglected to attempt the first part of the request thus losing two marks as there was never an expression for d in terms of a. The last part of the question was invariably correct.

Answer: **(i)** $d = -\frac{3a}{64}$ **(ii)** $S = \frac{8a}{3}$

Question 12

This was the last question on the pure mathematics section and, as such, candidates should expect to tackle an unstructured situation with minimum guidance. It was therefore to the credit of a large number of candidates that they saw that headway could only be achieved in solving the differential equation by using partial fractions. Those candidates who appreciated the need to use partial fractions nearly always did so correctly. Some never considered a value for C, assuming it to be zero. A surprisingly common error was to give the second numerator as –1 rather than $-x$, despite the initial fractions being set up correctly. A complete solution also required the use of logarithms to combine terms and deal with the arbitrary constant. Only the very ablest candidates were able to write their arbitrary constant as a logarithm and apply a logarithm law to write the solution in the form $y = \mathrm{f}(x)$ as required. Weaker candidates, on the other hand, simply separated the variables and usually gained another mark for $\ln y$, but were able to score no further marks.

Answer: $y = \frac{Ax}{\sqrt{1+x^2}}$

Section B: Probability

Question 13

This question was intended to be an easy start to the probability section, however, for many candidates this was not the case. Most candidates found the means correctly but a large number could not find the standard deviations correctly. In some cases, there was clearly a numerical slip but in most cases a failure to produce the right answer resulted from the use of an incorrect formula. Candidates should be aware that the small number of marks available for calculations of this kind suggests that an answer obtained directly from the calculator is acceptable but equally, it is essential that the formulae for the mean and standard deviation are perfectly known even if only as a check on a calculator answer. Comments were, in general, made on their results but quite a few compared only one of the two statistics found. Candidates might be advised to note that if mean and standard deviation have to be found, comments must be made on both to receive credit and that the results have to be correct if the comment is to be valid. Very few correct solutions to the second part of this question were seen. Most candidates did not appear to know where to start and often attempted informal methods, but these did not consider all of the cases needed.

Answer: **(a)** Boys' mean = 14.8 Boys' standard deviation = 1.21
Girls' mean = 14.7 Girls' standard deviation = 2.29 **(b)** 1200

Question 14

The first part of the question was very well done indeed and the value of *A* found correctly. All candidates made a good effort at the explanation, some providing the equation of the line, though a few could have achieved greater clarity. The second part of the question posed greater difficulty. Those who used a Venn Diagram were in general successful, although it appears that informal methods were used to determine the number in the intersection set. A number of candidates tried to use the formal probability laws of union and intersection but very few were successful, as were those smaller number who attempted to draw a tree diagram. Perhaps candidates might be encouraged to think carefully about an appropriate choice of method before tackling questions involving probability.

Answer: **(a)** $A = 4$ **(b) (i)** 0.5 **(ii)** 0.9

Question 15

Candidates were aided in the first part by the given answer but, although it was sometimes unclear as to the reasoning involved and which region of the normal distribution table was being considered, this part of the question was well done. The second part of the question was found challenging to all but the most able candidates, although almost all candidates were able to make a start on the question. Most seemed to be aiming at a single probability but there was often lack of clarity over profits and probabilities. Many candidates adopted the approach of considering the number of bars out of 100 sold at each profit and obtained probabilities in that way with great success in the main. There were reasonable attempts to find z values and hence x values but confusion over which tail to use often led to sign errors and these prevented a completely successful conclusion.

Answer: **(ii)** 110

Question 16

This was the last question and was intended to stretch the most able candidates. The first part allowed candidates to make reasonable attempts at a solution and it was pleasing to see many correct solutions though in many cases rather obviously aided by the answer being given. The second part was a different matter, however, with many candidates struggling to make progress. Largely this was due to confusion between probabilities of batch acceptance and expectation, a confusion not helped by a lack of clear explanation of what candidates intended. The most able candidates produced full and accurate arguments however, demonstrating at the same time a number of valid routes to the correct conclusion.

Answer: **(ii)** 9.07

MATHEMATICS

Paper 9794/02

Pure Mathematics and Mechanics

Key message

In order to do well in this paper, candidates need to ensure that all working is clearly shown, especially when asked to show a given result, and that they conform to the rubric on the front of the paper and use a value of 10 ms^{-1} for g in the mechanics section.

General comments

This paper proved more accessible than the June 2010 paper and there were some very high scoring scripts seen. It was good to see almost all candidates make a good attempt at the first three questions and parts of the remainder of the Pure mathematics section, although most found the Mechanics section more challenging. The standard of communication was in general very good, though solutions were often missing important details and graph sketching did cause some difficulties for many candidates.

Comments on specific questions

Section A: Pure Mathematics

Question 1

This question was generally answered very well, though many candidates omitted necessary details of their substitution in part **(i)**. Some candidates showed some confusion between roots, factors and solutions; although this did not result in loss of marks, it did often lead candidates to present a good solution to both parts in part **(i)**.

Answers: **(ii)** $(x-4)(x+2)(x+2)$

Question 2

(i) This part was generally answered very well, with most candidates scoring both marks.

(ii) As in **Question 1**, many candidates failed to included enough detail in their working towards a given answer, scoring 3 out of 4 as a result. In particular, the simplification of $\sqrt{125}$ to $5\sqrt{5}$ was often done at the same time as some multiplication, which failed to show clearly how the given answer was obtained.

Answers: **(i)** $61-28\sqrt{3}$

Question 3

Most candidates were aware of the general method of integration by parts and carried it out with some success. Although most managed to choose which term to integrate and which to differentiate, the integration of sin3*x* and/or cos3*x* was often not done correctly. Many candidates lost the final mark for omitting to include an arbitrary constant of integration.

Answer: $-\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x+c$

Question 4

(i) The standard of graph sketching was not good in this part. Most candidates knew the correct shape for a linear transformation of cosine, but not secant. Graphs of sine and arcsine were very common. Candidates with a good knowledge of graph shapes generally performed the transformations very well.

(ii) This was generally answered very well. Candidates seemed familiar with the techniques involved and presented them very well. Errors were mostly seen in the presentation of the solutions, either giving the angles in degrees or giving solutions outside the given range.

Answers: **(ii)** $x = 0.841$ or $x = \pi$

Question 5

(i) Solutions to this part were very good. Many methods were seen, including methods that involved finding many values for both drugs until one concentration dropped below one, rather than the much quicker method of logarithms. All answers that showed the necessary relationship were awarded full marks regardless of their sophistication.

(ii) The graph sketching was again very poor. While many candidates drew graphs of approximately the right shape in the first quadrant, the behaviour at $t = 0$ and as $t \to \infty$ was not clear and many candidates failed to show the relationship between the two drugs that was given in part **(i)**.

(iii) Nearly all candidates realised that it was necessary to add together two values for Coldcure. Most unfortunately then found the concentrations at $t = 10$ and $t = 20$, rather than 10 and 30. Even those candidates who found the correct values frequently rounded off during their calculations resulting in an inaccurate answer, so 1 out of 2 marks was fairly common here.

Answers: **(iii)** 3.10

Question 6

(i) Most candidates seemed to have some idea of what was required here, but not many managed to follow it through correctly, often ending up with an integrand that would not integrate nicely. Some of the stronger candidates managed to find the integral by inspection, which was, of course, acceptable, although some working was required for the evaluation.

(ii) The concept that gradient is zero at a turning point was understood by almost all candidates, as was the process of differentiation by the product rule, even if the details were not correct. Many gave their solutions either as decimal approximations or with $\sqrt{1}$ left unsimplified, which demonstrated a lack of understanding of the phrase "exact coordinates".

Answers: **(i)** 0.5 **(ii)** $(1, e^{-0.5})$ and $(-1, -e^{-0.5})$

Question 7

(i) (a) Most candidates correctly stated that f does not have an inverse. Clear and straightforward reasons such as "f is many-to-one" or "f is not one-to-one" were much less common than descriptive, and insufficient, answers about f being quadratic, curving back on itself or having many different powers of *x*.

(b) This was often not attempted by weaker candidates. While some candidates gave good solutions, many found the range of values for which f is negative, or found the turning point but were not sure what to do with it.

(ii) (a) This rearrangement was often correct and most candidates provided sufficient detail for both marks. Occasional confusion was seen between $\sin(x^2)$ and $\sin^2 x$, but most candidates seemed happy with both the notation and manipulation of the double angle formulae.

(b) The graph of $y = \mathrm{gh}(x)$ was very rarely seen entirely correct. The most common error was to draw the curve with cusps at $x = 0$, π and/or 2π, suggesting a function involving the modulus of a trigonometric function rather than a linear transformation which preserves continuity. That being said, most candidates attempting the graph did recognise that the period would be π and produced nicely symmetrical graphs.

Answers: **(i)(b)** either $\mathrm{f}(x) \geq -1$ or $[-1, \infty)$

Question 8

(i) (a) The first three marks were awarded very often as most candidates knew the method for finding $\frac{dy}{dx}$ from parametric equations. However, very few candidates correctly derived the given result. Those that did used either the double angle formulae, or substitutions based on the $t = \tan\frac{1}{2}\theta$ method, and did so with great skill. Examiners were disappointed that so few candidates considered the values of θ for which the gradient is undefined, given that none of the preceding calculus or manipulation was necessary to obtain these values.

(b) The method for obtaining the harmonic form was generally well known and accurately performed.

(c) Most candidates were aware of the sign change method for demonstrating the existence of a root. However, many failed to substitute the values correctly, or failed to rearrange the equation into the form $\mathrm{f}(\theta) = 0$ and became confused when both values apparently gave $\mathrm{f}(\theta) = 2$.

(ii) The first mark of this part was awarded for correctly finding $\frac{dy}{dx}$ and most candidates managed this as in part **(i)(a)**, though more errors were made in the differentiation here. Most candidates then did not know how to find $\frac{d^2y}{dx^2}$ for parametric equations and many did a lot of work that could not be awarded any marks. Those that did know the appropriate method usually performed it very well; Examiners were very pleased to see that most of these candidates simplified their expressions as they went along rather than maintaining unpleasant forms with fractional coefficients.

Answers: **(i)(a)** at least two of $\theta = \ldots -2\pi, 0, 2\pi \ldots$. **(b)** $A = \sqrt{2}, \alpha = \frac{1}{4}\pi$ **(ii)** $y = \frac{3}{4}$

Question 9

As the last question in the Pure mathematics section many candidates found it very difficult to get far with this question. Part **(i)** was certainly the most accessible, being attempted by all but the weakest candidates, though not always with success. In both part **(i)** and part **(ii)** many candidates failed to give enough detail for the given answer.

Part **(iii)** was rarely attempted, though good answers were seen. There were many ways to tackle this, either differentiating with respect to *t*, *k* or *a*, but the details of the implicit differentiation were often incorrect.

Part **(iv)** unfortunately contained an inconsistency. The context of the problem only allows for values of *k* which are greater than the given value of *a*, so that *a* = 1 implies that *k* must be 2. However, none of the small handful of candidates that tried to find values of *k* noticed this, and almost all of those produced the value of $\frac{da}{dt}$ given in the mark scheme. Full marks were awarded for either $k = -1+\sqrt{3}$ or $-1-\sqrt{3}$ followed by $\frac{da}{dt} = 1+\sqrt{3}$ or $1-\sqrt{3}$ respectively. A minority of candidates gave some manipulation of their expression for $\frac{da}{dt}$, but did not try to find another possible value of *k*, so could not be awarded any marks.

Answers: **(iii)** $\frac{da}{dt} = 1$ **(iv)** $\frac{da}{dt} = 1 \pm \sqrt{3}$

Section B: Mechanics

Question 10

(i) Many correct answers were seen to this part, using a wide variety of methods including scalar product, Pythagoras' theorem, trigonometry and congruent triangles. Much ingenuity was displayed here and it was very pleasing to see the creative attempts of many candidates.

(ii) Candidates that tried to resolve in the directions given were usually successful, but most of those that made a good attempt resolved vertically and horizontally which led them to simultaneous equations and a lot more work than the three marks available would suggest. Many candidates made erroneous assumptions about the tensions in the strings being equal and gained no marks at all, or simply did not attempt this at all.

(iii) Very few correct answers were seen to this part. Even the candidates that correctly found the magnitudes of the *x* and *y* components of the tension often left out the negative sign on the former.

Question 11

This was the most consistently well done question in ***Section B***. Most candidates seemed familiar with the general equations for projectile motion and could apply them in this context.

(i) Most candidates made some attempt at this part; a few missed out on a mark for lack of detail, but the solutions were generally very clear. The presentation of this work was often a little bare, with Examiners left to decide for themselves to what the various expressions referred.

(ii) This question often led to far more work than was necessary and many candidates did not appear to be aware of the symmetry that made this question very simple to answer. Many also failed to realise that a numerical answer was required, instead leaving their answer in terms of *V* for which the mark was not awarded.

(iii) Many candidates produced good work here, even if they could not quite find a value of *h* or keep track of the details well enough to obtain the correct value. Those candidates that did not find 100 m in part **(ii)** could not obey the instruction "Hence find…", but were awarded full marks for creating and solving simultaneous equations in terms of *h* and *V*.

Answers: **(ii)** 100 m **(iii)** $V = 10\sqrt{10}$ ms^{-1}, $h = 21$ m

Question 12

(i) Very few correct diagrams were seen here. The most common error was to include on the diagram the working for part **(ii)**, i.e. the components of weight parallel and perpendicular to the slope. This was condoned as long as it was clear that this is what had been done. However, it was very common to see diagrams with one of these components included, without a normal reaction force or with friction.

(ii) This was very well answered by the majority of candidates.

(iii) Most candidates were aware of Newton's experimental law and the conservation of linear momentum and were able to use them to find the velocity of P, but many made sign errors when forming their equations which led to incorrect answers. The most common error that led to no solution was to find the speed of separation as 4 ms^{-1}, but not to express this in terms of the final velocities of the particles.

Answers: **(ii)** $v = 5t$ **(iii)** $v = 7 - 5T$

Question 13

This question was undoubtedly the hardest of the paper and few candidates made more than a passing attempt at any part of it. However, some completely correct solutions were seen and many of the stronger candidates produced good solutions to at least parts **(i)** and **(ii)**.

(i) Most solutions to this part lacked necessary detail, some appearing to be little more than reverse engineered solutions from the given answer. Candidates who were awarded all three marks had a clear explanation for $T = mg$ from considering the equilibrium of B and a diagram showing the various components required for resolving forces on A perpendicular to the slope.

(ii) Almost all candidates that gave a solution for this simply substituted $\alpha = 45$ into the expression in part **(i)**. While this certainly shows the boundary condition to be true, it is not sufficient to show the given inequality and could only be awarded 1 out of the 3 marks. Very few good solutions were seen here; full marks required a careful consideration of the values of $\cos\alpha$ and $\sin\alpha$ for $\alpha \le 45$.

(iii) Very few solutions were seen here; although some candidates did consider the signs of either the friction or the normal reaction force only a few did so convincingly. The most common insufficient approach was to show that $\mu = \dfrac{2\sin\alpha - \cos\alpha}{2\cos\alpha - \sin\alpha}$ and then conclude that, since μ is positive, the denominator must be positive, omitting the case where both denominator and numerator are negative.

(iv) The few candidates that attempted this part generally produced very nice solutions. Occasional abuses were seen when cancelling, but for the most part this was done very well.

Answers: **(iv)** $\mu = \dfrac{2\tan\alpha - 1}{2 - \tan\alpha}$, max value of μ is 1.