

Mathematics and Further Mathematics Pre-U June 2010

The following question papers for Mathematics and Further Mathematics are the first papers to be taken by Pre-U students at the end of the two-year course. This also means that they are the first live question papers to be set for Pre-U candidates and in common with all new Pre-U examination questions were subjected to the same rigorous question paper setting procedure, involving subject experts and experienced teachers as well as assessment professionals.

Setting a new standard is always a challenging activity for those involved and CIE was aware that Mathematics in particular might be a difficult standard to gauge owing to the wide spread of ability in this subject. Extra work was therefore commissioned to assess the level of difficulty of the examination papers and the outcome reassured CIE that the papers were appropriate for the cohort.

However, it became evident as soon as Paper 1 had been sat that the level of difficulty might have been too high, and this was confirmed after Paper 2 had been taken. The grade boundaries for these papers were set at a level that reflected A level grades A and E at D3 and P3 respectively. Additional statistical analysis led to further work to ensure fairness to all candidates.

CIE is releasing these papers according to its usual practice but would like to point out that future examination papers will include more accessible marks for those in the middle of the distribution.



MATHEMATICS

9794/02

Paper 2 Pure Mathematics and Mechanics

May/June 2010

3 hours

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF20)



READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use 10 m s^{-2} .

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

Section A: Pure Mathematics (80 marks)

- 1 Find the exact value of

$$\int_1^4 (10x^{\frac{3}{2}} - 3x^{\frac{1}{2}}) dx. \quad [3]$$

- 2 Solve the inequality

$$\log_3(2x^2 - x) - \log_3(2x^2 - 3x + 1) > 1. \quad [5]$$

- 3 An arithmetic progression has 13th term equal to 60 and 31st term equal to 141.

- (i) Find the first term and common difference of the progression. [3]

A second arithmetic progression has first term 1.5 and common difference 3.

- (ii) (a) Write down the first four terms of each progression. [1]

- (b) Prove that the two progressions have an infinite number of terms in common. [2]

- 4 (i) Show that

$$\cos^4 x - \sin^4 x \equiv 2 \cos^2 x - 1. \quad [2]$$

- (ii) Hence find the solutions of

$$\cos^4 x - \sin^4 x = \cos x,$$

where $0^\circ \leq x \leq 360^\circ$. [4]

- 5 It is given that

$$y = \frac{1}{x+i} + \frac{1}{x-i},$$

where x and y are real and positive, and $i^2 = -1$.

- (i) Show that

$$x = \frac{1 \pm \sqrt{1-y^2}}{y} \quad \text{and} \quad y \leq 1. \quad [4]$$

- (ii) Deduce that

$$xy < 2. \quad [2]$$

- (iii) Indicate the region in the x - y plane defined by

$$y \leq 1 \quad \text{and} \quad xy < 2. \quad [3]$$

6 (i) Express $\frac{x-1}{x^2+2x+1}$ in the form $\frac{A}{x+1} + \frac{B}{(x+1)^2}$, where A and B are integers. [2]

(ii) Find the quotient and remainder when $2y^2 + 1$ is divided by $y + 1$. [2]

(iii) A curve in the x - y plane passes through the point $(0, 2)$ and satisfies the differential equation

$$(2y^2 + 1)(x^2 + 2x + 1)\frac{dy}{dx} = (x - 1)(y + 1).$$

By solving the differential equation find the equation of the curve in implicit form. [6]

7 Let $y = (x - 1)\left(\frac{2}{x^2} + t\right)$ define y as a function of x ($x > 0$), for each value of the real parameter t .

(i) When $t = 0$,

(a) determine the set of values of x for which y is positive and an increasing function, [3]

(b) locate the stationary point of y , and determine its nature. [2]

(ii) It is given that $t = 2$ and $y = -2$.

(a) Show that x satisfies $f(x) = 0$, where $f(x) = x^3 + x - 1$. [1]

(b) Prove that f has no stationary points. [2]

(c) Use the Newton-Raphson method, with $x_0 = 1$, to find x correct to 4 significant figures. [4]

8 The point F has coordinates $(0, a)$ and the straight line D has equation $y = b$, where a and b are constants with $a > b$. The curve C consists of points equidistant from F and D .

(i) Show that the cartesian equation of C can be expressed in the form

$$y = \frac{1}{2(a-b)}x^2 + \frac{1}{2}(a+b). \quad [3]$$

(ii) State the y -coordinate of the lowest point of the curve and prove that F and D are on opposite sides of C . [2]

(iii) (a) The point P on the curve has x -coordinate $\sqrt{a^2 - b^2}$, where $|a| > |b|$. Show that the tangent at P passes through the origin. [4]

(b) The tangent at P intersects the line D at the point Q . In the case that $a = 12$ and $b = -8$, find the coordinates of P and Q . Show that the length of PQ can be expressed as $p\sqrt{q}$, where $p = 2q$. [5]

- 9 (i) Show that

$$\int_1^a x^n \ln x \, dx = \frac{a^{n+1}}{(n+1)^2} ((n+1) \ln a - 1) + \frac{1}{(n+1)^2},$$

where $n \neq -1$ and $a > 1$.

[6]

- (ii) (a) Determine the x -coordinate of the point of intersection of the curves $y = x^2 \ln x$ and $y = x \ln 2^x$, where $x > 0$. [2]
- (b) Find the exact value of the area of the region enclosed between these two curves, the line $x = 1$ and their point of intersection. Express your answer in the form $b + c \ln 2$, where b and c are rational. [4]
- (iii) The curve $y = (x^3 \ln x)^{0.5}$, for $1 < x < e$, is rotated through 2π radians about the x -axis. Determine the value of the resulting volume of revolution, giving your answer correct to 4 significant figures. [3]

Section B: Mechanics (40 marks)

- 10 A particle is projected from a point P on an inclined plane, up the line of greatest slope through P , with initial speed V . The angle of the plane to the horizontal is θ .

- (i) If the plane is smooth, and the particle travels for a time $\frac{2V}{g} \cos \theta$ before coming instantaneously to rest, show that $\theta = \frac{1}{4}\pi$. [4]
- (ii) If the same plane is given a roughened surface, with a coefficient of friction 0.5, find the distance travelled before the particle comes instantaneously to rest. [5]

- 11 Two forces \mathbf{F}_1 and \mathbf{F}_2 are given by

$$\mathbf{F}_1 = 13\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}, \quad \mathbf{F}_2 = -2\mathbf{i} + 6\mathbf{j} + \mathbf{k},$$

in which the units of the components are newtons. A third force, \mathbf{F}_3 , of magnitude 6 N acts parallel to the vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

- (i) Find the two possible resultants of \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 , and show that they have the same magnitude. [5]

A particle, P , of mass 2 kg is initially at rest at the origin. The only forces acting on P are \mathbf{F}_1 and \mathbf{F}_2 .

- (ii) Find the magnitude of the acceleration of P . [3]
- (iii) Find the time taken for P to travel 60 m. [2]

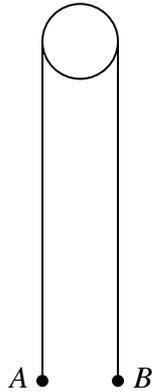
- 12** A particle moves along a straight line under the action of a variable force. The acceleration is given by

$$a = \begin{cases} 30 - 6t, & \text{for } 0 \leq t \leq 10 \\ 6t - 90, & \text{for } 10 \leq t \leq 20 \end{cases}$$

where time t is measured in seconds and a in m s^{-2} . The particle is at rest at the origin at $t = 0$.

- (i) (a) Find the velocity v of the particle in terms of t . Verify that $v = 0$ when $t = 10$ and $t = 20$. [7]
 (b) Sketch the velocity-time graph for the motion. [2]
- (ii) Calculate the total distance travelled by the particle. [4]

- 13** A light inextensible string passes over a fixed smooth light pulley. Particles A and B , of masses 2 kg and 3 kg respectively, are attached to the ends so that the portions of the string below the axis of the pulley are vertical (see diagram). The centre of the horizontal axis of the pulley is 4 m above ground level.



The particles are released from rest 1 m above ground level with the string taut.

- (i) Determine the acceleration of both particles prior to the impact of B with the ground. [3]
 (ii) Determine the greatest height attained by A above ground level. [3]
 (iii) If B rebounds after impact to a first maximum height of 0.05 m, determine the coefficient of restitution between B and the ground. [2]

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