Mathematics and Further Mathematics Pre-U June 2010

The following question papers for Mathematics and Further Mathematics are the first papers to be taken by Pre-U students at the end of the two-year course. This also means that they are the first live question papers to be set for Pre-U candidates and in common with all new Pre-U examination questions were subjected to the same rigorous question paper setting procedure, involving subject experts and experienced teachers as well as assessment professionals.

Setting a new standard is always a challenging activity for those involved and CIE was aware that Mathematics in particular might be a difficult standard to gauge owing to the wide spread of ability in this subject. Extra work was therefore commissioned to assess the level of difficulty of the examination papers and the outcome reassured CIE that the papers were appropriate for the cohort.

However, it became evident as soon as Paper 1 had been sat that the level of difficulty might have been too high, and this was confirmed after Paper 2 had been taken. The grade boundaries for these papers were set at a level that reflected A level grades A and E at D3 and P3 respectively. Additional statistical analysis led to further work to ensure fairness to all candidates.

CIE is releasing these papers according to its usual practice but would like to point out that future examination papers will include more accessible marks for those in the middle of the distribution.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics and Probability **May/June 2010**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

Section A: Pure Mathematics (79 marks)

1   Solve the equation $2^x = 4^{2x+1}$. [3]

2   The equation $x^3 - 5x + 3 = 0$ has a root between $x = 0$ and $x = 1$.

   (i) The equation can be rearranged into the form $x = \mathrm{g}(x)$ where $\mathrm{g}(x) = px^3 + q$. State the values of p and q. [1]

   (ii) By considering $|\mathrm{g}'(x)|$, show that the iterative form $x_{n+1} = \mathrm{g}(x_n)$ with a suitable starting value converges to the root between $x = 0$ and $x = 1$. [You are not required to find this root.] [2]

3   Let $\mathrm{f}(x) = x^2(x - 2)$ and $\mathrm{g}(x) = 2x - 1$ for all real x.

   (i) Sketch the graph of $y = \mathrm{f}(x)$ and explain briefly why the function f has no inverse. [2]

   (ii) Write down $\mathrm{g}^{-1}(x)$. [1]

   (iii) On the same diagram, sketch the graphs of $y = \mathrm{f}(x - 1) - 3$ and $y = \mathrm{g}^{-1}(x)$ and state the number of real roots of the equation $\mathrm{f}(x - 1) - 3 = \mathrm{g}^{-1}(x)$. [3]

4   Using the substitution $u = 1 + \sqrt{x}$, or otherwise, find $\displaystyle\int \frac{1}{1 + \sqrt{x}}\,\mathrm{d}x$ giving your answer in terms of x.

[5]

5   The parametric equations of a curve are $x = \dfrac{1}{1 + t^2}$ and $y = \dfrac{t}{1 + t^2}$, $t \in \mathbb{R}$.

   (i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [5]

   (ii) Hence find the coordinates of the stationary points of the curve. [2]

6   A geometric progression with common ratio r consists of positive terms. The sum of the first four terms is five times the sum of the first two terms.

   (i) Find an equation in r and deduce that $r = 2$. [3]

   (ii) Given that the fifth term is 192, find the value of the first term. [1]

   (iii) Find the smallest value of n such that the sum of the first n terms of the progression exceeds 10^{64}. [3]

7 Let $f(x) = \dfrac{1 + x^2}{\sqrt{4 - 3x}}$.

    (i) Obtain in ascending powers of x the first three terms in the expansion of $\dfrac{1}{\sqrt{4 - 3x}}$ and state the values of x for which this expansion is valid. [5]

    (ii) Hence obtain an approximation to $f(x)$ in the form $a + bx + cx^2$ where a, b and c are constants. [2]

    (iii) Use your approximation to estimate $\displaystyle\int_0^{0.1} f(x)\, dx$. [2]

8 The points A and B have position vectors $\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ respectively, relative to the origin O. The point C is on the line OA extended so that $\overrightarrow{AC} = 2\overrightarrow{OA}$ and the point D is on the line OB extended so that $\overrightarrow{BD} = 3\overrightarrow{OB}$. The point X is such that $OCXD$ is a parallelogram.

    (i) Show that a vector equation of the line AX is $\mathbf{r} = \mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(5\mathbf{i} + \mathbf{j} + 7\mathbf{k})$ and find an equation of the line CD in a similar form. [5]

    (ii) Prove that the lines AX and CD intersect and find the position vector of their point of intersection. [4]

9 A curve has equation $x^2 - xy + y^2 = 1$.

    (i) Find $\dfrac{dy}{dx}$ in terms of x and y. [4]

    (ii) Find the coordinates of the points on the curve in the second and fourth quadrants where the tangent is parallel to $y = x$. [5]

10 (a) Solve the equation

$$(2 + \mathrm{i})z = (4 + \mathrm{i}n).$$

    Give your answer in the form $a + \mathrm{i}b$, expressing a and b in terms of the real constant n. [4]

    (b) The roots of the equation $z^2 + 8z + 25 = 0$ are denoted by z_1 and z_2.

        (i) Find z_1 and z_2 and show these roots on an Argand diagram. [3]

        (ii) Find the modulus and argument in radians of each of $(z_1 + 1)$ and $(z_2 + 1)$. [3]

11 **(i)** Write down an identity for $\tan 2\theta$ in terms of $\tan\theta$ and use this result to show that

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$ [4]

(ii) Given that $0 < \theta < \frac{1}{2}\pi$ and $\theta = \sin^{-1}\left(\frac{1}{\sqrt{10}}\right)$, show that $\tan 3\theta = \frac{13}{9}$. [3]

(iii) Show that the solutions of the equation

$$\tan(3\sin^{-1}x) = \frac{13}{9}$$

for $0 < x < 2\pi$ are

$$x = \frac{\sqrt{10}}{10} \qquad \text{and} \qquad x = \frac{\sqrt{10}\left(1 + 3\sqrt{3}\right)}{20}.$$ [4]

Section B: Probability (41 marks)

12 **(a)** Events A and B are such that $P(A' \cap B') = \frac{1}{6}$.

(i) Find $P(A \cup B)$. [2]

(ii) Given that $P(A \mid B) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, find $P(A \cap B)$ and $P(A)$. [3]

(b) In playing the UK Lottery, a set of 6 different integers is chosen irrespective of order from the integers 1 to 49 inclusive. How many different sets of 6 integers can be chosen? [2]

13 A survey was conducted into the annual salary offered for 19 different jobs in 2008. The results were as follows, in thousands of pounds.

15	16	18	19	21	36	36	38	41	41
43	47	51	55	56	60	62	64	110	

(a) **(i)** Find the median and interquartile range of this set of data. [3]

(ii) Show that £110 000 is an outlier and discuss briefly how this outlier might be treated. [2]

It was decided to undertake a further study to see if self-esteem was correlated with level of annual salary. A random sample of 11 employees was taken and self-esteem was rated on a scale of 1 to 10 with the highest self-esteem being 10.

The results were as follows.

Salary in £10 000's	1	2	3	4	5	6	7	8	9	10	11
Self-esteem	4	3	5	1	7	7	8	5	10	7	9

(b) **(i)** Calculate the product-moment correlation coefficient and comment on the result. [3]

(ii) "The best way to increase self-esteem is to pay people more money." Comment on this claim. [2]

14 (a) In a game show contestants are asked up to five questions in succession to qualify for the next round. An incorrect answer eliminates a contestant from the game show.

Let X denote the number of questions correctly answered by a contestant. The probability distribution of X is given below.

x	0	1	2	3	4	5
P($X = x$)	0.30	0.25	0.20	0.16	0.06	0.03

    (i) Find the expected number of correctly answered questions and the variance of the distribution. [3]

   (ii) Find the probability that a randomly selected contestant will correctly answer 3 or more questions. [1]

  (iii) Each show had two contestants. Find the probability that both the contestants will correctly answer at least one question. [2]

(b) In a promotion, a newspaper included a token in every copy of the newspaper. A proportion, 0.002, are winning tokens and occur randomly. A reader keeps buying copies of the newspaper until he buys one with a winning token and then stops.

Let Y denote the number of copies bought.

    (i) Explain briefly why this situation may be modelled by a geometric distribution and write down a formula for P($Y = y$). [2]

   (ii) Find the probability that the reader gets a winning token with the twentieth copy bought. [2]

  (iii) Find the probability that the reader will not have to buy more than three copies in order to get a winning token. [2]

15 A manufacturer produces components designed with length L mm such that $12 < L < 15$. The Quality Control department finds that 15% of the components sampled are longer than 15 mm while 8% are shorter than 12 mm. Assume that L is normally distributed with mean μ and standard deviation σ.

    (i) Calculate μ and σ. [6]

   (ii) The shortest 5% of components are rejected. Find the minimum length which a component may have before it is rejected. [3]

  (iii) It was found in a random sample that 10% of components were longer than 16 mm. Determine whether this finding is consistent with the assumption that L is normally distributed with the μ and σ found in part **(i)**. [3]

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