UNIT A9

Recommended Prior Knowledge Units A1 to A4.

Context The work in this unit is applied to transformations in unit S10 and so this topic has been left to unit 9 in the course. Matrices could be introduced earlier in the course if desired.

Outline Rectangular, not square, matrices are used initially to help students appreciate how to add and subtract matrices and to find the product of two matrices of appropriate order. When students have gained competence in these skills, the focus moves on to considering 2 × 2 matrices and their algebra, leading to finding the determinant of a matrix and the inverse of a non-singular matrix.

	Learning Outcomes	Suggested Teaching Activities	Resources
37	Display information in the form of a matrix of any order; solve problems involving the calculation of the sum and product (where appropriate) of two matrices and interpret the results; calculate the product of a scalar quantity and a matrix; use the algebra of 2 × 2 matrices including the zero and identity 2 × 2 matrices; calculate the determinant and inverse of a non-singular matrix. ($\mathbf{A}^{-1}$ denotes the inverse of **A**.)	Start with an example such as a 3 × 4 matrix of numbers of 4 items in a shopping order on three separate weeks, multiplied by a 4 × 1 matrix of prices. Then extend the 4 × 1 matrix to a 4 × 2 matrix to include the new prices after an increase. Use this example to discuss the layout and principles of finding the product of two matrices. You could use combining with the shopping order of a neighbour to demonstrate addition of matrices, and similarly doubling an order to demonstrate multiplication of a matrix by a scalar quantity. Apply these principles to rectangular matrices of different shapes, including using them to solve problems, before focusing more on 2 × 2 matrices. Discuss their algebra compared with the four operations using numbers, for instance that order of multiplication matters with **AB** being different from **BA** in general. Give the students some products which have the	http://www.sosmath.com/matrix/matrix0/matrix0.html has an introduction to matrix algebra

		identity matrix as answer, then lead on to calculating the determinant and inverse of a non-singular matrix. Show students the use of inverse matrices in solving simultaneous equations [they will also be used in S10 to find the coordinates of points after an inverse transformation]. .	

UNIT N9

Recommended Prior Knowledge Units N1, A1 to A4

Context This unit is mostly a 'stand-alone' unit and may be studied earlier in the O level course if wished, using examples that the students have studied so far.

Outline Knowledge and understanding of set language and Venn diagrams is gradually built up, starting with simple examples of first one then two sets. This leads on to considering combinations of the sets and using Venn diagrams and sets to solve problems.

	Learning Outcomes	Suggested Teaching Activities	Resources
2	Use set language and set notation, and Venn diagrams, to describe sets and represent relationships between sets as follows: Definition of sets, e.g. $A = \{x : x \text{ is a natural number}\}$ $B = \{(x, y) : y = mx + c\}$ $C = \{x : a \leq x \leq b\}$ $D = \{a, b, c... \}$ Notation: Union of A and B $A \cup B$ Intersection of A and B $A \cap B$ Number of elements in set A n(A) "...is an element of..." $\in$ "...is not an element of..." $\notin$ Complement of set A A' The empty set $\varnothing$ The universal set E A is a subset of B $A \subseteq B$ A is a proper subset of B $A \subset B$ A is not a subset of B $A \not\subseteq B$ A is not a proper subset of B $A \not\subset B$	Start with a list of items such as some colours, which you write in set form, as in D in the learning outcomes. Then write a set for colours of the rainbow. Use the brackets notation and also ask questions about elements of sets to introduce this language and the symbols $\in$, $\notin$ and n(A). Draw a Venn diagram to show the two sets, together with the universal set of colours. Use an example such as $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$ to draw a Venn diagram with mutually exclusive sets. Use other examples to give the range of different types of Venn diagrams and of the notation of describing the elements of a set. Give the students practice in the language and diagrams used. Then use the diagrams drawn already to discuss the meaning of different areas of the Venn diagram and introduce the rest of the required set language and notation. Give students practice in this, including using Venn diagrams to solve problems to find the number of elements in a set.	chapter 1 at http://assets.cambridge.org/0521539021/sample/0521539021WS.pdf

UNIT S9

Recommended Prior Knowledge Units S1 to S8.

Context Trigonometry is extended from right-angled triangles to include acute and obtuse angled triangles. Column vectors have been met previously in unit S5 in work on translations; the concept of vectors is now extended to cover the principal basics of vector geometry.

Outline Initially, right-angled trigonometry is used to find an unknown length in an acute angled triangle by splitting it into two right-angled triangles. The sine and cosine rules are derived and used to find unknown lengths and angles, with the sine and cosine function definitions being extended to include obtuse angles. The area formula ½ *ab* sin *C* for the area of a triangle is obtained and used.

Attention then moves to vectors, starting with column vectors used in translations. From these, addition of vectors and the magnitude of vectors is discussed before moving on to more general representations of vectors as line segments, and position vectors. Sums and differences of coplanar vectors are used in geometrical problems.

	Learning Outcomes	Suggested Teaching Activities	Resources
34	Extend sine and cosine functions to angles between 90° and 180°; solve problems using the sine and cosine rules for any triangle and the formula ½ *ab* sin *C* for the area of a triangle.	Give the students a right-angled triangle where they are required to find a distance using the sine function, and solve this problem. Then give them an acute-angled triangle where they need to find a length for which the sine rule would be appropriate. Ask them how they can solve this [if they do not recommend dividing it into appropriate right-angled triangles then give them the hint by drawing in the appropriate perpendicular height]. When this has been solved together, give the same situation using letters rather than numbers and use the same method to obtain the sine rule. After practice in using the sine rule to obtain sides and angles in acute- angled triangles, introduce an obtuse angled triangle requiring use of the obtuse angle. Show that this can be solved using the supplementary angle, and then extend the sine and cosine functions to include obtuse	http://www.catcode.com/trig/trig08.html has interactive pages on extending sine and cosine functions such as 'sine and cosine Do "the wave"' http://www.ex.ac.uk/cimt/mepres/allgcse/bka4.pdf sections 4.8 to 4.9 is about using trigonometry in non-right angled triangles.

		angles. You could demonstrate that sin θ = sin (180 – θ) and cos θ = –cos (180 – θ) when θ is obtuse using calculator values or extend the definitions to generate the sine and cosine waves at least as far as 180°. The cosine rule and the formula for the area of a triangle may be developed similarly to the sine rule activity above. Give practice in using these, including situations where students have to decide which (or both) of the sine and cosine rules they need to use.	
39	Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a**; add vectors and multiply a vector by a scalar. Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$. (Vectors will be printed as $\overrightarrow{AB}$ or **a** and their magnitudes indicated by modulus signs, e.g. | $\overrightarrow{AB}$ | or |**a**|. In all their answers to questions candidates are expected to indicate **a** in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{AB}$ or a̲.) Represent vectors by directed line segments; use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors; use position vectors.	Revise the work on translations by asking students to find the image of a point after a translation and then after a further translation, asking them to give the vector for the combined transformation. Generalise to representing vectors by directed line segments, to adding and subtracting column vectors and to multiplying a column vector by a scalar. Use Pythagoras' theorem to find the magnitude of a column vector. Introduce the notation $\overrightarrow{AB}$ and **a** for describing vectors and modulus signs to indicate magnitude. Discuss the relationship between vectors **a** and *k***a**. Use position vectors and show the sum and difference of two vectors. Use vectors to solve problems and demonstrate properties of plane figures e.g. that the diagonals of a parallelogram bisect each other, or that the medians of a triangle intersect, dividing the medians in the ratio 2:1.	http://standards.nctm.org/document/eexamples/chap7/7.1/part2.htm has interactive work about vector sums. Go to http://www.standards.nctm.org/ and click the search button to find resources on other topics from this site. http://www.ex.ac.uk/cimt/mepres/allgcse/bkc19.pdf is a chapter about vectors.