

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2013 series

4037 ADDITIONAL MATHEMATICS

4037/13

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Accuracy mark for a correct result or statement independent of method marks.

When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol $\surd$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.

B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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1 (i) ${}^6C_2 (2^4) (px)^2$ or $\left(\frac{6}{2}\right) 2^4 (px)^2$ $240p^2 = 60$ $p = \frac{1}{2}$ (ii) coefficients of the terms needed $(-1) {}^6C_1 (2)^5 p + (3 \times 60)$ $= 84$	B1 M1 A1 [3] M1 B1 A1 [3]	Seen or implied, unsimplified M1 for their coefficient of $x^2 = 60$ and attempt to solve M1 for realising that 2 terms are involved B1 for $(-1) {}^6C_1 (2)^5 p$ or $-192p$, using their p.
2 $\lg \frac{y^2}{5y+60} = \lg 10$ Or $\lg y^2 = \lg 10 (5y + 60)$ $y^2 - 50y - 600 = 0$ leading to $y = -10, 60$ y must be positive so $y = 60$	B1 B1 M1 DM1 A1 [5]	B1 for $2 \lg y = \lg y^2$ B1 for $1 = \lg 10$ or equivalent, allow when seen M1 for use of $\log A - \log B = \log A/B$ or $\log A + \log B = \log AB$ DM1 for forming a 3 term quadratic equation and an attempt to solve A1 for $y = 60$ only

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3 $\tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$ $= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$ $= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}$ $= \frac{\sin^4 \theta}{\cos^2 \theta}$ $= \sin^4 \theta \sec^2 \theta$ Alt solution 1 Using $\tan^2 \theta = \sin^2 \theta \sec^2 \theta$ $\begin{aligned} \text{LHS} &= \sin^2 \theta \sec^2 \theta - \sin^2 \theta \\ &= \sin^2 \theta (\sec^2 \theta - 1) \\ &= \sin^2 \theta \tan^2 \theta \\ &= \sin^4 \theta \sec^2 \theta \end{aligned}$ Alt solution 2 $\begin{aligned} \text{RHS} &= \sin^4 \theta \sec^2 \theta \\ &= \frac{\sin^2 \theta \sin^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} \\ &= \tan^2 \theta - \sin^2 \theta \end{aligned}$	M1 M1 M1 A1 [4] M1 M1 M1 A1 M1 M1 M1 A1	Marks are awarded only if they can lead to a complete proof for the methods other than those shown below M1 for dealing with tan and a fraction M1 for factorising M1 for use of identity $\cos^2 \theta + \sin^2 \theta = 1$ A1 for all correct M1 use of $\tan^2 x = \sin^2 x \sec^2 x$ M1 for factorising M1 for use of identity A1 for all correct M1 for splitting $\sin^4 \theta$ and use of identity M1 for multiplication M1 for writing as two terms and cancelling A1 for all correct
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4 (i) $\frac{dy}{dx} = \frac{(x+3)^2 2e^{2x} - e^{2x} 2(x+3)}{(x+3)^4}$ $= \frac{2e^{2x}(x+2)}{(x+3)^3}, A = 2$ Alt solution $\frac{dy}{dx} = e^{2x} (-2(x+3)^{-3}) + 2e^{2x}(x+3)^{-2}$ $= \frac{2e^{2x}(x+2)}{(x+3)^3}, A = 2$ (ii) $x = -2, y = e^{-4}$	M1 A2, 1, 0 A1 [4] M1 A2,1,0 A1 B1, B1 [2]	M1 for attempt at quotient rule –1 for each error Must be convinced of correct simplification e.g. sight of $(x+3-1)$ or $(x+2)(x+3)$ M1 for attempt at product rule –1 for each error Must be convinced of correct simplification e.g. sight of $(x+3-1)$ or $(x+2)(x+3)$ Accept $1/e^4$
5 (i) $f^2(x) = f(2x^3)$ $= 2(2x^3)^3 \text{ or } 2\left(2\left(\frac{1}{2}\right)^3\right)^3$ $= 2^{-5}$ Alt method $f\left(\frac{1}{2}\right) = \frac{1}{4} \quad f\left(\frac{1}{4}\right) = 2^{-5}$ (ii) $f'(x) = g'(x)$ $6x^2 = 4 - 10x$ Leading to $(3x-1)(x+2) = 0$ $x = \frac{1}{3}, -2$	M1 A1 [2] M1 A1 B1 B1 M1 A1 [4]	M1 for $= 2(2x^3)^3$ or $2\left(2\left(\frac{1}{2}\right)^3\right)^3$ For 2^{-5} only M1 for f of their $f\left(\frac{1}{2}\right)$ For 2^{-5} only B1 for $6x^2$ B1 for $4 - 10x$ M1 for solution of quadratic equation obtained from differentiation of both A1 for both

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7 (i) $2t^2 - 2(t^2 - t + 1)$ Leading to, $t = \frac{3}{2}$ (ii) $A = \begin{pmatrix} 6 & 2 \\ 7 & 3 \end{pmatrix}, A^{-1} = \frac{1}{4} \begin{pmatrix} 3 & -2 \\ -7 & 6 \end{pmatrix}$ $\begin{pmatrix} 6 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 3 & -2 \\ -7 & 6 \end{pmatrix} \begin{pmatrix} 10 \\ 11 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, leading to $x = 2, y = -1$	B1 M1 A1 [3] B1, B1 B1 M1 A1 [5]	Correct determinant seen unsimplified M1 for simplification and solution A1 for solution of $\det A=1$ only, not $1/\det A=1$ B1 for $\frac{1}{4}$, B1 for matrix B1 for dealing correctly with the factor of 2 M1 for pre-multiplying their $\begin{pmatrix} 10 \\ 11 \end{pmatrix}$ by their A^{-1} to obtain a column matrix Allow $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ for A1
8 (i) $\frac{1}{2}(4^2)\sin\theta = 7.5$ $\sin\theta = \frac{15}{16}, \theta = 1.215 \dots$ (ii) $\sin\frac{\theta}{2} = \frac{\frac{1}{2}CD}{4}, (CD = 4.567)$ Arc length = $6(1.215)$ Perimeter = $2 + 2 + 6(1.215) + \text{their } CD$ = awrt 15.9 (iii) Area = $\frac{1}{2}6^2(1.215) - 7.5$ = 14.4 (awrt)	M1 A1 [2] M1 B1 M1 A1 [4] B1 M1 A1 [3]	M1 for attempt to find the area of the triangle and equate to 7.5 A1 for solution to obtain the given answer Solution must include 1.2153.... or 1.2154 M1 for attempt to find CD B1 for arc length M1 for sum of 4 appropriate lengths B1 for sector area M1 for subtraction of the 2 areas

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9 (a) (i) $6(1 - \cos^2 x) = 5 + \cos x$ $6 \cos^2 x + \cos x - 1 = 0$ $(3 \cos x - 1)(2 \cos x + 1) = 0$ $x = 70.5^\circ \quad x = 120^\circ$ (ii) $\cos x = \sin y$ $\sin y = \frac{1}{3}$ only so $y = 19.5^\circ, 160.5^\circ$ (b) $\cot z (4 \cot z - 3) = 0$ $\cot z = 0, \quad z = \frac{\pi}{2}$ $\cot z = \frac{3}{4}, \tan z = \frac{4}{3}$ so $z = 0.927$	M1 M1 A1, A1 [4] DM1 $\sqrt{A1}, \sqrt{A1}$ [3] M1 B1 M1 A1 [4]	M1 for use of $\sin^2 x = (1 - \cos^2 x)$ correctly M1 for solution of a 3 term quadratic in $\cos$ and attempt at solution of a trig equation A1 for each correct solution DM1 for relating $\cos x$ and $\sin y$ or other correct method of solution M1 for attempt to use a factor B1 for $\frac{\pi}{2}$ (1.57) M1 dealing with $\cot$ and attempt at solution										
10 (i) $\lg s$ (ii) <table border="1"><tr><td>$\lg s$</td><td>0.3</td><td>0.6</td><td>0.78</td><td>0.9</td></tr><tr><td>$\lg t$</td><td>1.4</td><td>0.8</td><td>0.44</td><td>0.19</td></tr></table> (iii) <u>No marks in this part unless $\lg t$ v $\lg s$ graph is used</u> Gradient : $n = -2$ (allow $-2.1 \rightarrow -1.9$) Intercept : $\log k$, or other method $k = 100$ (allow $90 \rightarrow 120$) Alt method Using simultaneous equations, points used must lie on the plotted line. (iv) When $t = 4$, $\lg t = 0.6$ so $\lg s = 0.69$ $s = 4.9$ (allow $4.8 \rightarrow 5.2$)	$\lg s$	0.3	0.6	0.78	0.9	$\lg t$	1.4	0.8	0.44	0.19	B1 [1] M1 DM1 A1 [3] M1A1 M1, A1 [4] M2 A1, A1 M1 A1 [2]	Allow in table or on graph if no contradiction <u>No marks for graph unless $\lg t$ against $\lg s$ (or $\ln t$ against $\ln s$)</u> M1 for 3 or more points correct DM1 for a line through 3 or 4 correct points A1 all points correct with a straight line extending at least from first point to last point M1 calculates gradient A1 for $n = -2$ M1 for use of intercept and dealing with logarithm correctly (can use another point) Must attempt to solve 2 valid equations. $k = 100$ and $n = -2$ M1 for valid method using either the correct graph or using $\lg t = n \lg s + \lg k$ or $t = ks^n$ using their n and their k
$\lg s$	0.3	0.6	0.78	0.9								
$\lg t$	1.4	0.8	0.44	0.19								

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11 (i) $\left[e^{2x} + \frac{5}{4}e^{-2x} \right]_0^k$ $\left(e^{2k} + \frac{5}{4}e^{-2k} \right) - \left(1 + \frac{5}{4} \right) = 3$ $e^{2k} + \frac{5}{4}e^{-2k} - \frac{12}{4} = 0$ $4e^{4k} - 12e^{2k} + 5 = 0$ (ii) $4y^2 - 12y + 5 = 0$ leading to $e^{2k} = \frac{5}{2}, e^{-2k} = \frac{1}{2}$ $k = 0.458, -0.347$	B1, B1 M1 M1 A1 [5] M1 M1 A1, A1 [4]	B1 for each term integrated correctly, allow unsimplified M1 for application of limits to an integral of the form $Ae^{2x} \pm Be^{-2x}$ M1 for equating to $\frac{3}{4}$ and attempt to rearrange to obtain a 3 term equation. Must be using an integral of the form $Ae^{2x} \pm Be^{-2x}$ Answer given, so must be convinced M1 for solution of quadratic equation M1 for solving equations involving exponentials A1 for each
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