UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Ordinary Level

MARK SCHEME for the November 2004 question paper

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
OW -1,2	This is deducted from A or B marks when essential working is omitted.
PA -1	This is deducted from A or B marks in the case of premature approximation.
S -1	Occasionally used for persistent slackness – usually discussed at a meeting.
EX -1	Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

November 2004

GCE O LEVEL

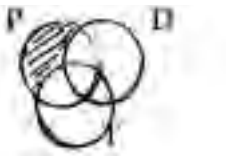
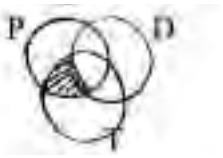
MARK SCHEME

MAXIMUM MARK: 80

SYLLABUS/COMPONENT: 4037/01

ADDITIONAL MATHEMATICS
(Paper 1)

UNIVERSITY of CAMBRIDGE
International Examinations

1	**OA** = **i** + 9**j** **OB** = 5**i** – 3**j** **OC** = k(**i** + 3**j**) **AB** or **BA** = 4**i** – 12**j** **AC** or **CA** = (k – 1)**i** + (3k – 9)**j** **CB** or **BC** = (5 – k)**i** – (3k + 3)**j** Ratio of **i** to **j** is the same → k = 2 [or m = –3 = (3k + 3) ÷(k – 5) = (3k – 9) ÷ (k – 1) → M1 A1 → k = 2 M1 A1] [or (1, 9) → (5, –3) $y = -3x + 12$ M1 A1 subs (k, 3k) or solve with $y = 3x$ M1A1]	M1 A1 M1 A1 **[4]**	For | one relevant vector |. Must be "–" Any 2 of these correct unsimplified. Using ratio idea (not DM). Co. If use of **AB** = **a**–**b** and correct answer obtained, allow full marks. Drawing M2 A2
2 **(i)**	 $P \cap D' \cap T'$ or $P \cap (D \cup T)'$ or $D' \cap T'$ or $(D \cup T)'$	B1 B1	Co.co
(ii)	 $P \cap D' \cap T$	B1 B1 **[4]**	Co.co
3	Change to powers of 2. $2^{3x} \div 2^{y} = 2^{6} \rightarrow 3x - y = 6$ Change to powers of 3. $3^{4x} \times 3^{-2y+2} = 3^{4} \rightarrow 4x - 2y + 2 = 4$ → x = 5 and y = 9	M1 A1 M1 A1 A1 **[5]**	Needs to try all terms as powers of 2 and have ±. Co Needs to try all terms as powers of 3 and have ±. Co with bracket sorted. Co
4	 Let OR = r $\frac{1}{2}r^2 (4/3) - \frac{1}{2}7^2 (4/3) = 48$ → r = 11 If x + 7 for r allow A1 for x = 4. Perimeter = 11 × (4/3) + 7 × (4/3) + 2 (11 – 7) → 32	M1 A1 A1 M1 A1√ A1 **[6]**	Any use of $\frac{1}{2}r^2\theta$ for M mark. A1 unsimplified, but complete. Co. M1 for any use of $s = r\theta$. A1√ unsimplified and √ on his r or x only. Co – allow anything rounding to 32.0

5		$(a + x)(1 - 2nx + \ldots) = 3 - 41x + bx^2$		
		term in $x^3 = {}_nC_2 (\pm 2x)^2$	B1	Wherever it comes.
		$\rightarrow a = 3$	B1	Co – anywhere
		$1 - 2an = -41 \rightarrow n = 7$	M1 A1	Must use 2 terms.
		Coeff of x^2 is $3 \times 84 - 1 \times 14$	M1	Must use sum of 2 products.
		$\rightarrow 238$	A1	Co.
			[6]	
6		$f(x) = 5 + 3\cos 4x$		
	(i)	$a = 3$, period $= \frac{1}{2}\pi$	B1 B1	Co. allow 90° for period.
	(ii)	max/min $x = \pi/4$ or $2\pi/4$ or $3\pi/4$		
		$\rightarrow$ max of 8	B1	When "8" is used as stationary value.
		$\rightarrow$ min of 2	B1	When "2" is used as stationary value.
		$(\pi/4, 2)$ $(2\pi/4, 8)$ $(3\pi/4, 2)$	B2, 1√	√ for 5 ± his "a". [B0 if degrees here]
			[6]	Ignore inclusion of max/min at 0 or π.
7	**(a)**	$8 \times 8!$ or $\frac{8}{9} \times 9!$ or $9! - 8!$	M1	
		$\rightarrow 322\,560$	A1 **[2]**	Must be ${}_nC_r$ – knows what to do. Ans only is ok for 2 marks.
	(b)	2G, 1B ${}_5C_2 \times {}_3C_1 = 10 \times 3 = 30$	M1 A1	Needs to be a product of ${}_nC_r$'s. Co.
		3G, 0B ${}_5C_3 = 10$	B1	Anywhere.
		total = sum of these = 40	A1	Co.
			[4]	
8	**(i)**	$y = (3x + 11)/(x - 3)$		
		Makes x the subject.	M1	Good algebra in making x the subject.
		$f^{-1}(x) = (3x + 11)/(x - 3)$	A1	
		f and f^{-1} are the same functions.		
		$\rightarrow$ Graph has y = x as line of symmetry.	B1	Co accept any mention of y = x.
			[3]	
	(ii)	$g(x) = \frac{1}{2}(x - 3)$		
		$g^{-1}(x) = 2x + 3$	B1	Anywhere.
		$\rightarrow 2x + 3 = (3x + 11)/(x - 3)$		
		$\rightarrow 2x^2 - 6x - 20 = 0 \rightarrow x = -2$ or 5	M1 A1	Algebra must lead to quadratic. Co.
			[3]	
	(iii)	$gf(x) = -2 \rightarrow f(x) = g^{-1}(-2)$	B1	However obtained.
		$\rightarrow x = -2$	**[1]**	

9	**(a)**	$s^2 = 3c^2 + 4s$ Use of $s^2 + c^2 = 1$ $\rightarrow 4s^2 - 4s - 3 = 0 \rightarrow s = -½$ or 3/2 $\rightarrow x = 210°$ and $330°$	M1 DM1 A1 A1√ **[4]**	Used to eliminate cos completely. Scheme for quadratic. Co. √ for 2nd value from incorrect sine. (A1√ not given for extra values **in the range**, but could be given if soln of quadratic led to 2 values of sine<1.)

	(b)	cot = 1/tan used
		tan 2y = 4
		
		2y = 1.326 → y = 0.66
		or 2y = π + 1.326 or 2π + 1.326
		→ y = 2.23
		→ y = 3.80 or 3.81
			M1
			
			A1
			
			A1√
			A1√
			[4]
				Use of cot = 1/tan even if "2" removed incorrectly. Not for tan and 2y split.
				Co (must be radians) – not for 0.67.
				
				For (i) + ½π
				For (i) + π or (ii) + ½π
				[S–1 for extra values in the range]
				[sc All answers in degrees B1.]

10		$y = x^3 \ln x$		
	(i)	$dy/dx = 3x^2 \ln x + x^3 (1/x)$ $= 3x^2 \ln x + x^2$	M1 A1 **[2]**	M1 correct "uv". A1 ok unsimplified.
	(ii)	$dy/dx = 0 \quad \ln x = -⅓$	M1 A1 **[2]**	Not DM – setting his dy/dx to 0 + attempt to solve.
	(ii)	$\delta y = dy/dx \times \delta x = (e^2 + 3e^2)p$ $= 4e^2p$ or 29.6p	M1 A1 **[2]**	Use of small increases. Allow for use of dy/dt. Δx = p essential for M mark. Alg expression with "p" ok for M1.
	(iii)	$d/dx (x^3 \ln x) = x^2 + 3x^2 \ln x$ Integrating → $x^3 \ln x = ⅓x^3 + \int 3x^2 \ln x dx$ $\int x^2 \ln x dx = ⅓(x^3 \ln x - ⅓x^3)$	M1 A1 A1 **[3]**	∫ is reverse of diff used. A1 needs $⅓x^3$ co Integration by parts ok. M1 A1 A1.
11		4y = 3x + 1 and xy = 28x – 27y Sim equations. $\rightarrow x^2 - 10x + 9 = 0$ or $y^2 - 8y + 7 = 0$ → (9, 7) [(1, 1) was given] P(1, 1), Q(9, 7) → gradient of PQ = ¾ Gradient of perp bisector is –4/3 M (mid-point of PQ) = (5, 4) Eqn of perp bis. y–4 = – 4/3(x – 5) 3y + 4x = 32 meets y = 4x at R(2, 8) Area of ΔPQ = ½ × PQ × MR = ½ × 10 × 5 (or matrix method) → 25	M1 DM1 A1 M1 M1 A1 M1 M1 A1 **[9]**	Complete elimination of x or y Soln of quadratic (by scheme) Co for (9, 7) Use of $m_1m_2 = -1$ with his PQ Use of $(½(x_1 + x_2), ½(y_1 + y_2))$ Co – unsimplified ok. Simulataneous eqns. Must be with a perp Any correct method. Co.

12	**EITHER**		
(a)	$N = 20\,000e^{-0.05n}$		
(i)	n = 10, N = 12 130 or more places.	B1 **[1]**	Co
(ii)	$2000 = 20\,000e^{-0.05n}$		
	$e^{-0.05n} = 0.1$	M1	Isolating exponential – or taking logs to get 3 terms.
	Take logs	M1	Taking logs.
	n = 45.1 → 2006	A1	Co. needs 2006, not 2005.
		[3]	
(b)	Put $y = 3^x$	M1	Used.
	$3^{x+1} = 3y$ or $3^{x-1} = \frac{1}{3}y$	A1 A1	For each expression.
	$3y - 2 = 8y/3 \rightarrow y = 6$	A1	Co.
	$3^x = 6$, $x\lg 3 = \lg 6 \rightarrow x = 1.63$	M1 A1	Taking logs for his 3^x. co.
	[or ÷ by 3^{x-1} M1 A1 $3^{x-1} = 2$ M1 A1]	**[6]**	
	[or ÷ by 3^{x+1} M1 A1 $3^{x+1} = 18$ M1 A1]		
12	**OR**		
	$y = e^{\frac{x}{2}} + 3e^{\frac{-x}{2}}$		
(i)	$dy/dx = \frac{1}{2}e^{\frac{x}{2}} - \frac{3}{2}e^{\frac{-x}{2}}$	B1 B1	Anywhere –
	= 0 when $e^x = 3$	M1	Setting his dy/dx to 0 and reasonable attempt at making e^x the subject.
	$y = \sqrt{3} + 3 \div \sqrt{3} = 2\sqrt{3}$	A1	Co. Decimal check – A0.
		[4]	
(ii)	$d^2y/dx^2 = \frac{1}{4}e^{\frac{x}{2}} + \frac{3}{4}e^{\frac{-x}{2}} > 0$, MIN	M1 A1	M1 Reasonable attempt by any method.
		[2]	A1 Correct deduction but needs second differential correct.
(iii)	$\int\left(e^{\frac{x}{2}} + 3e^{\frac{-x}{2}}\right)dx = 2e^{\frac{x}{2}} - 6e^{\frac{-x}{2}}$	M1 A1	Knowing to integrate for area + any attempt with exponentials. A1 co.
	[ ] at 1 – [ ] at 0	M1	DM0 if [ ] at 0 is ignored.
	= 4 + 2√e - 6√e = 3.66	A1	Co.
		[4]	

DM1 for quadratic equation.

Formula Equation must be set to 0. Formula must be correct and correctly used, but allow for numerical and algebraic errors.

Brackets Equation must be set to 0. Must be an attempt to get two linear brackets. Each bracket must then be equated to 0 and solved.