

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2013 series

4037 ADDITIONAL MATHEMATICS

4037/11

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\surd$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\sqrt{}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
OW –1,2	This is deducted from A or B marks when essential working is omitted.
PA –1	This is deducted from A or B marks in the case of premature approximation.
S –1	Occasionally used for persistent slackness – usually discussed at a meeting.
EX –1	Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

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1	(i)		B1	correct shape for $y = \cos x - 1$
	(ii)		B1	all correct
			B1	correct shape for $y = \sin 2x$
			B1	all correct
	(iii)	3	B1	
2	Either	gradient = 1	B1	
		intercept = 2	B1	
		$\ln b = \text{gradient}$ or $\ln A = \text{intercept}$	M1	M1, need to equate either gradient to $\ln b$ or intercept to $\ln A$
		$b = e$ or 2.72	A1	
		$A = e^2, A = 7.39$	A1	
	Or	$e^4 = Ab^2$ and $e^{10} = Ab^8$	[B1 B1	B1 for each equation
	leading to	$b^6 = e^6$ or $e^4 = e^2 A$ or $e^{10} = e^8 A$	M1	M1 for attempt to solve for either A or b
		$b = e$ or 2.72	A1	
		$A = e^2, A = 7.39$	A1]	
	Or	$10 = 8 \ln b + \ln A$	[B1	
		$4 = 2 \ln b + \ln A$	B1	
	leading to	$\ln b = 1$ or $6 = 3 \ln A$	M1	M1 for attempt to solve for either A or b
	$b = e$ or 2.72	A1		
	$A = e^2, A = 7.39$	A1]		

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3	(i)	${}^{14}C_6 = 3003$	B1	
	(ii)	${}^5C_3 \times {}^9C_3 = 840$	M1 A1	M1 for product of 2 combinations
	(iii)	Either $3003 - {}^9C_6 = 2919$ Or 1M + 5W: $5 \times {}^9C_5 = 630$ 2M + 4W: ${}^5C_2 \times {}^9C_4 = 1260$ 3M + 3W: 840 (part (ii)) 4M + 2W: ${}^5C_4 \times {}^9C_2 = 180$ 5M + 1W: $1 \times {}^9C_1 = 9$ Total: 2919	M1 B1 A1 [B2 1 0 B1]	M1 for 3003 – number of committees containing no men B1 for 9C_6 –1 each error B1 for correct final answer
4	(i)	2	B1	
	(ii)	$\log_4 y^2 - \log_4 (5y - 12) (= \log_4 2)$	B1	B1 for power
		$\log_4 \left(\frac{y^2}{5y - 12} \right) (= \log_4 2)$	M1	correct division
		$y^2 - 10y + 24 = 0$	M1	attempt at solution of a 3 term quadratic
		$y = 4, 6$	A1	A1 for both
5	(i)	$x + \frac{6}{x} (+c)$	B1 B1	B1 for each term
	(ii)	$\left(3k + \frac{6}{3k} \right) - \left(k + \frac{6}{k} \right) (= 2)$	M1	correct use of limits
		$2k^2 - 2k - 4 = 0$	M1	attempt to obtain a 3 term quadratic from 2 brackets equated to 2
		leading to $k = 2$	DM1 A1	DM1 or solution of quadratic dependent on 2 nd M1

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6	(i) $A^{-1} = \frac{1}{13} \begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}$ (ii) Either $\begin{pmatrix} a & b \\ c & -1 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 7 & 5 \\ 17 & d \end{pmatrix}$ $= \frac{1}{13} \begin{pmatrix} 52 & 25+d \\ 13 & -15+2d \end{pmatrix}$ leading to $a = 4, c = 1$ and $b = 2, d = 1$ Or $\begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} a & b \\ c & -1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 17 & d \end{pmatrix}$ $2a - c = 7, 3a + 5c = 17, a = 4, c = 1$ $2b + 1 = 5, 3b - 5 = d, b = 2, d = 1$	B1 B1 M1 DM1 A3,2,1,0 [M1 DM1 A3,2,1,0]	B1 for matrix, B1 for multiplying by a correct determinant evidence of multiplication of both sides by A^{-1} DM1 for attempt to equate like elements –1 each error M1 for evidence of matrix multiplication DM1 for attempt to equate like elements –1 each error
7	(i) $\tan B = \frac{\sqrt{5+1}}{\sqrt{5-2}}$ $= \frac{\sqrt{5+1}}{\sqrt{5-2}} \times \frac{\sqrt{5+2}}{\sqrt{5+2}}$ $= 7 + 3\sqrt{5}$ (ii) $(7 + 3\sqrt{5})^2 + 1 = \sec^2 B$ $\sec^2 B = 95 + 42\sqrt{5}$ Or $\sec^2 B = \frac{1}{\cos^2 B} = \frac{(\sqrt{5+1})^2 + (\sqrt{5-2})^2}{(\sqrt{5-2})^2}$ $\sec^2 B = \frac{15 - 2\sqrt{5}}{9 - 4\sqrt{5}} \times \frac{9 + 4\sqrt{5}}{9 + 4\sqrt{5}}$ $\sec^2 B = 95 + 42\sqrt{5}$	B1 M1 A1 M1 M1 $\sqrt{A1}$ $\sqrt{A1}$ [M1 M1 A1 A1]	attempt at rationalisation (Allow if inverse is used) M1 for attempt to use the correct identity M1 for simplification to give 3 or 4 terms cao A1 for 95, A1 for $42\sqrt{5}$ M1 for attempt to use to find BC^2 M1 for use of $\sec B = \frac{1}{\cos B}$ A1 for 95, A1 for $52\sqrt{5}$

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8	(i)	Either $\tan \frac{\theta}{2} = \frac{8}{6}$	M1	M1 for use of trig to obtain half angle
		$\frac{\theta}{2} = 0.927\dots$		Can use $\sin \frac{\theta}{2} = \frac{8}{10}$ or $\cos \frac{\theta}{2} = \frac{6}{10}$
		$\theta = 1.855$	A1	A1 Allow if done in degrees and converted
		Or Area of triangle $MEF = 48$	[M1]	M1 for a complete method to find the obtuse angle
		$\frac{1}{2} \times 10^2 \times \sin \theta = 48$		
	(ii)	$\theta = 1.287, \pi - 1.287$		
		$\theta = 1.855$	A1]	
		Or $16^2 = 10^2 + 10^2 - (2 \times 10 \times \cos \theta)$	[M1]	M1 for use of the cosine rule, need to see working as answer given
		$\theta = 1.855$	A1]	
		radius = 10	B1	B1 for the radius, allow anywhere
(iii)		$P = (10 \times 1.855) + 10 + 10 + 16$	M1 M1	M1 for use of arc length M1 for method, must be arc +3 sides
		$= 54.6 \text{ or } 54.5 \text{ or } 54.55$	A1	
		$A = 256 - 2 \left(\frac{1}{2} \times 8 \times 6 \right) - \frac{1}{2} 10^2 (1.855)$	M1 M1	M1 for area of sector M1 for a correct plan to obtain the required area
		$= 115.25 \text{ or } 115.3 \text{ or } 115$ awrt 115	A1	

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9	(i)	$\overrightarrow{AP} = \frac{3}{4}(\mathbf{b} - \mathbf{a})$	B1	
		$\overrightarrow{OP} = \mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a}), \text{ or}$	M1	M1 for attempt at vector addition
		$\overrightarrow{OP} = \mathbf{a} - \frac{1}{4}(\mathbf{b} - \mathbf{a}),$		
		$= \frac{1}{4}(\mathbf{a} + 3\mathbf{b})$	A1	Answer given
	(ii)	$\overrightarrow{OQ} = \frac{2}{5}\mathbf{c}, \text{ or } \overrightarrow{QC} = \frac{3}{5}\mathbf{c} \text{ or } \overrightarrow{CQ} = -\frac{3}{5}\mathbf{c}$	B1	B1 for $\overrightarrow{OQ}$, $\overrightarrow{QC}$ or $\overrightarrow{CQ}$
		$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	M1	M1 for correct vector addition/subtraction
10		$= \frac{2}{5}\mathbf{c} - \frac{\mathbf{a}}{4} - \frac{3\mathbf{b}}{4}$	A1	
	(iii)	$2\mathbf{c} - \frac{5\mathbf{a}}{4} - \frac{15\mathbf{b}}{4} = 6(\mathbf{c} - \mathbf{b})$	M1	M1 for use of <i>their</i> vectors and attempt to get $k\mathbf{c}$
		$\mathbf{c} = \frac{9\mathbf{b} - 5\mathbf{a}}{16}$	A1	
10	(i)	When $x = 2, y = -5$	B1	B1 for $y = -5$
		$\frac{dy}{dx} = 3x^2 - 8x + 1$	M1	M1 for attempt to differentiate
		when $x = 2, \frac{dy}{dx} = -3$	DM1	DM1 for attempt at tangent equation – must be tangent with use of $x = 2$
		Tangent: $y + 5 = -3(x - 2)$ ($y = 1 - 3x$)	A1	allow unsimplified
	(ii)	$1 - 3x = x^3 - 4x^2 + x + 1$	M1	M1 for equating tangent and curve equations
		$x(x - 2)^2 = 0$	DM1	DM1 for attempt to solve resulting cubic equation
		Meets at (0, 1)	A1 A1	A1 for each coordinate

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(iii)	Grad of perp = $\frac{1}{3}$ Midpoint (1, -2) Perp bisector $y + 2 = \frac{1}{3}(x - 1)$	√B1 M1 M1 A1	√B1 on <i>their</i> gradient in (i) only M1 for attempt to find the midpoint M1 for attempt at line equation – must be perp bisector A1 allow unsimplified
11 (a)	$\sin\left(x + \frac{\pi}{3}\right) = -\frac{1}{2}$ $x + \frac{\pi}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}$ $x = \frac{5\pi}{6}, \frac{3\pi}{2}$	B1 B1 B1 B1	B1 for $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ B1 for first correct solution B1 for a second correct solution with all solutions in radians and with no excess solutions within the range
(b)	$\tan y - 2 = \frac{1}{\tan y}$ $\tan^2 y - 2 \tan y - 1 = 0$ $\tan y = 1 \pm \sqrt{2}$ $y = 67.5^\circ, 157.5^\circ$	B1 M1 A1 DM1 A1 A1	B1 for a correct equation M1 for attempt to obtain a 3 term quadratic equation A1 for a correct equation equated to zero DM1 for solution of quadratic A1 for first correct solution A1 for a second correct solution with all solutions in degrees and with no excess solutions within the range.