

# UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Ordinary Level

## MARK SCHEME for the June 2005 question paper

### 4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



## Mark Scheme Notes

Marks are of the following three types:

- M     Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
  - A     Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
  - B     Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
  - The symbol  $\surd$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
  - Note:     B2 or A2 means that the candidate can earn 2 or 0.  
              B2/1/0 means that the candidate can earn anything from 0 to 2.



The following abbreviations may be used in a mark scheme or used on the scripts:

|     |                                                                                                                                     |
|-----|-------------------------------------------------------------------------------------------------------------------------------------|
| AG  | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)                                              |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)                                         |
| CWO | Correct Working Only – often written by a 'fortuitous' answer                                                                       |
| ISW | Ignore Subsequent Working                                                                                                           |
| MR  | Misread                                                                                                                             |
| PA  | Premature Approximation (resulting in basically correct work that is insufficiently accurate)                                       |
| SOS | See Other Solution (the candidate makes a better attempt at the same question)                                                      |

### Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through  $\sqrt{\phantom{x}}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
- OW –1,2 This is deducted from A or B marks when essential working is omitted.
- PA –1 This is deducted from A or B marks in the case of premature approximation.
- S –1 Occasionally used for persistent slackness – usually discussed at a meeting.
- EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.



**JUNE 2005**

**GCE O LEVEL**

**MARK SCHEME**

**MAXIMUM MARK: 80**

**SYLLABUS/COMPONENT: 4037/01**

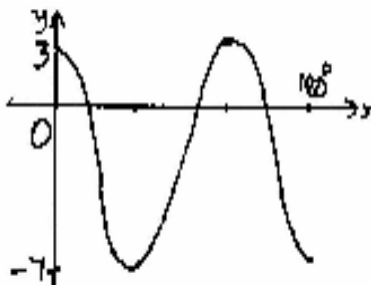
**ADDITIONAL MATHEMATICS  
(Paper 1)**



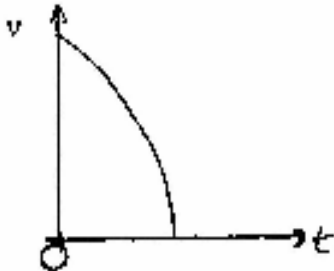
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|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                |                                                                                                                                                                                                                                                                                                                                                |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>1</b> <math>A^2 = \begin{pmatrix} 2 &amp; 1 \\ -1 &amp; 1 \end{pmatrix}^2 = \begin{pmatrix} 3 &amp; 3 \\ -3 &amp; 0 \end{pmatrix}</math></p> <p><math>(A^2)^{-1} = \frac{1}{9} \begin{pmatrix} 0 &amp; -3 \\ 3 &amp; 3 \end{pmatrix}</math></p> <p><u>or</u> <math>A^{-1}</math> first followed by squaring</p>                                                                                                                                                                                                                                                        | <p><b>B2,1</b></p> <p><b>B1√, B1√</b><br/><b>[4]</b></p>                                       | <p>One off for each error.</p> <p>B1√ for 1÷9. B1√ for rest. √ from his attempt at <math>A^2</math>.</p> <p>If <math>\begin{pmatrix} 4 &amp; 1 \\ 1 &amp; 1 \end{pmatrix}</math> used, could get last 2 marks.</p>                                                                                                                             |
| <p><b>2</b> 9 CDs → 4 Beatles, 3 Abba, 2 Rolling</p> <p>(i) <math>{}_8C_3 = (8 \times 7 \times 6) \div (3 \times 2 \times 1) = 56</math></p> <p>(ii) 2B 2A <math>{}_4C_2 \times {}_3C_2 = 18</math><br/> 2B 2R <math>{}_4C_2 \times 1 = 6</math><br/> 2A 2R <math>{}_3C_2 \times 1 = 3</math><br/> → Total of 27</p>                                                                                                                                                                                                                                                         | <p><b>M1 A1</b><br/><b>[2]</b></p> <p><b>M1</b><br/><b>M1</b><br/><b>A1</b><br/><b>[3]</b></p> | <p>2 if correct without working<br/> <math>{}_9C_3</math> M0. <math>4 \times {}_8C_3</math> gets M1 A0</p> <p>One correct product with <math>{}_nC_r</math>s<br/> 3 products added – even if <math>{}_nP_r</math><br/> CAO</p>                                                                                                                 |
| <p><b>3</b> <math>\cos \theta = \frac{\sqrt{1 - \sin^2 \theta}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}</math></p> <p><math>\frac{s}{s-c} = \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{2} - 1}</math></p> <p>× top and bottom by <math>(\sqrt{2} + 1)</math></p> <p>→ <math>1 + \sqrt{2}</math></p>                                                                                                                                                                                                                            | <p><b>M1 A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b><br/><b>[5]</b></p>          | <p>Use of <math>s^2 + c^2 = 1</math> to obtain cos as a surd – or correctly from <math>90^\circ</math> triangle.</p> <p>Correct algebra – getting rid of <math>\sqrt{3}</math></p> <p>Correct technique used to rationalise the denominator.</p> <p>This form ok. No need for <math>a =</math>, <math>b =</math>. (decimals get no credit)</p> |
| <p><b>4</b> <math>\vec{OA} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}</math>, <math>\vec{OB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}</math>, <math>\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}</math></p> <p><math>\vec{AC} = \frac{3}{5} \vec{AB} = \begin{pmatrix} 12/5 \\ 9/5 \end{pmatrix}</math></p> <p><math>\vec{OC} = \vec{OA} + \vec{AC} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 12/5 \\ 9/5 \end{pmatrix} = \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix}</math></p> <p><math>OC = \sqrt{\left(\frac{9}{25} + \frac{16}{25}\right)} = 1</math></p> | <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1 A1</b></p> <p><b>M1 A1</b><br/><b>[6]</b></p>       | <p>Use of <math>\mathbf{b} - \mathbf{a}</math> or <math>\mathbf{a} - \mathbf{b}</math> – not for <math>\mathbf{a} + \mathbf{b}</math></p> <p>CAO – not for negative of this. Could be implied by correct <math>\vec{OC}</math>.</p> <p>Any correct method ok. CAO</p> <p>Correct method on his OC. Answer was given.</p>                       |

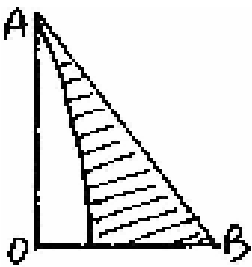
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|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|------|---|-----|------|------|------|------|---------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>5</b> <math>f(x) = A + 5\cos Bx</math></p> <p>(i) <math>A = -2</math></p> <p>(ii) Amplitude = 5</p> <p>(iii) <math>B = 3</math></p> <p>(iv) Range 3 to <math>-7</math></p>                                                                                                                                                                                                                                                         | <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>                                                                          | <p>CAO</p> <p>CAO</p> <p>CAO</p> <p><math>-3</math> to <math>7</math> implied somewhere – table ok – even if no graph</p>                                                                                                                                           |      |      |   |     |      |      |      |      |                                                                                                   |                                                                                                                                                                                                                                                                     |
|                                                                                                                                                                                                                                                                                                                                                         | <p><b>B2,1</b></p> <p><b>[6]</b></p>                                                                                                         | <p>Needs <math>1\frac{1}{2}</math> oscillations – over-rides rest. <math>\sqrt{\quad}</math> on <math>3</math> and <math>-7</math></p> <p>Start at max – finishes at second min.</p> <p>Curves – but be tolerant</p>                                                |      |      |   |     |      |      |      |      |                                                                                                   |                                                                                                                                                                                                                                                                     |
| <p><b>6 (i)</b> <math>-7 \leq f(x) \leq 8</math></p> <p><b>(i)</b> <math>0 \leq g(x) \leq 8</math></p> <p><b>(ii)</b> <math>-7 \leq h(x) \leq 2</math></p> <p>f yes      g no      h no</p>                                                                                                                                                                                                                                              | <p><b>B1</b></p> <p><b>B1 B1</b></p> <p><b>B1 B1</b></p> <p><b>B2,1</b></p> <p><b>[7]</b></p>                                                | <p>CAO Allow <math>&lt;</math> for <math>\leq</math></p> <p>CAO As above</p> <p>CAO As above</p> <p>Loses one for each wrong decision. (answer f on its own – allow B2)</p>                                                                                         |      |      |   |     |      |      |      |      |                                                                                                   |                                                                                                                                                                                                                                                                     |
| <p><b>7 (a)</b> <math>I = I_0(1 + \alpha)^t</math></p> <p>Subs and divides <math>1.031 = 1.0025^t</math></p> <p><math>t = \lg 1.031 \div \lg 1.0025 = 12.3</math></p> <p><b>(b)</b> <math>1 = \log 10</math></p> <p>LHS = <math>\lg 10(8 - x)</math></p> <p><math>80 - 10x = 3x + 2</math></p> <p><math>\rightarrow x = 6</math></p>                                                                                                     | <p><b>M1</b></p> <p><b>M1 A1</b></p> <p><b>[3]</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>[4]</b></p> | <p>Sub + division before taking logs. (or <math>\lg I = \lg I_0 + t \lg(1 + \alpha)</math> + use)</p> <p>Taking logs. CAO to 3 sf or more.</p> <p>Anywhere in the question.</p> <p>Putting any 2 logs together</p> <p>Complete elimination of 3 logs</p> <p>CAO</p> |      |      |   |     |      |      |      |      |                                                                                                   |                                                                                                                                                                                                                                                                     |
| <p><b>8</b></p> <table><tr><td>lgx</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>lgy</td><td>3.28</td><td>2.40</td><td>1.49</td><td>0.60</td></tr></table> <p>(i) Knows what to do. Pts within <math>\frac{1}{2}</math> square.</p> <p>(ii) Gradient = <math>\pm n</math>      <math>n = -0.88</math> to <math>-0.92</math></p> <p>log <math>k = y</math>-intercept      <math>k = 14\,000</math> to <math>16\,000</math></p> | lgx                                                                                                                                          | 1                                                                                                                                                                                                                                                                   | 2    | 3    | 4 | lgy | 3.28 | 2.40 | 1.49 | 0.60 | <p><b>M1 A2,1</b></p> <p><b>[3]</b></p> <p><b>B1 A1</b></p> <p><b>B1 A1</b></p> <p><b>[4]</b></p> | <p>For part (ii) – use of sim eqns is ok if points used are on line, not from table.</p> <p>Knows what to do.</p> <p>Accuracy within <math>\frac{1}{2}</math> square.</p> <p>B1 even if just stated without graph.</p> <p>B1 even if just stated without graph.</p> |
| lgx                                                                                                                                                                                                                                                                                                                                                                                                                                      | 1                                                                                                                                            | 2                                                                                                                                                                                                                                                                   | 3    | 4    |   |     |      |      |      |      |                                                                                                   |                                                                                                                                                                                                                                                                     |
| lgy                                                                                                                                                                                                                                                                                                                                                                                                                                      | 3.28                                                                                                                                         | 2.40                                                                                                                                                                                                                                                                | 1.49 | 0.60 |   |     |      |      |      |      |                                                                                                   |                                                                                                                                                                                                                                                                     |

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| <p><b>9 (i)</b> <math>x^2 + 2x + k = 3kx - 1</math><br/> <math>\rightarrow x^2 + (2 - 3k)x + (k + 1) = 0</math><br/> Uses <math>b^2 - 4ac =, &gt; \text{ or } &lt; 0</math><br/> <math>\rightarrow 9k^2 - 16k</math></p> <p>End-points of 0 and 16/9<br/> Use of <math>b^2 - 4ac &lt; 0</math><br/> Solution set <math>0 &lt; k &lt; 16/9</math></p> <p><b>(ii)</b> Same case with <math>k = 1</math><br/> No intersection since <math>k</math> inside the range</p> <p>Special case. Solves simultaneous. eqns <math>\rightarrow \sqrt{-7}</math>. B1</p>                  | <p><b>M1</b><br/><b>A1</b></p> <p><b>DM1</b><br/><b>M1</b><br/><b>A1</b> [5]</p> <p><b>B1</b><br/><b>B1</b>√ [2]</p> | <p>Any use of <math>b^2 - 4ac</math><br/> This quadratic only.</p> <p>Solution of this quadratic <math>\rightarrow 2</math> values<br/> Definite recognition of – ve.<br/> CAO</p> <p>NB No intersection on its own<br/> without <math>k = 1</math> gets no credit.</p>                                                       |
| <p><b>10 (i)</b> <math>x = -a \rightarrow -2a^3 + 2a^2 + 13a + 12</math><br/> <math>x = a \rightarrow 2a^3 + 2a^2 - 13a + 12</math><br/> <math>-2a^3 + 2a^2 + 13a + 12</math><br/> <math>= 3(2a^3 + 2a^2 - 13a + 12)</math><br/> <math>2a^3 + a^2 - 13a + 6 = 0</math></p> <p><b>(ii)</b> Tries <math>a = 2</math> : fits ok. ( or <math>-3, \frac{1}{2}</math> )<br/> <math>\div (x - 2) \rightarrow 2a^2 + 5a - 3</math></p> <p>Solution <math>\rightarrow a = -3</math> and <math>\frac{1}{2}</math></p> <p>If factors left as final answer, loses the last 2 marks.</p> | <p><b>M1</b></p> <p><b>M1 A1</b> [3]</p> <p><b>M1A1</b><br/><b>M1</b></p> <p><b>M1 A1</b> [5]</p>                    | <p>For either of these – ignore simple algebraic and numeric slips</p> <p>Allow M1 if 3 wrong side. Answer given.</p> <p>Tries a search for first value<br/> Must be <math>(x - )</math> for M. CAO for A mark.<br/> CAO for both.</p> <p>T &amp; I : M1 A1 for first value,<br/> A1 for second value,<br/> A2 for third.</p> |
| <p><b>11</b> <math>a = -2 - 2t</math></p> <p><b>(i)</b> <math>v = -2t - t^2 (+ c)</math><br/> <math>v = 0</math> when <math>t = 4 \rightarrow c = 24</math><br/> if <math>t = 0, v = 24 \text{ ms}^{-1}</math></p> <p><b>(ii)</b> <math>s = -t^2 - t^3/3 + \dots (24t) \dots</math><br/> Put <math>t = 4 \rightarrow 58\frac{2}{3} \text{ m}</math></p> <p><b>(iii)</b></p>                                                                                                              | <p><b>M1 A1</b><br/><b>DM1</b><br/><b>A1</b> [4]</p> <p><b>M1A1</b>√</p> <p><b>A1</b> [3]</p> <p><b>B1</b> [1]</p>   | <p>T<br/> Attempt at <math>\int</math>. Ignore omission of <math>c</math><br/> Attempt at <math>c</math>.<br/> CAO</p> <p>Attempt at <math>\int</math>. “24t” not needed.<br/> CAO</p> <p>Curve necessary.</p>                                                                                                                |

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|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>12 EITHER</b> <math>y = 8 - e^{-2x}</math></p>  <p> <math>x = 0, y = 7</math><br/> <math>dy/dx = -2e^{-2x}</math><br/> At <math>x = 0, m = -2</math> </p> <p>Tangent crosses y-axis at <math>(3\frac{1}{2}, 0)</math></p> <p> <math>y = 0, x = \frac{1}{2}\ln 8</math> or 1.04<br/> Area of triangle = <math>\frac{1}{2} \times 3.5 \times 7 = 12.25</math><br/> <math>\int</math> curve = <math>[8x - \frac{1}{2}e^{-2x}]</math><br/> From 0 to his "x" <math>[4\ln 8 - 4] - [0 - 0.5]</math><br/> <br/> <math>12.25 - (4\ln 8 - 3.5) = 7.43</math> </p> | <p><b>M1</b><br/><b>A1</b></p> <p><b>M1 A1</b><br/><b>[4]</b></p> <p><b>B1</b><br/><b>M1</b><br/><b>M1 A1</b><br/><b>DM1</b></p> <p><b>A1</b><br/><b>[6]</b></p>                | <p>For differential.<br/>CAO for gradient of -2.</p> <p>Any method ok providing calculus used. Numeric gradient for M1.</p> <p>Anywhere in the question.<br/>Even if no integration later.<br/>Attempt at <math>\int</math>. CAO<br/>DM0 if value at 0 assumed to be 0.</p> <p>CAO</p>                                                                                                                                                                                         |
| <p><b>12 OR</b></p> <p>(i) Perimeter of square + circumference<br/>= 2 m<br/> <math>\rightarrow 4x + 2\pi r = 2</math><br/> <math>\rightarrow r = \frac{1-2x}{\pi}</math><br/> <math>\rightarrow A = x^2 + \pi \left(\frac{1-2x}{\pi}\right)^2</math><br/> <math>\rightarrow A = \frac{(\pi+4)x^2 - 4x + 1}{\pi}</math></p> <p>(ii) <math>\frac{dA}{dx} = \frac{1}{\pi}(2\pi x + 8x - 4)</math><br/> = 0 when <math>x = \frac{4}{2\pi + 8} = 0.28</math> m<br/> A = 0.14</p> <p>(iii) <math>\frac{d^2A}{dx^2} = \frac{1}{\pi}(2\pi + 8)</math> +ve <math>\rightarrow</math> MIN</p>                                                              | <p><b>M1</b><br/><b>A1</b></p> <p><b>M1</b><br/><b>A1</b><br/><b>[4]</b></p> <p><b>M1 A1</b><br/><b>DM1</b><br/><b>A1</b><br/><b>[4]</b></p> <p><b>M1 A1</b><br/><b>[2]</b></p> | <p>Allow for <math>\pi d</math> or <math>\pi r</math> and for <math>2x</math> or <math>4x</math><br/>CAO – in any form</p> <p>Needs <math>\pi r^2</math> and <math>r^2</math> (both)</p> <p>CAO – answer given</p> <p>Attempt at diff. A0 if <math>\pi</math> missing, but can then gain rest of marks.<br/>Sets his differential to 0.<br/>CAO – 2 sig figures sufficient.</p> <p>Any valid method ok. Needs correct algebraic <math>\frac{d^2A}{dx^2}</math> for A mark.</p> |
| <p>DM1 for quadratic equation. Equation must be set to 0 if using formula or factors.</p> <p><u>Formula</u><br/>Must be correct<br/>– ignore arithmetic and algebraic slips.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |

Factors

Must attempt to put quadratic into 2 factors  
Each factor then equated to 0.