



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4037/01, 0606/01

Paper 1

October/November 2009

MARK SCHEME

Maximum Mark : 80

IMPORTANT NOTICE

Mark Schemes have been issued on the basis of **one** copy per Assistant examiner and **two** copies per Team Leader.



UNIVERSITY of CAMBRIDGE
International Examinations

[Turn over

<p>1(i) $2a^3 - 7a^2 + 7a^2 + 16 = 0$ leading to $a^3 = -8$, $a = -2$</p> <p>(ii) $2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16$ $= 21$</p>	<p>M1 A1 [2]</p> <p>M1 A1 [2]</p>	<p>M1 for use of $x = a$</p> <p>M1 for substitution of $x = -\frac{1}{2}$</p>
<p>2 (i) (ii)</p> $\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$	<p>B1,B1 [2]</p> <p>B2,1,0 [2]</p>	<p>B1 for each matrix, must be in correct order</p> <p>-1 for each error</p>
<p>3 $4(2k+1)^2 = 4(k+2)$ $4k^2 + 3k - 1 = 0$ leading to $k = \frac{1}{4}, -1$</p>	<p>M1 A1</p> <p>M1 A1 [4]</p>	<p>M1 for use of '$b^2 - 4ac$' Correct quadratic equation</p> <p>M1 for correct attempt at solution A1 for both 1 values</p>
<p>4 $(13-3y)^2 + 3y^2 = 43$ (or $x^2 + \frac{(13-x)^2}{3} = 43$) $6(2y^2 - 13y + 21) = 0$ (or $2(2x^2 - 13x + 20) = 0$) $(2y-7)(y-3) = 0$ (or $(2x-5)(x-4) = 0$) $y = 3$ or $\frac{7}{2}$ $\left(x = \frac{5}{2}$ or $4\right)$ (or $x = 4$ or $\frac{5}{2}$ $\left(y = \frac{7}{2}$ or $3\right)$)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1,A1 [5]</p>	<p>M1 for eliminating one variable</p> <p>A1 for correct quadratic</p> <p>M1 for correct attempt at solving quadratic</p> <p>A1 for each correct pair</p>
<p>5 (i) $(3+\sqrt{2})^2 + (3-\sqrt{2})^2 = 22$ $AC = \sqrt{22}$</p> <p>(ii) $\tan A = \frac{3-\sqrt{2}}{3+\sqrt{2}}$</p> $\frac{(3-\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{11-6\sqrt{2}}{7}$	<p>M1</p> <p>A1 [2]</p> <p>M1</p> <p>M1, A1 [3]</p>	<p>M1 for use of Pythagoras</p> <p>M1 for correct ratio</p> <p>M1 for rationalising</p>

<p>6 (i) $3x^2 - 10x - 8 = 0$ $(3x + 2)(x - 4) = 0$ critical values $-\frac{2}{3}, 4$ $A = \{x : -\frac{2}{3} \leq x \leq 4\}$</p> <p>(ii) $B = \{x : x \geq 3\}$ $A \cap B = \{x : 3 \leq x \leq 4\}$</p>	<p>M1 A1 A1 [3]</p> <p>B1 B1 [2]</p>	<p>M1 for attempt to solve quadratic A1 for critical values B1 for values of x that define B. B1 for attempt to combine the sets correctly and correct use of notation</p>
<p>7 (i) ${}^{13}C_8 = 1287$</p> <p>(ii) 7 teachers, 1 student : 6 6 teachers, 2 students ${}^7C_6 \times {}^6C_2 : 105$ 5 teachers, 3 students ${}^7C_5 \times {}^6C_3 : 420$ 531</p>	<p>M1, A1 [2]</p> <p>B1 B1 B1 B1 [4]</p>	<p>M1 for correct C notation</p>
<p>8 (i) When $t = 0, N = 1000$</p> <p>(ii) $\frac{dN}{dt} = -1000ke^{-kt}$ when $t = 0, \frac{dN}{dt} = -20$ leading to $k = \frac{1}{50}$</p> <p>(iii) $500 = 1000e^{-kt}$ $t = -50 \ln \frac{1}{2}$ leading to 34.7 mins</p>	<p>B1 [1]</p> <p>M1 B1 A1 [3]</p> <p>M1 M1 A1 [3]</p>	<p>M1 for differentiation B1 for $\frac{dN}{dt} = -20$ stated M1 for attempt to formulate equation using half life M1 for a correct attempt at solution</p>
<p>9 (i) $20 \times -2(1 - 2x)^{19}$</p> <p>(ii) $x^2 \frac{1}{x} + 2x \ln x$</p> <p>(iii) $\frac{x(2 \sec^2(2x+1)) - \tan(2x+1)}{x^2}$</p>	<p>B1, B1 [2]</p> <p>M1 B1 A1 [3]</p> <p>M1 B1 A1 [3]</p>	<p>B1 for 20 and $(1 - 2x)^{19}$ B1 for -2 M1 for attempt to differentiate a product. B1 for $\frac{1}{x}$ M1 for attempt to differentiate a quotient. B1 for differentiation of $\tan(2x+1)$</p>

<p>10 (i) $\frac{dy}{dx} = 9x^2 - 4x + 2$ at P grad = 7 tangent $y - 3 = 7(x - 1)$</p> <p>(ii) at Q, $7x - 4 = 3x^3 - 2x^2 + 2x$ leading to $3x^3 - 2x^2 - 5x + 4 = 0$</p> $(x - 1)(3x^2 + x - 4) - 0$ $(x - 1)(3x + 4)(x - 1) = 0$ leading to $x = -\frac{4}{3}, y = -\frac{40}{3}$	<p>M1 A1 M1 A1 [4]</p> <p>M1 B1,M1 M1 A1 [5]</p>	<p>M1 for differentiation and attempt to find gradient M1 for attempt to find tangent equation, allow unsimplified</p> <p>M1 for equating tangent and curve equations B1 for realising $(x - 1)$ is a factor and attempt to factorise M1 for factorisation and attempt to solve quadratic A1 for both</p>
<p>11 (a) $\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$ $= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$ $= \frac{1}{\cos \theta \sin \theta}$ $= \operatorname{cosec} \theta \sec \theta$</p> <p>(b)(i) $\tan x = 3 \sin x$ $\frac{\sin x}{\cos x} = 3 \sin x$ $\sin x - 3 \sin x \cos x = 0$ leading to $\cos x = \frac{1}{3}, \sin x = 0$ $x = 70.5^\circ, 289.5^\circ$ and $x = 180^\circ$</p> <p>(ii) $2 \cot^2 y + 3 \operatorname{cosec} y = 0$ $2(\operatorname{cosec}^2 y - 1) + 3 \operatorname{cosec} y = 0$ $2 \operatorname{cosec}^2 y + 3 \operatorname{cosec} y - 2 = 0$ $(2 \operatorname{cosec} y - 1)(\operatorname{cosec} y + 2) = 0$ leading to $\sin y = -\frac{1}{2}, y = \frac{7\pi}{6}, \frac{11\pi}{6}$</p>	<p>M1 M1 A1 [3]</p> <p>M1 A1√A1 B1 [4]</p> <p>M1 M1 A1,A1 [5]</p>	<p>M1 for attempt to get in terms of sin and cos and attempt to get one fraction M1 for use of identity</p> <p>M1 for use of $\tan x = \frac{\sin x}{\cos x}$ and correct attempt to solve</p> <p>√A1 on their $x = 70.5^\circ$ B1 for $x = 180^\circ$</p> <p>M1 for use of correct identity M1 for attempt to solve quadratic M1 for dealing with cosec</p>

<p>12 EITHER</p> <p>(i) $\pi r^2 h = 1000$, leading to</p> $h = \frac{1000}{\pi r^2}$ <p>(ii) $A = 2\pi r h + 2\pi r^2$ leading to given answer</p> $A = 2\pi r^2 + \frac{2000}{r}$ <p>(iii) $\frac{dA}{dr} = 4\pi r - \frac{2000}{r^2}$ when $\frac{dA}{dr} = 0$, $4\pi r = \frac{2000}{r^2}$ leading to $r = 5.42$</p> <p>(iv) $\frac{d^2 A}{dr^2} = 4\pi + \frac{4000}{r^3}$</p> <p>+ ve when $r = 5.42$ so min value</p> <p>$A_{\min} = 554$</p>	<p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>M1 for attempt to use volume</p> <p>M1 for attempt to use surface area GIVEN ANSWER</p> <p>M1 for attempt to differentiate and set to 0 DM1 for solution</p> <p>M1 for second derivative method or gradient method'</p> <p>A1 for minimum, can be given if r incorrect but + ve</p>
<p>12 OR (i) $y = x + \cos 2x$</p> $\frac{dy}{dx} = 1 - 2 \sin 2x$ <p>when $\frac{dy}{dx} = 0$, $\sin 2x = \frac{1}{2}$ leading to $x = \frac{\pi}{12}, \frac{5\pi}{12}$</p> <p>(ii) Area = $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} x + \cos 2x dx$</p> $= \left[\frac{x^2}{2} + \frac{1}{2} \sin 2x \right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$ $= \frac{\pi^2}{12}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1, A1</p> <p>[6]</p> <p>M1</p> <p>A1, A1</p> <p>DM1</p> <p>A1</p> <p>[5]</p>	<p>M1 for attempt to differentiate</p> <p>M1 for setting to 0 and attempt to solve M1 for correct order of operations</p> <p>M1 for attempt to integrate</p> <p>A1 for each term correct DM1 for correct use of limits (Trig terms cancel out)</p>