UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the October/November 2010 question paper for the guidance of teachers

0606 ADDITIONAL MATHEMATICS

0606/13 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 $\sec x - \cos x = \dfrac{1}{\cos x} - \cos x$ $= \dfrac{1-\cos^2 x}{\cos x} = \sin x \dfrac{\sin x}{\cos x}$ $= \sin x \tan x$ (Alt: $\dfrac{\sec^2 x - 1}{\sec x} = \dfrac{\tan^2 x}{\sec x} = \dfrac{\sin x}{\cos x}\tan x \cos x$)	M1 M1 A1 [3] M1 M1 A1	M1 for dealing with sec and fractions M1 for use of trig identity M1 for dealing with sec and fractions M1 for use of trig identity
2 **(i)** ${}^7P_4 = 840$ **(ii)** $4 \times {}^6P_3$ or $\dfrac{4}{7} \times 840$ 480	B1, B1 [2] M1 A1 [2]	B1 for ${}^7P_4$ only M1 for a valid method
3 $mx + 2 = x^2 + 12x + 18$ $x^2 + (12 - m)x + 16 = 0$ $(12 - m)^2 = 4 \times 16$ leading to $m = 4, 20$ Alt scheme: $m = 2x + 12$ $(2x + 12)x + 2 = x^2 + 12x + 18$ $x = \pm 4$ so $m = 4, 20$	M1 M1 M1, A1 [4] M1 M1 M1 A1 [4]	M1 for equation in x only, allow unsimplified M1 for use of '$b^2 - 4ac$' M1 for solution of quadratic M1 for equating gradients M1 for elimination of m M1 for x and subsequent calculation for m
4 $f(2) = 8 + 4k - 10 - 3$ $f(-1) = -1 + k + 5 - 3$ $(4k - 5) = 5(k + 1)$ leading to $k = -10$	M1 M1 M1 A1 [4]	M1 for use of $x = 2$ M1 for use of $x = -1$ M1 for attempt to link the two remainders
5 $a = b^2, 2a - b = 3$ $2b^2 - b - 3 = 0$ or $4a^2 - 13a + 9 = 0$ leading to $a = \dfrac{9}{4}$, $b = \dfrac{3}{2}$	B1, B1 M1 A1, A1 [5]	 M1 for solution of equations leading to a quadratic. Final A1 – correct pair only.

6 $x = 2$ or -4 or $-\frac{1}{3}$	B1	B1 for spotting a solution
Either $(x-2)(3x^2+13x+4)$	M1	M1 for attempt to get quadratic factor
or $(x+4)(3x^2-5x-2)$		
or $(3x+1)(x^2+2x-8)$	A1	A1 for correct quadratic factor
$(x-2)(x+4)(3x+1)$	M1, A1	M1 for dealing with quadratic factor A1 for correct factors
$x = 2, -4, -\frac{1}{3}$	A1	A1 for all solutions
	[6]	
7 **(i)** Graph of modulus function	B1 B1 B1 [3]	B1 for shape B1 for 5 marked on y axis B1 for $\frac{5}{3}$ marked on x axis
(ii) Straight line graph	B1 [1]	B1 for straight line with greater gradient
(iii) $8x = \pm(3x-5)$	M1	M1 for attempt to deal with modulus
leading to $x = \frac{5}{11}$ or 0.455 **only**	M1, A1 [3]	M1 for solution
8 **(a)** **(i)** $f_{min} = -10$, occurs when $x = -2$	B1 B1 [2]	
(ii) e.g. $x \geqslant -2$	B1 [1]	Allow any suitable domain that makes f a 1:1 function
(b) **(i)** $x = \left(\frac{y}{2}-1\right)$, leading to	M1	M1 for a valid method of finding the inverse function
$g^{-1}(x) = 2(x+1)$	A1 [2]	
(ii) $\frac{x^2-x}{2} - 1 = 2(x+1)$	M1	M1 for correct order
leading to $x^2 - 5x - 6 = 0$	DM1	DM1 for solution of quadratic
solution $x = 6$ and -1	A1 [3]	

9 **(a)** $\int x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + 9\,\mathrm{d}x = \frac{3}{5}x^{\frac{5}{3}} - \frac{9}{2}x^{\frac{4}{3}} + 9x(+c)$	M1 A2,1,0 [3]	M1 for expansion and attempt to integrate –1 each error
(b) **(i)** $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{x^2+6} + x\left(\frac{2x}{2\sqrt{x^2+6}}\right)$	M1 A2,1,0 [3]	M1 for attempt to differentiate a product. –1 each error
(ii) $\int \frac{x^2+3}{\sqrt{x^2+6}}\,\mathrm{d}x = \frac{1}{2}x\sqrt{x^2+6}$	M1 A1 [2]	M1 for use of their answer to **(i)**
10 **(i)** $t = \sqrt{\mathrm{e}^5 - 1}$ or $t^2 + 1 = \mathrm{e}^5$ $t = 12.1$	B1 B1 [2]	
(ii) distance = ln 10 – ln 5 = ln 2 or 0.693	M1 A1 [2]	M1 for $s_3 - s_2$
(iii) $v = \frac{2t}{t^2+1}$, $v = 0.8$	M1, A1 [2]	M1 for attempt to differentiate
(iv) $a = \frac{(t^2+1)2 - 2t(2t)}{(t^2+1)^2}$ When $t = 2$, $a = -\frac{6}{25}$, or –0.24	M1, A1 A1 [3]	M1 for attempt to differentiate a product or quotient A1 all correct, allow unsimplified
11 **(i)** $\tan x = \frac{4}{3}$, $x = 53.1°, 233.1°$	M1 A1, √A1 [3]	M1 for an equation in tan Follow through on their first answer +180°
(ii) $11 \sin y + 1 = 4(1 - \sin^2 y)$ $(4 \sin y - 1)(\sin y + 3) = 0$ $\sin y = \frac{1}{4}$, $y = 14.5°, 165.5°$	M1 M1 A1,√A1 [4]	M1 for use of correct identity M1 for dealing with quadratic Follow through on their 14.5
(iii) $\cos\left(2z + \frac{\pi}{3}\right) = -\frac{1}{2}$ $2z + \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}$ so $z = \frac{\pi}{6}, \frac{\pi}{2}$	B1 M1 A1, A1 [4]	M1 for correct order of operations

12 EITHER		
(i) $3 = A\sin\frac{\pi}{6} + B\cos\frac{\pi}{4}$, $3 = \frac{1}{2}A + \frac{1}{\sqrt{2}}B$	M1 A1	M1 for attempt at substitution A1 for correct equation
$\frac{dy}{dx} = 2A\cos 2x - 3B\sin 3x$	M1	M1 for attempt to differentiate
$-4 = 2A\cos\frac{2\pi}{3} - 3B\sin\pi$	A1	A1 for all correct
$A = 4$, $B = \sqrt{2}$	A1, A1 [6]	A1 for each
(ii) $A = \int_0^{\frac{\pi}{3}} 4\sin 2x + B\cos 3x\, dx$	M1	M1 for attempt to integrate
$= \left[-2\cos 2x + \frac{B}{3}\sin 3x\right]_0^{\frac{\pi}{3}}$	A2,1,0	–1 each error
$= \left(-2\cos\frac{2\pi}{3} + \frac{B}{3}\sin\pi\right) - (-2)$, $= 3$	DM1,A1 [5]	DM1 for use of limits
12 OR		
(i) $\frac{dy}{dx} = 8x - 6x^2$	M1	M1 for differentiation
Grad at $A = 2$, perp grad $= -\frac{1}{2}$	M1	M1 for use of $m_1m_2 = -1$
At A, $y = 2$	B1	B1 for y coordinate
Equation of normal: $y - 2 = -\frac{1}{2}(x-1)$	DM1	DM1 for finding equation of normal
C (0, 2.5)	A1 [5]	A1 **answer given**
(ii) B (2,0)	B1	B1 for coords of B
$A = \frac{1}{2}(2.5+2)1 + \int_1^2 4x^2 - 2x^3 dx$	M1	M1 for area of trapezium
$= 2.25 + \left[\frac{4x^3}{3} - \frac{x^4}{2}\right]_1^2$	M1 A1 DM1	M1 for attempt to integrate A1 all integration correct DM1 for correct use of limits
$= \frac{49}{12}$ or 4.08	A1 [6]	