CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/01**

Paper 1

October/November 2003

2 hours

Additional Materials: Answer Booklet/Paper
Electronic calculator
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **6** printed pages and **2** blank pages.

MCS UCH190 S47473/1

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \tfrac{1}{2} bc \sin A.$$

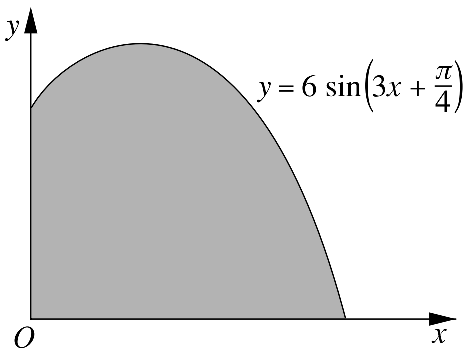
1 Find the values of k for which the line $x + 3y = k$ and the curve $y^2 = 2x + 3$ do not intersect. [4]

2 Without using a calculator, solve the equation $\dfrac{2^{x-3}}{8^{-x}} = \dfrac{32}{4^{\frac{1}{2}x}}$. [4]

3 The expression $x^3 + ax^2 + bx - 3$, where a and b are constants, has a factor of $x - 3$ and leaves a remainder of 15 when divided by $x + 2$. Find the value of a and of b. [5]

4 A rectangular block has a square base. The length of each side of the base is $(\sqrt{3} - \sqrt{2})$ m and the volume of the block is $(4\sqrt{2} - 3\sqrt{3})$ m^3. Find, without using a calculator, the height of the block in the form $(a\sqrt{2} + b\sqrt{3})$ m, where a and b are integers. [5]

5



The diagram shows part of the curve $y = 6\sin\left(3x + \dfrac{\pi}{4}\right)$. Find the area of the shaded region bounded by the curve and the coordinate axes. [6]

6 In this question, **i** is a unit vector due east and **j** is a unit vector due north.

A plane flies from P to Q. The velocity, in still air, of the plane is $(280\mathbf{i} - 40\mathbf{j})$ km h^{-1} and there is a constant wind blowing with velocity $(50\mathbf{i} - 70\mathbf{j})$ km h^{-1}. Find

(i) the bearing of Q from P, [4]

(ii) the time of flight, to the nearest minute, given that the distance PQ is 273 km. [2]

7 A small manufacturing firm produces four types of product, A, B, C and D. Each product requires three processes – assembly, finishing and packaging. The number of minutes required for each type of product for each process and the cost, in \$ per minute, of each process are given in the following table.

	Number of minutes				Cost per minute (\$)
Process \ Type	A	B	C	D	
Assembly	8	6	6	5	0.60
Finishing	5	4	3	2	0.20
Packaging	3	3	2	2	0.50

The firm receives an order for 40 of type A, 50 of type B, 50 of type C and 60 of type D. Write down three matrices such that matrix multiplication will give the total cost of meeting this order. Hence evaluate this total cost. [6]

8 Given that $y = \frac{\ln x}{2x+3}$, find

(i) $\frac{dy}{dx}$, [3]

(ii) the approximate change in y as x increases from 1 to $1 + p$, where p is small, [2]

(iii) the rate of change of x at the instant when $x = 1$, given that y is changing at the rate of 0.12 units per second at this instant. [2]

9 **(a)** Solve, for $0° < x < 360°$, the equation $4\tan^2 x + 8\sec x = 1$. [4]

(b) Given that $y < 4$, find the largest value of y such that $5\tan(2y + 1) = 16$. [4]

10 The function f is given by $\mathrm{f}: x \mapsto 5 - 3e^{\frac{1}{2}x}$, $x \in \mathbb{R}$.

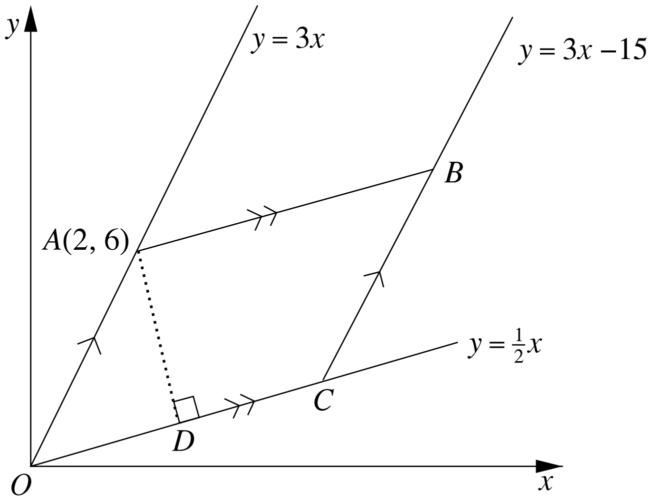
(i) State the range of f. [1]

(ii) Solve the equation $\mathrm{f}(x) = 0$, giving your answer correct to two decimal places. [2]

(iii) Sketch the graph of $y = \mathrm{f}(x)$, showing on your diagram the coordinates of the points of intersection with the axes. [2]

(iv) Find an expression for f^{-1} in terms of x. [3]

11 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a parallelogram *OABC* where *O* is the origin and *A* is the point (2, 6). The equations of *OA*, *OC* and *CB* are $y = 3x$, $y = \frac{1}{2}x$ and $y = 3x - 15$ respectively. The perpendicular from *A* to *OC* meets *OC* at the point *D*. Find

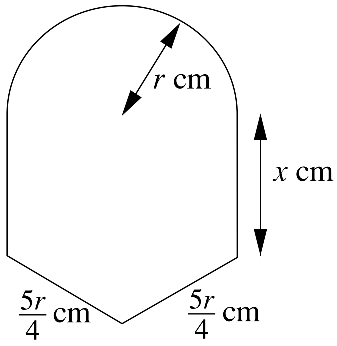
(i) the coordinates of *C*, *B* and *D*, [8]

(ii) the perimeter of the parallelogram *OABC*, correct to 1 decimal place. [3]

[Question 12 is printed on the next page.]

12 Answer only **one** of the following two alternatives.

EITHER



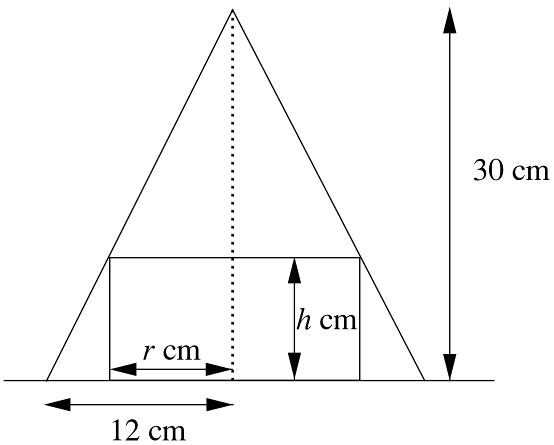
A piece of wire, 125 cm long, is bent to form the shape shown in the diagram. This shape encloses a plane region, of area A cm^2, consisting of a semi-circle of radius r cm, a rectangle of length x cm and an isosceles triangle having two equal sides of length $\frac{5r}{4}$ cm.

(i) Express x in terms of r and hence show that $A = 125r - \frac{\pi r^2}{2} - \frac{7r^2}{4}$. [6]

Given that r can vary,

(ii) calculate, to 1 decimal place, the value of r for which A has a maximum value. [4]

OR



The diagram shows the cross-section of a hollow cone of height 30 cm and base radius 12 cm and a solid cylinder of radius r cm and height h cm. Both stand on a horizontal surface with the cylinder inside the cone. The upper circular edge of the cylinder is in contact with the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by $V = \pi(30r^2 - \frac{5}{2}r^3)$. [4]

Given that r can vary,

(ii) find the volume of the largest cylinder which can stand inside the cone and show that, in this case, the cylinder occupies $\frac{4}{9}$ of the volume of the cone. [6]

[The volume, V, of a cone of height H and radius R is given by $V = \frac{1}{3}\pi R^2 H$.]

BLANK PAGE

BLANK PAGE