UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the May/June 2006 question paper

0606 ADDITIONAL MATHEMATICS

0606/02 Paper 2, maximum raw mark 80

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

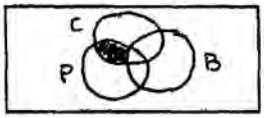
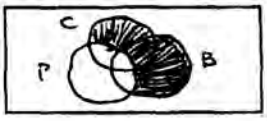
MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 [4]	$dy/dx = (3x-1) \times 1/x + 3 \ln x$ $dy/dt = [\, dy/dx \,]_{x=1} \times dx/dt \Rightarrow dy/dt = 2 \times 3 = 6$ units/s	M1 A1 M1 A1
2 [4]	$\begin{pmatrix} 5 & 3 & 0 \\ 4 & 1 & 2 \\ 4 & 0 & 4 \\ 2 & 1 & 4 \\ 1 & 1 & 6 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ or $\left(\begin{array}{cc\|ccc} 3 & 1 & 0 & & \\ & & & & \end{array}\right) \begin{pmatrix} 5 & 4 & 4 & 2 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 0 & 2 & 4 & 4 & 6 \end{pmatrix}$ Column $\Leftarrow$ 18 13 12 7 4 $\Rightarrow$ Row	B1 B1 M1 A1
3 [5]	(i) $(0.28)^2 + p^2 = 1 \Rightarrow p = 0.96$ (ii) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (12-q)\mathbf{i} + 24\mathbf{j}$ $\frac{24}{0.96} = \frac{12-q}{0.28} \Rightarrow q = 5$	M1 A1 B1 M1 A1
4 [6]	(a) $d(e^{\tan x})/dx = e^{\tan x} \times \sec^2 x$ (b) $\int e^{1-2x} dx = e^{1-2x} \times -\frac{1}{2}$ $\left[\ \right]_0^{1/2} = -\frac{1}{2} - (-\frac{1}{2}e) = \frac{1}{2}(e-1)$ [≈ 0.859]	M1 A1 M1 A1 DM1 A1
5 [6]	(i) $1 + \sqrt{15} = \frac{1}{2}(\sqrt{5} + \sqrt{3}) \times AB$ $AB = \frac{2(1+\sqrt{15})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = 4\sqrt{3} - 2\sqrt{5}$ (ii) $(\sqrt{5}+\sqrt{3})^2 = 5+3+2\sqrt{15}$ or $(4\sqrt{3}-2\sqrt{5})^2 = 48+20-16\sqrt{15}$ $BC^2 = (\sqrt{5}+\sqrt{3})^2 + (4\sqrt{3}-2\sqrt{5})^2 = 76 - 14\sqrt{15}$	M1 M1 A1 B1 M1 A1
6 [6]	(a) (i)  (ii)  (b) (i) Max $n(F \cap S) = 10$ Min $n(F \cap S) = 16 + 10 - 20 = 6$ (ii) Max $n(F \cup S) = 20$ Min $n(F \cup S) = 16$	B1 B1 B1 B1 B1 B1

7 [7]	(a) Positions for A,B X . . . X . . or . X . . . X . or . . X . . . X ⇒ 3	B1
	Arrangements of A,B ⇒ 2	B1
	Total number of arrangements = $3 \times 2 \times 5! = 720$	M1 A1
	(b) 3 different ⇒ ${}^6C_3 = 20$	B1
	2 of 1 kind + 1 ⇒ $6 \times 5 = 30$ Total 50	B1 DB1
8 [7]	(i) ${}^8C_r x^r (k/x^3)^{8-r}$ ⇒ $r - 3(8-r) = 0$ ⇒ $r = 6$	M1 A1
	${}^8C_6 k^2 = 252$ ⇒ $k^2 = 9$ ⇒ $k = 3$	M1 A1
	(ii) $(x + 3/x^3)^8$ ⇒ $\ldots + 8x^7(3/x^3) + \ldots$	B1
	$(1 - x^4/4)(x^8 + 24x^4 + 252 + \ldots)$ ⇒ Coefficient of $x^4 = 24 - 63 = -39$	M1 A1√
9 [7]	(i) Area $= 2(2x^2 + 2xh + xh) = 120$ ⇒ $h = (60 - 2x^2)/3x$	M1 A1
	(ii) $V = 2x^2h = 40x - 4x^3/3$	A1
	(iii) $dV/dx = 40 - 4x^2 = 0$ ⇒ $x = \sqrt{10}$	M1 A1
	$h = (60 - 20)/3\sqrt{10} = 4x/3$	M1 A1
10 [9]	(a) $a^2 + b^2 = \sec^2x + 2\sec x \operatorname{cosec} x + \operatorname{cosec}^2x + \sec^2x - 2\sec x \operatorname{cosec} x + \operatorname{cosec}^2x$	
	$= 2\sec^2x + 2\operatorname{cosec}^2x$ $= 2/\cos^2x + 2/\sin^2x$	B1 B1
	[Or $\sec x = 1/\cos x$, $\operatorname{cosec} x = 1/\sin x$ B1 ⇒ $2/\cos^2x + 2/\sin^2x$ B1]	
	$2/\cos^2x + 2/\sin^2x = (2\sin^2x + 2\cos^2x)/\cos^2x \sin^2x$	
	Use $\cos^2x + \sin^2x = 1$ ⇒ $2/\cos^2x \sin^2x = 2\sec^2x \operatorname{cosec}^2x$	M1 A1
	(b) $\cot y = \cos y / \sin y$	B1
	$2\cos y = 3\sin^2y$ Use $\sin^2y = 1 - \cos^2y$ ⇒ $3\cos^2y + 2\cos y - 3 = 0$	M1
	$\cos y = (-2 \pm \sqrt{40})/6$ ⇒ $y = 0.77$ or 5.52	M1 A1 A1

11 [9]	(i) [BOD is diameter.] Realises most, or all, of $\angle DOA = \angle OAC = \angle ACO = \angle COB$	M1
	All these angles $= \pi - 2.5$ or $2.5 - \angle AOC$ [or $\angle COB = x$, $\angle AOC = 2.5 - x$]	
	$\angle$s of $\Delta = \pi$ or $\angle$s on $BOD = \pi$ or $\angle DOA = \pi - 2.5 = 2.5 - \angle AOC = \angle COB$	M1
	to find $\angle AOC$ directly [or find $x = \pi - 2.5$ & hence $\angle AOC$] $= 5 - \pi$ (≈ 1.86)	A1
	(ii) Sector $= \frac{1}{2} \times 12^2 \times (5 - \pi)$ ($\approx 133.8 \sim .9$)	M1
	Sector $-\Delta$(any method) $= (\ ) - \frac{1}{2} \times 12^2 \times \sin(5 - \pi) \approx 64.7 \sim .9$	M1 A1
	(iii) Length AC (any method) [$2 \times 12 \times \sin\{\frac{1}{2}(5 - \pi)\} \approx 19.2$]	M1
	Length AC + length of arc $= (\ ) + 12(5 - \pi) \approx 41.5 \sim .6$	M1 A1
12 EITHER [10]	(i) $a = 2, b = -2$ $c = -5$	B1 B1
	(ii) $x = 2$ $y = -5$ [if calculus used M1 and A1 for $(2, -5)$]	B1√ B1√
	(iii) $[f^2(x) = 8x^4 - 64x^3 + 136x^2 - 32x - 3]$ $f^2(0) = f\{f(0)\} = f(3) = -3$	M1 A1
	(iv) 2	B1
	(v) $x = 2(y-2)^2 - 5 \Rightarrow y = 2 \pm \sqrt{\frac{x+5}{2}}$	M1 A1
	$g^{-1}: x \mapsto 2 - \sqrt{\frac{x+5}{2}}$ only	A1
OR [10]	(i) $10 - x^2 + 6x \geq 15 \Rightarrow x^2 - 6x + 5 \leq 0$	
	$(x-5)(x-1) = 0 \Rightarrow 1, 5$ $1 \leq x \leq 5$	M1A1 A1
	(ii) Attempt to incorporate $(x \pm 3)^2$ $\Rightarrow a = 19, b = -3$	M1 A1
	(iii) $x = 3$ $y = 19$ [if calculus used M1 and A1 for $(3, 19)$]	B1√ B1√
	(iv) $gf(x) = 2(10 - x^2 + 6x) - k$ $[2x^2 - 12x + (k - 20) = 0]$	B1
	$12^2 = 8(k - 20)$ or $2(x-3)^2 - 18 + k - 20 = 0$ $k = 38$	M1 A1